Chapter 2

Coordinate and Time Systems

GPS satellites orbit the earth over time. GPS surveys are conducted mostly on land. To describe the GPS observation (distance) as a function of GPS orbit (satellite position) and the measuring position (station location), suitable coordinate and time systems must be defined.

2.1 Geocentric Earth-Fixed Coordinate Systems

The Earth-Centred Earth-Fixed (ECEF) coordinate system is useful for describing the location of a station on the earth's surface. This system is a right-handed Cartesian system (x, y, z). Its origin and the earth's centre of mass coincide, while its z-axis and the mean rotational axis of the earth coincide; the x-axis points to the mean Greenwich meridian, while the y-axis is directed to complete a right-handed system (cf. Fig. 2.1). In other words, the z-axis points to a mean pole of the earth's rotation. The mean pole, defined by international convention, is called the Conventional International Origin (CIO). Then the xy-plane is called mean equatorial plane, and the xz-plane is called mean zero-meridian.

The ECEF coordinate system is also known as the Conventional Terrestrial System (CTS). The mean rotational axis and mean zero-meridian used here are necessary. The true rotational axis of the earth changes its direction with respect to the earth's body all the time. If such a pole were used to define a coordinate system, the coordinates of the station would also be continuously changing. Because the

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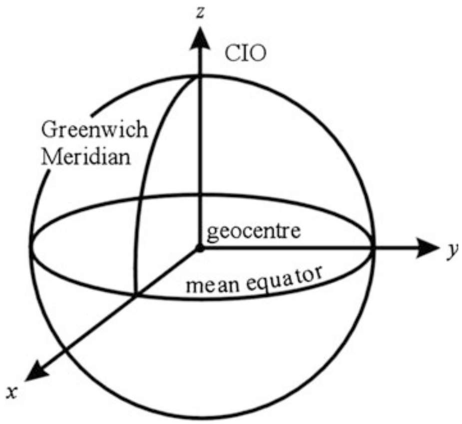


Fig. 2.1 Earth-centred earth-fixed coordinates

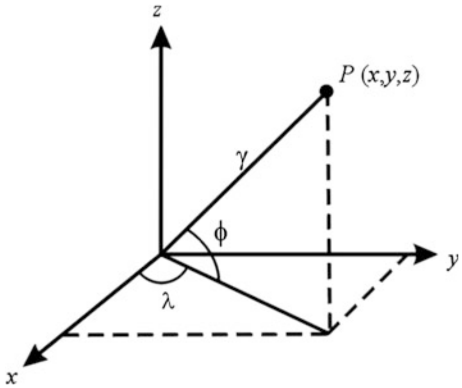


Fig. 2.2 Cartesian and spherical coordinates

survey is made in our true world, the polar motion obviously must be taken into account, and this will be discussed later.

Of course, the ECEF coordinate system can be represented by a spherical coordinate system (r, ϕ, λ), where r is the radius of the point (x, y, z), and ϕ and λ are the geocentric latitude and longitude, respectively (cf. Fig. 2.2). λ is counted eastward from the zero-meridian. The relationship between (x, y, z) and (r, ϕ, λ) is obvious:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\cos\phi\,\cos\lambda \\ r\cos\phi\,\sin\lambda \\ r\,\sin\phi \end{pmatrix}, \quad \text{or} \quad \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \tan\lambda = y/x \\ \tan\phi = z/\sqrt{x^2+y^2} \end{cases}. \tag{2.1}$$

An ellipsoidal coordinate system (φ, λ, h) may also be defined based on the ECEF coordinates; however, geometrically, two additional parameters are needed to define the shape of the ellipsoid (cf. Fig. 2.3). φ, λ, and h are geodetic latitude, longitude, and height, respectively. The ellipsoidal surface is a rotational ellipse.

