Preface

Logica Universalis (or *Universal Logic*, *Logique Universelle*, *Universelle Logik*, in vernacular languages) is not a new logic, but a general theory of logics, considered as mathematical structures. The name was introduced about ten years ago, but the subject is as old as the beginning of modern logic: Alfred Tarski and other Polish logicians such as Adolf Lindenbaum developed a general theory of logics at the end of the 1920s based on consequence operations and logical matrices. Talking about the papers of Tarski dealing with this topic, John Etchemendy says: "What is most striking about these early papers, especially against their historical backdrop, is the extraordinary generality and abstractness of the perspective adopted" [4]. After the second world war, this line of work was pursued mainly in Poland and became a bit of an esoteric subject. Jerzy Łoś's fundamental monograph on logical matrices was never translated in English and the work of Roman Suszko on abstract logics remained unknown outside of Poland during many years.

Things started to change during the 1980s. Logic, which had been dominated during many years by some problems related to the foundations of mathematics or other metaphysical questions, was back to reality. Under the impulsion of artificial intelligence, computer science and cognitive sciences, new logical systems were created to give an account to the variety of reasonings of everyday life and to build machines, robots, programs that can act efficiently in difficult situations, for example that can smoothly process inconsistent and incomplete information. John McCarthy launched non-monotonic logic, few years later Jean-Yves Girard gave birth to linear logic. Logics were proliferating: each day a new logic was born. By the mid eighties, there were more logics on earth than atoms in the universe. People began to develop general tools for a systematic study of this huge amount of logics, trying to put some order in this chaotic multiplicity. Old tools such as consequence operations, logical matrices, sequent calculus, Kripke structures, were revived and reshaped to meet this new goal. For example sequent calculus was the unifying instrument for substructural logics. New powerful tools were also activated, such as labelled deductive systems by Dov Gabbay.

Amazingly, many different people in many different places around the world, quite independently, started to work in this new perspective of a general theory of logics, writing different monographs, each one presenting his own way to treat the problem: Norman Martin's emphasis was on Hilbert systems [9], Richard Epstein's,

This research was supported by a grant of the Swiss National Science Foundation.

on semantical tools, in particular relational structures and logical matrices [5], Newton da Costa's, on non truth-functional bivalent semantics [7], John Cleave's, on consequence and algebra [3], Arnold Koslow's, on Hertz's abstract deductive systems [8]. This was also the time when was published a monograph by Ryszard Wójcicki on consequence operations making available for the first time to a wide public the main concepts and results of Polish logic [10], and the time when Dov Gabbay edited a book entitled *What is a logical system?* gathering a collection of papers trying to answer this question in many different ways [6]. Through all these publications, the generality and abstractness of Tarski's early work was being recovered. It is surrounded by this atmosphere that I was doing my PhD [2] and that I coined in the middle of a winter in Poland the expression "universal logic" [1], by analogy to the expression "universal algebra".

The present book contains recent works on universal logic by first-class researchers from all around the world. The book is full of new and challenging ideas that will guide the future of this exciting subject. It will be of interest for people who want to better understand what logic is. It will help those who are lost in the jungle of heterogeneous logical systems to find a way. Tools and concepts are provided here for those who want to study classes of already existing logics or want to design and build new ones.

In Part I, different frameworks for a general theory of logics are presented. Algebra, topology, category theory are involved. The first paper, written by myself, is a historical overview of the different logical structures and methods which were proposed during the XXth century: Tarski's consequence operator and its variants in particular Suszko's abstract logic, structures arising from Hertz and Gentzen's deductive systems, da Costa's theory of valuation, etc. This survey paper presents and explains many concepts that are used in other papers of the book. The following paper, by Marta García-Matos and Jouko Väänänen, gives a hint of how *abstract model theory* can be used for developing universal logic. Although abstract logic and abstract model theory are expressions which look similar, they refer to two different traditions. Abstract logic has been developed by Suszko in the context of the Polish tradition focusing on a general theory of zero-order logics (i.e. propositional logics). On the other hand, the aim of abstract model theory has been the study of classes of higher order logics. The combination of abstract model theory with abstract logic is surely an important step towards the development of universal logic. It is also something more than natural if we think that both theories have their origins in the work of Alfred Tarksi. Steffen Lewitzka's approach is also model-theoretical, but based on topology. He defines in a topological way logic-homomorphims between abstract logics, which are mappings that preserve structural properties of logics. And he shows that those model-theoretical abstract logics together with a strong form of logic-homomorphisms give rise to the notion of institution. Then comes the work of Ramon Jansana which is a typical example of what is nowadays called *abstract algebraic logic*, the study of algebraization of logics, a speciality of the Barcelona logic group. Within this framework, abstract logics are considered as generalized matrices and are used as models for logics. Finally, Pierre Ageron's paper deals with logics for which the law of *self-deductibility* does not hold. According to this law, a formula is always a consequence of itself, it was one of the basic axioms of Tarski's consequence operator. Ageron shows here how to develop logical structures without this law using tools from category theory.

