Contents

1 Mechanics 1
1.1 Point Mechanics 1
1.1.1 Basic Concepts of Mechanics and Kinematics 1
1.1.2 Newton's Law of Motion 3
1.1.3 Simple Applications of Newton's Law 6
1.1.4 Harmonic Oscillator in One Dimension 13
1.2 Mechanics of Point Mass Systems 17
1.2.1 The Ten Laws of Conservation 17
1.2.2 The Two-Body Problem 19
1.2.3 Constraining Forces and d'Alembert's Principle 20
1.3 Analytical Mechanics 24
1.3.1 The Lagrange Function 24
1.3.2 The Hamilton Function 26
1.3.3 Harmonic Approximation for Small Oscillations 28
1.4 Mechanics of Rigid Bodies 33
1.4.1 Kinematics and Inertia Tensor 33
1.4.2 Equations of Motion 36
1.5 Continuum Mechanics 42
1.5.1 Basic Concepts 42
1.5.2 Stress, Strain and Hooke's Law 47
1.5.3 Waves in Isotropic Continua 49
1.5.4 Hydrodynamics 50

2 Electricity and Magnetism 61
2.1 Vacuum Electrodynamics 61
2.1.1 Steady Electric and Magnetic Fields 61
2.1.2 Maxwell's Equations and Vector Potential 66
2.1.3 Energy Density of the Field 68
2.1.4 Electromagnetic Waves 68
2.1.5 Fourier Transformation 69
2.1.6 Inhomogeneous Wave Equation 70

2.1.7 Applications . . . 71
2.2 Electrodynamics in Matter . . . 76
2.2.1 Maxwell's Equations in Matter . . . 76
2.2.2 Properties of Matter . . . 77
2.2.3 Wave Equation in Matter . . . 79
2.2.4 Electrostatics at Surfaces . . . 80
2.3 Theory of Relativity . . . 83
2.3.1 Lorentz Transformation . . . 84
2.3.2 Relativistic Electrodynamics . . . 87
2.3.3 Energy, Mass and Momentum . . . 89

3 Quantum Mechanics . . . 93
3.1 Basic Concepts . . . 93
3.1.1 Introduction . . . 93
3.1.2 Mathematical Foundations . . . 94
3.1.3 Basic Axioms of Quantum Theory . . . 96
3.1.4 Operators . . . 98
3.1.5 Heisenberg's Uncertainty Principle . . . 100
3.2 Schrödinger's Equation . . . 101
3.2.1 The Basic Equation . . . 101
3.2.2 Penetration . . . 102
3.2.3 Tunnel Effect . . . 104
3.2.4 Quasi-classical WKB Approximation . . . 105
3.2.5 Free and Bound States in the Potential Well . . . 106
3.2.6 Harmonic Oscillators . . . 107
3.3 Angular Momentum and the Structure of the Atom . . . 110
3.3.1 Angular Momentum Operator . . . 110
3.3.2 Eigenfunctions of L^2 and L_z . . . 111
3.3.3 Hydrogen Atom . . . 112
3.3.4 Atomic Structure and the Periodic System . . . 115
3.3.5 Indistinguishability . . . 116
3.3.6 Exchange Reactions and Homopolar Binding . . . 118
3.4 Perturbation Theory and Scattering . . . 120
3.4.1 Steady Perturbation Theory . . . 120
3.4.2 Unsteady Perturbation Theory . . . 122
3.4.3 Scattering and Born's First Approximation . . . 124

4 Statistical Physics . . . 129
4.1 Probability and Entropy . . . 129
4.1.1 Canonical Distribution . . . 129
4.1.2 Entropy, Axioms and Free Energy . . . 132
4.2 Thermodynamics of the Equilibrium . . . 135
4.2.1 Energy and Other Thermodynamic Potentials . . . 135
4.2.2 Thermodynamic Relations . . . 138

