

# Preface

The aim of this book is to communicate some results on solving linear differential equations that have been achieved in the last two decades. The key concept is the factorization of a differential equation or the corresponding differential operator, and the resulting decomposition into unique objects of lower order. Although more than 100 years old, these results had been forgotten for almost a century before they were reawakened.

Several new developments have entailed novel interest in this subject. On the one hand, methods of differential algebra lead to a better understanding of the basic problems involved. Instead of dealing with individual equations, the corresponding differential operators are considered as elements of a suitable ring where they generate an ideal. This proceeding is absolutely necessary if partial differential equations and operators are investigated. In particular the concept of a Janet basis for the generators of an ideal and the Loewy decomposition of the ideal corresponding to the given equations are of fundamental importance. In order to apply these results for solving concrete problems, the availability of computer algebra software is indispensable due the enormous size of the calculations usually involved.

Proceeding along these lines, for large classes of linear differential equations – ordinary as well as partial – a fairly complete theory for obtaining its solutions in closed form has been achieved. Whenever feasible, constructive methods for algorithm design are given, and the possible limits of decidability are indicated. This proceeding may serve as a model for dealing with other problems in the area of differential equations.

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