The papers of Part II deal with a central problem of universal logic: the question of identity between logical structures. A logic, like classical logic, is not a given structure, but a class of structures that can be identified with the help of a given criterion. According to this criterion, we say that structures of a given class are equivalent, congruent or simply identical. Although this question may at first look trivial, it is in fact a very difficult question which is strongly connected to the question of what a logical structure is. In other words, it is not possible to try to explain how to identify different logical structures without investigating at the same time the very nature of logical structures. This is what makes the subject deep and fascinating. Three papers and seven authors are tackling here the problem, using different strategies. Caleiro and Gonçalves's work is based on concepts from category theory and they say that two logics are the same, *equipollent* in their terminology, when there exist uniform translations between the two logical languages that induce an isomorphism on the corresponding theory spaces. They gave several significative illustrations of equipollent and non equipollent logics. Mossakowski, Goguen, Diaconescu and Tarlecki also use category theory, more specifically their work is based on the notion of *institution*. They argue that every plausible notion of equivalence of logics can be formalized using this notion. Lutz Straßburger's paper is proof-theoretically oriented, he defines identity of proofs via *proof nets* and identity of logics via pre-orders.

In part III, different tools and concepts are presented that can be useful for the study of logics. The papers by Arnon Avron and by Carlos Caleiro et al. both deal with a concept very popular in the Polish tradition, the concept of logical matrices, the basic tool for many-valued logics. In his paper Avron studies the notion of *non-deterministic matrices* which allows to easily construct semantics for proof systems and can be used to prove decidability. This tool can be applied to a wide range of logics, in particular to logics with a formal consistency operator. Caleiro, Carnielli, Coniglio and Marcos discuss Suszko's thesis, according to which any logic is bivalent, and present some techniques which permit to construct in a effective way a *bivalent semantics*, generally not truth-functional, from a many-valued matrix. Their paper is illustrated by some interesting examples, including Belnap's four-valued logic. Then comes a paper by David Makinson, one of the main responsible for the revival of Tarski's consequence operator at the beginning of the 1980s. He used it at the main tool, on the one hand for the development together with Carlos Alchourrón and Peter Gärdenfors, of theory change (universally known today under the acronym AGM), on the other hand as a basis for a general theory of non monotonic logics. In both cases, Makinson's use of Tarski's theory was creative, he kept the original elegant abstract spirit, but widened and extended the basic underlying concepts. Here again he is innovative defining within classical propositional logic two new concepts, *logical friendliness and sympathy*, which lead to some consequence relations with non standard properties. The paper by Lloyd Humberstone is no less original and brilliant, he studies the very interesting phenomenon of *logical discrimination*. The question he examines is in which circumstances, discrimination, i.e. distinction between formulas, is correlated with the strength of a logic. The work of Humberstone is a very good example of the philosophical import of universal logic. By a careful examination of a phenomenon like discrimination, that requires a precise mathematical framework, one can see to which extent a statement with philosophical flavor saying that discrimination is inversely proportional to strength is true or not.

References

[1] J.-Y. Beziau, *Universal Logic*. Proceedings of the 8th International Colloquium - Logica'94, T. Childers and O. Majer (eds), Czech Academy of Sciences, Prague, 2004, pp.73-93.

[2] J.-Y. Beziau, *Recherches sur la Logique Universelle*. PhD Thesis, Department of Mathematics, University Denis Diderot, Paris, 1995.

[3] J.P. Cleave, *A study of logics*. Clarendon, Oxford, 1991.

[4] J.Etchemendy, *Tarski on truth and logical consequence*. The Journal of Symbolic Logic, **58** (1988), 51-79.

[5] R.L. Epstein, *The semantic foundations of logic*. Kluwer, Dordrecht, 1990.

[6] D. Gabbay (ed), *What is a logical system?* Clarendon, Oxford, 1994.

[7] N. Grana, *Sulla teoria delle valutazioni di N.C.A. da Costa*. Liguori, Naples, 1990.

[8] A. Koslow, *A structuralist theory of logic*. Cambridge University Press, New York, 1992.

[9] N.M. Martin, *Systems of logic*. Cambridge University Press, New York, 1989.

[10] R. Wójcicki, *Theory of logical calculi – Basic theory of consequence operations*. Kluwer, Dordrecht, 1988.

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January 31th, 2005.

Preface to the Second Edition

The first edition of this book was a success and was sold out in a short time. In this second edition you will find the same authors and table of contents, but most of the papers have been extended and improved.

Universal logic is making its way. A new journal entitled *Logica Universalis* has been created and soon a book series dedicated to the subject will also be launched by Birkhäuser: *Studies in Universal Logic*.

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February, 2007.

http://www.springer.com/978-3-7643-8353-4

Logica Universalis
Towards a General Theory of Logic
Beziau, J.-Y. (Ed.)
2007, XII, 246 p., Softcover
ISBN: 978-3-7643-8353-4
A product of Birkhäuser Basel