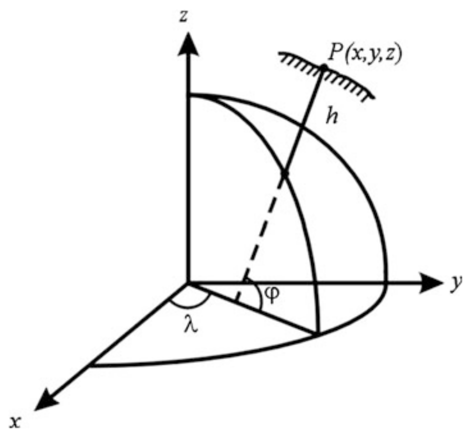


Fig. 2.3 Ellipsoidal coordinate system

The ellipsoidal system is also called the geodetic coordinate system. Geocentric longitude and geodetic longitude are identical. The two geometric parameters can be the semi-major radius (denote by a) and the semi-minor radius (denote by b) of the rotating ellipse, or the semi-major radius and the flattening (denote by f) of the ellipsoid. They are equivalent sets of parameters. The relationship between (x, y, z) and (φ, λ, h) is (cf., e.g., Torge 1991):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (N+h)\cos\varphi\,\cos\lambda \\ (N+h)\cos\varphi\,\sin\lambda \\ (N(1-e^2)+h)\sin\varphi \end{pmatrix}, \tag{2.2}$$

or

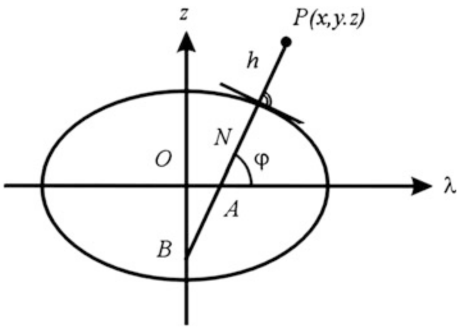
$$\begin{cases} \tan\varphi = \frac{z}{\sqrt{x^2+y^2}}\left(1 - e^2\frac{N}{N+h}\right)^{-1} \\ \tan\lambda = \frac{y}{x} \\ h = \frac{\sqrt{x^2+y^2}}{\cos\varphi} - N \end{cases}, \tag{2.3}$$

where

$$N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}. \tag{2.4}$$

N is the radius of curvature in the prime vertical, and e is the first eccentricities. The geometric meaning of N is shown in Fig. 2.4. In Eq. 2.3, φ and h must be solved by iteration; however, the iteration process converges quickly, since $h \ll N$. The flattening and the first eccentricities are defined as:

Fig. 2.4 Radius of curvature in the prime vertical



$$f = \frac{a-b}{a}, \quad \text{and} \quad e = \frac{\sqrt{a^2 - b^2}}{a}. \tag{2.5}$$

In cases where $\varphi = \pm 90°$ or h is very large, the iteration formulas of Eq. 2.3 are unstable. Alternatively, using (cf. Lelgemann 2002)

$$\operatorname{ctan}\varphi = \frac{\sqrt{x^2 + y^2}}{z + \Delta z},$$

$$\Delta z = e^2 N \sin\varphi = \frac{ae^2 \sin\varphi}{\sqrt{1 - e^2 \sin^2\varphi}},$$

may lead to a stably iterated result of φ. Δz and $e^2 N$ are the lengths of $\overline{OB}$ and $\overline{AB}$ (cf. Fig. 2.4), respectively. h can be obtained by using Δz, i.e.

$$h = \sqrt{x^2 + y^2 + (z + \Delta z)^2} - N.$$

The two geometric parameters used in the World Geodetic System 1984 (WGS-84) are (a = 6,378,137 m, f = 1/298.2572236). In International Terrestrial Reference Frame 1996 (ITRF-96), the two parameters are (a = 6,378,136.49 m, f = 1/298.25645). ITRF uses the International Earth Rotation Service (IERS) Conventions (cf. McCarthy 1996). In PZ-90 (Parameters of the Earth Year 1990) coordinate system of GLONASS, the two parameters are (a = 6,378,136 m, f = 1/298.2578393).