4.2.3 Alternatives to the Canonical Probability Distribution 140
4.2.4 Efficiency and the Carnot Cycle 141
4.2.5 Phase Equilibrium and the Clausius-Clapeyron Equation 143
4.2.6 Mass Action Law for Gases 146
4.2.7 The Laws of Henry, Raoult and van't Hoff 147
4.2.8 Joule-Thomson Effect 149
4.3 Statistical Mechanics of Ideal and Real Systems 150
4.3.1 Fermi and Bose Distributions 150
4.3.2 Classical Limiting Case $\beta\mu \to -\infty$ 152
4.3.3 Classical Equidistribution Law 155
4.3.4 Ideal Fermi-Gas at Low Temperatures $\beta\mu \to +\infty$ 156
4.3.5 Ideal Bose-Gas at Low Temperatures $\beta\mu \to 0$ 157
4.3.6 Vibrations 160
4.3.7 Virial Expansion of Real Gases 161
4.3.8 Van der Waals' Equation 162
4.3.9 Magnetism of Localised Spins 164
4.3.10 Scaling Theory 169

5 Fractals in Theoretical Physics 189
5.1 Non-random Fractals 190
5.2 Random Fractals: The Unbiased Random Walk 192
5.3 'A Single Length' 194
5.3.1 The Concept of a Characteristic Length 194
5.3.2 Higher Dimensions 195
5.3.3 Additional Lengths that Scale with $\sqrt{t}$ 195
5.4 Functional Equations and Scaling: One Variable 196
5.5 Fractal Dimension of the Unbiased Random Walk 197
5.6 Universality Classes and Active Parameters 197
5.6.1 Biased Random Walk 197
5.6.2 Scaling of the Characteristic Length 198
5.7 Functional Equations and Scaling: Two Variables 200
5.8 Fractals and the Critical Dimension 201
5.9 Fractal Aggregates 207
5.10 Fractals in Nature 210

6 Dynamical Systems and Chaos 215
6.1 Basic Notions and Framework 215
6.1.1 Phase Space 215
6.1.2 Continuous-Time Dynamical Systems 216
6.1.3 Flows and Phase Portraits 217
6.1.4 Insights from Dynamical Systems Theory 217
6.1.5 Some Examples 218

6.2 Fixed Points and Linear Stability Analysis . . . 221
6.2.1 Fixed Points and Stability Matrix . . . 221
6.2.2 Flow Linearization Theorem . . . 222
6.2.3 The Different Types of Fixed Points . . . 223
6.2.4 Constructing the Phase Portrait . . . 225
6.2.5 Application: Anharmonic Oscillators . . . 227
6.2.6 The Origin of Bifurcations . . . 230
6.3 Attractors, Bifurcations and Normal Forms . . . 231
6.3.1 Attractors . . . 231
6.3.2 Conservative *Versus* Dissipative Systems . . . 232
6.3.3 The Different Types of Bifurcations . . . 233
6.3.4 Normal Forms and Structural Stability . . . 235
6.4 Discrete-Time Dynamical Systems . . . 235
6.4.1 Discrete-Time Evolution Equations . . . 235
6.4.2 Linear Stability Analysis . . . 236
6.4.3 Attractors and Bifurcations . . . 237
6.4.4 Discretization by Poincaré Sections . . . 237
6.5 Lyapunov Exponents and Deterministic Chaos . . . 239
6.5.1 Lyapunov Exponents . . . 239
6.5.2 Deterministic Chaos . . . 240
6.5.3 Ergodic Theory . . . 242
6.6 Routes to Chaos . . . 243
6.6.1 Period Doubling and Subharmonic Cascade . . . 243
6.6.2 Intermittency . . . 245
6.6.3 Ruelle-Takens Scenario . . . 246
6.6.4 Hamiltonian Systems and KAM Theorem . . . 246
6.7 Conclusion . . . 248
6.8 Problems . . . 249
6.9 Further Reading . . . 250

Appendix A: Elementary Particles . . . 253

Appendix B: Answers to Questions . . . 263

Index . . . 267

http://www.springer.com/978-3-662-53683-4

From Newton to Mandelbrot
A Primer in Theoretical Physics
Stauffer, D.; Stanley, H.E.; LESNE, A.
2017, XIV, 270 p. 84 illus., Hardcover
ISBN: 978-3-662-53683-4