

# Contents

<b>1</b>	<b>Introduction</b> .....	1
1.1	Selection Sort and Quicksort .....	1
1.2	Recurrence Equations .....	5
1.3	Symbolic Sums .....	7
1.4	Generating Functions .....	8
1.5	Asymptotic Estimates .....	12
1.6	The Concrete Tetrahedron .....	13
1.7	Problems .....	15
<b>2</b>	<b>Formal Power Series</b> .....	17
2.1	Basic Facts and Definitions .....	17
2.2	Differentiation and Division .....	19
2.3	Sequences of Power Series .....	24
2.4	The Transfer Principle .....	29
2.5	Multivariate Power Series .....	31
2.6	Truncated Power Series .....	35
2.7	Problems .....	38
<b>3</b>	<b>Polynomials</b> .....	43
3.1	Polynomials as Power Series .....	43
3.2	Polynomials as Sequences .....	46
3.3	The Tetrahedron for Polynomials .....	48
3.4	Polynomials as Solutions .....	50
3.5	Polynomials as Coefficients .....	53
3.6	Applications .....	55
3.7	Problems .....	58

<b>4</b>	<b>C-Finite Sequences</b> .....	63
4.1	Fibonacci Numbers .....	63
4.2	Recurrences with Constant Coefficients .....	66
4.3	Closure Properties .....	70
4.4	The Tetrahedron for C-finite Sequences .....	74
4.5	Systems of C-finite Recurrences .....	78
4.6	Applications .....	80
4.7	Problems .....	84
<b>5</b>	<b>Hypergeometric Series</b> .....	87
5.1	The Binomial Theorem .....	87
5.2	Basic Facts and Definitions .....	90
5.3	The Tetrahedron for Hypergeometric Sequences .....	94
5.4	Indefinite Summation .....	98
5.5	Definite Summation .....	103
5.6	Applications .....	107
5.7	Problems .....	110
<b>6</b>	<b>Algebraic Functions</b> .....	113
6.1	Catalan Numbers .....	113
6.2	Basic Facts and Definitions .....	116
6.3	Puiseux Series and the Newton Polygon .....	119
6.4	Closure Properties .....	123
6.5	The Tetrahedron for Algebraic Functions .....	125
6.6	Applications .....	130
6.7	Problems .....	133
<b>7</b>	<b>Holonomic Sequences and Power Series</b> .....	137
7.1	Harmonic Numbers .....	137
7.2	Equations with Polynomial Coefficients .....	139
7.3	Generalized Series Solutions .....	144
7.4	Closed Form Solutions .....	148
7.5	The Tetrahedron for Holonomic Functions .....	152
7.6	Applications .....	157
7.7	Problems .....	160

**Appendix** ..... 165

    A.1 Basic Notions and Notations ..... 165

    A.2 Basic Facts from Computer Algebra ..... 167

    A.3 A Collection of Formal Power Series Identities ..... 168

    A.4 Closure Properties at One Glance ..... 169

    A.5 Software ..... 171

    A.6 Solutions to Selected Problems ..... 174

    A.7 Bibliographic Remarks ..... 189

**References** ..... 193

**Subject Index** ..... 197



<http://www.springer.com/978-3-7091-0444-6>

The Concrete Tetrahedron  
Symbolic Sums, Recurrence Equations, Generating  
Functions, Asymptotic Estimates

Kauers, M.; Paule, P.

2011, IX, 203 p., Softcover

ISBN: 978-3-7091-0444-6