The relation between the geocentric and geodetic latitude ϕ and φ may be given by (cf. Eqs. 2.1 and 2.3):

$$\tan\phi = \left(1 - e^2 \frac{N}{N+h}\right) \tan\varphi. \tag{2.6}$$

2.2 Coordinate System Transformations

Any Cartesian coordinate system can be transformed to another Cartesian coordinate system through three successive rotations if their origins are the same and if they are both right-handed or both left-handed systems. These three rotation matrices are:

$$R_1(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix},$$

$$R_2(\alpha) = \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix}, \tag{2.7}$$

$$R_3(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where α is the rotating angle, which has a positive sign for a counter-clockwise rotation as viewed from the positive axis to the origin. R_1, R_2, and R_3 are called the rotating matrix around the x-, y-, and z-axis, respectively. For any rotation matrix R, there are $R^{-1}(\alpha) = R^{\mathrm{T}}(\alpha)$ and $R^{-1}(\alpha) = R(-\alpha)$; that is, the rotation matrix is an orthogonal one, where R^{-1} and R^{T} are the inverse and transpose of the matrix R.

For two Cartesian coordinate systems with different origins and different length units, the general transformation can be given in vector (matrix) form as

$$X_{\mathrm{n}} = X_0 + \mu R X_{\mathrm{old}}, \text{ or,} \tag{2.8}$$

$$\begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \mu R \begin{pmatrix} x_{\mathrm{old}} \\ y_{\mathrm{old}} \\ z_{\mathrm{old}} \end{pmatrix},$$

where μ is the scale factor (or the ratio of the two length units), and R is a transformation matrix that can be formed by three suitably successive rotations. X_{n} and X_{old} denote the new and old coordinates, respectively; X_0 denotes the translation vector and is the coordinate vector of the origin of the old coordinate system in the new one.

If rotational angle α is very small, then one has $\sin\alpha \approx \alpha$ and $\cos\alpha \approx 0$. In such a case, the rotation matrix can be simplified. If the three rotational angles α_1, α_2, α_3 in R of Eq. 2.8 are very small, then R can be written as (cf., e.g., Lelgemann and Xu 1991):

$$R = \begin{pmatrix} 1 & \alpha_3 & -\alpha_2 \\ -\alpha_3 & 1 & \alpha_1 \\ \alpha_2 & -\alpha_1 & 1 \end{pmatrix}, \tag{2.9}$$

where α_1, α_2, α_3 are small rotating angles around the x-, y-, and z-axis, respectively. Using the simplified R, the transformation 2.8 is called the Helmert transformation.

As an example, the transformation from WGS-84 to ITRF-90 is given by McCarthy (1996):

$$\begin{pmatrix} x_{\mathrm{ITRF-90}} \\ y_{\mathrm{ITRF-90}} \\ z_{\mathrm{ITRF-90}} \end{pmatrix} = \begin{pmatrix} 0.060 \\ -0.517 \\ -0.223 \end{pmatrix} + \mu \begin{pmatrix} 1 & -0.0070'' & -0.0003'' \\ 0.0070'' & 1 & -0.0183'' \\ 0.0003'' & 0.0183'' & 1 \end{pmatrix} \begin{pmatrix} x_{\mathrm{WGS-84}} \\ y_{\mathrm{WGS-84}} \\ z_{\mathrm{WGS-84}} \end{pmatrix},$$

where $\mu = 0.999999989$, the translation vector has the unit of meter.

The transformations between the coordinate systems of GPS, GLONASS, and Galileo can be generally represented by Eq. 2.8 with the scale factor $\mu = 1$ (i.e. the length units used in the three systems are the same). A formula of velocity transformations between different coordinate systems can be obtained by differentiating the Eq. 2.8 with respect to the time.

2.3 Local Coordinate System

The local left-handed Cartesian coordinate system (x', y', z') can be defined by placing the origin to the local point $P_1(x_1, y_1, z_1)$, whose z'-axis points to the vertical, x'-axis is directed to the north, and y’ points to the east (cf. Fig. 2.5). The x' y'-plane is called the horizontal plane; the vertical is defined perpendicular to the ellipsoid. Such a coordinate system is also called a local horizontal coordinate system. For any point P_2, whose coordinates in the global and local coordinate system are (x_2, y_2, z_2) and (x', y , z'), respectively, one has relations of

