Preface

This book provides a picture of what can be done in differential equations with advanced methods and software tools of symbolic computation. It focuses on the symbolic-computational aspect of three kinds of fundamental problems in differential equations: transforming the equations, solving the equations, and studying the structure and properties of their solutions. Modern research on these problems using symbolic computation, or more restrictively using computer algebra, has become increasingly active since the early 1980s when effective algorithms for symbolic solution of differential equations were proposed, and so were computer algebra systems successfully applied to perturbation, bifurcation, and other problems. Historically, symbolic integration, the simplest case of solving ordinary differential equations, was already the target of the first computer algebra package SAINT in the early 1960s.

With 20 chapters, the book is structured into three parts with both tutorial surveys and original research contributions: the first part is devoted to the qualitative study of differential systems with symbolic computation, including stability analysis, establishment of center conditions, and bifurcation of limit cycles, which are closely related to Hilbert's sixteenth problem. The second part is concerned with symbolic solutions of ordinary and partial differential equations, for which normal form methods, reduction and factorization techniques, and the computation of conservation laws are introduced and used to aid the search. The last part is concentrated on the transformation of differential equations into such forms that are better suited for further study and application. It includes symbolic elimination and triangular decomposition for systems of ordinary and partial differential polynomials. A 1991 paper by Wen-tsün Wu on the construction of Gröbner bases based on Riquier–Janet's theory, published in China and not widely available to the western readers, is reprinted as the last chapter. This book should reflect the current state of the art of research and development in differential equations with symbolic computation and is worth reading for researchers and students working on this interdisciplinary subject of mathematics and computational science. It may also serve as a reference for everyone interested in differential equations, symbolic computation, and their interaction.

The idea of compiling this volume grew out of the Seminar on Differential Equations with Symbolic Computation (DESC 2004), which was held in Beijing, China in April 2004 (see http://www-calfor.lip6.fr/~wang/DESC2004) to facilitate the interaction between the two disciplines. The seminar brought together active researchers and graduate students from both disciplines to present their work and to report on their new results and findings. It also provided a forum for over 50 participants to exchange ideas and views and to discuss future development and cooperation. Four invited talks were given by Michael Singer, Lan Wen, Wen-tsün Wu, and Zhifen Zhang. The enthusiastic support of the seminar speakers and the

high quality of their presentations are some of the primary motivations for our endeavor to prepare a coherent and comprehensive volume with most recent advances on the subject for publication. In addition to the seminar speakers, several distinguished researchers who were invited to attend the seminar but could not make their trip have also contributed to the present book. Their contributions have helped enrich the contents of the book and make the book beyond a proceedings volume. All the papers accepted for publication in the book underwent a formal review-revision process.

DESC 2004 is the second in a series of seminars, organized in China, on various subjects interacted with symbolic computation. The first seminar, held in Hefei from April 24–26, 2002, was focused on geometric computation and a book on the same subject has been published by World Scientific. The third seminar planned for April 2006 will be on symbolic computation in education.

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