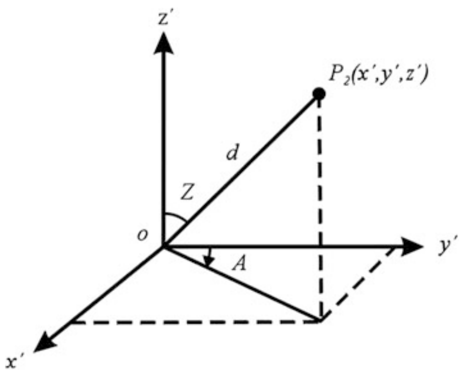


Fig. 2.5 Astronomical coordinate system

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = d \begin{pmatrix} \cos A \, \sin Z \\ \sin A \, \sin Z \\ \cos Z \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} d = \sqrt{x'^2 + y'^2 + z'^2} \\ \tan A = y'/x' \\ \cos Z = z'/d \end{pmatrix}, \tag{2.10}$$

where A is the azimuth, Z is the zenith distance, and d is the radius of the P_2 in the local system. A is measured from the north clockwise; Z is the angle between the vertical and the radius d.

The local coordinate system (x', y', z') can indeed be obtained by two successive rotations of the global coordinate system (x, y, z) by $R_2(90 - \varphi)R_3(\lambda)$ and then by changing the x-axis to a right-handed system. In other words, the global system must be rotated around the z-axis with angle λ, then around the y-axis with angle $90 - \varphi$, and then change the sign of the x-axis. The total transformation matrix R is then

$$R = \begin{pmatrix} -\sin\varphi\,\cos\lambda & -\sin\varphi\,\sin\lambda & \cos\varphi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\varphi\,\cos\lambda & \cos\varphi\,\sin\lambda & \sin\varphi \end{pmatrix}, \tag{2.11}$$

and there are:

$$X_{\text{local}} = R X_{\text{global}} \quad \text{and} \quad X_{\text{global}} = R^{\mathrm{T}} X_{\text{local}}, \tag{2.12}$$

where X_{local} and X_{global} are the same vector represented in local and global coordinate systems. (φ, λ) are the geodetic latitude and longitude of the local point.

If the vertical direction is defined as the plump line of the gravitational field at the local point, then such a local coordinate system is called an astronomic horizontal system (its x'-axis points to the north, left-handed system). The plump line of gravity g and the vertical line of the ellipsoid at point p generally do not coincide; however, the difference is very small, and is omitted in GPS in practice.

Combining Eqs. 2.10 and 2.12, the zenith angle and azimuth of a point P_2 (satellite) related to the station P_1 can be directly computed by using the global coordinates of the two points by

$$\cos Z = \frac{z'}{d} \quad \text{and} \quad \tan A = \frac{y'}{x'}, \tag{2.13}$$

where

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}, \\ x' &= -(x_2 - x_1)\sin\varphi\,\cos\lambda - (y_2 - y_1)\sin\varphi\,\sin\lambda + (z_2 - z_1)\cos\varphi, \\ y' &= -(x_2 - x_1)\sin\lambda + (y_2 - y_1)\cos\lambda \quad \text{and} \\ z' &= (x_2 - x_1)\cos\varphi\,\cos\lambda + (y_2 - y_1)\cos\varphi\,\sin\lambda + (z_2 - z_1)\sin\varphi. \end{aligned}$$

2.4
Earth-Centred Inertial Coordinate System

To describe the motion of the GPS satellites, an inertial coordinate system must be defined. The motion of the satellites follows Newtonian mechanics, and Newtonian mechanics is valid and expressed in an inertial coordinate system. The Conventional Celestial Reference Frame (CRF) is suitable for our purpose. The *xy*-plane of the CRF is the plane of the earth's equator; the coordinates are celestial longitude, measured eastward along the equator from the vernal equinox, and celestial latitude. The vernal equinox is a crossover point of the ecliptic and the equator. Thus, the right-handed earth-centred inertial (ECI) system uses the earth centre as the origin, the CIO as the *z*-axis, and its *x*-axis is directed to the equinox of J2000.0 (Julian date for 1 January 2000 at 12 h). Such a coordinate system is also called equatorial coordinates of date. Because of the motion (acceleration) of the earth's centre, ECI is indeed a quasi-inertial system, and the general relativistic effects must be taken into account. The system moves around the sun, however, without rotating with respect to the CIO. This system is also called the earth-centred space-fixed (ECSF) coordinate system.

An excellent figure has been given by Torge (1991) to illustrate the motion of the earth's pole with respect to the ecliptic pole (cf. Fig. 2.6). The earth's flattening, combined with the obliquity of the ecliptic, results in a slow turning of the equator on the ecliptic due to the differential gravitational effect of the moon and the sun. The slow circular motion with a period of about 26,000 years is called precession, and the other quicker motion with periods from 14 days to 18.6 years is called nutation. Taking the precession and nutation into account, the earth's mean pole (related to the mean equator) is transformed to the earth's true pole (related to the true equator). The *x*-axis of the ECI is pointed to the vernal equinox of date.

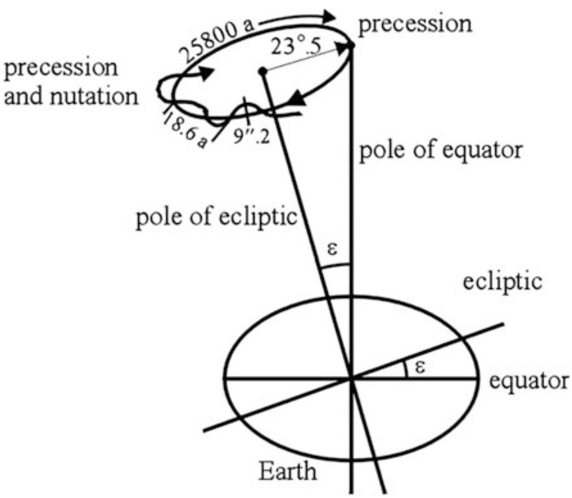


Fig. 2.6 Precession and nutation

The angle of the earth's rotation from the equinox of date to the Greenwich meridian is called Greenwich Apparent Sidereal Time (GAST) . Taking GAST into account (called the earth's rotation), the ECI of date is transformed to the true equatorial coordinate system. The difference between the true equatorial system and the ECEF system is the polar motion. Thus, we have transformed the ECI system in a geometric way to the ECEF system. Such a transformation process can be written as

$$X_{\mathrm{ECEF}} = R_{\mathrm{M}} R_{\mathrm{S}} R_{\mathrm{N}} R_{\mathrm{P}} X_{\mathrm{ECI}}, \tag{2.14}$$

where R_{P} is the precession matrix, R_{N} is the nutation matrix, R_{S} is the earth rotation matrix, R_{M} is the polar motion matrix, X is the coordinate vector, and indices ECEF and ECI denote the related coordinate systems.

Precession

The precession matrix consists of three successive rotation matrices (cf., e.g., Hofman-Wellenhof et al. 1997; Leick 1995; McCarthy 1996), i.e.

$$\begin{aligned} R_{\mathrm{P}} &= R_3(-z) R_2(\theta) R_3(-\zeta) \\ &= \begin{pmatrix} \cos z \cos\theta \cos\zeta - \sin z \sin\zeta & -\cos z \cos\theta \sin\zeta - \sin z \cos\zeta & -\cos z \sin\theta \\ \sin z \cos\theta \cos\zeta + \cos z \sin\zeta & -\sin z \cos\theta \sin\zeta + \cos z \cos\zeta & -\sin z \sin\theta \\ \sin\theta \cos\zeta & -\sin\theta \sin\zeta & \cos\theta \end{pmatrix}, \end{aligned} \tag{2.15}$$

where z, θ, ζ are precession parameters and

$$\begin{aligned} z &= 2306.''2181\,T + 1.''09468\,T^2 + 0.''018203T^3 \\ \theta &= 2004.''3109\,T - 0.''42665\,T^2 - 0.''041833T^3 \\ \zeta &= 2306.''2181\,T + 0.''30188\,T^2 + 0.''017998T^3 \end{aligned} \tag{2.16}$$

where T is the measuring time in Julian centuries (36,525 days) counted from J2000.0 (cf. Sect. 2.6 time systems).

Nutation

The nutation matrix consists of three successive rotation matrices (cf., e.g., Hoffman-Wellenhof et al. 1997; Leick 1995; McCarthy 1996), i.e.

$$\begin{aligned} R_N &= R_1(-\varepsilon - \Delta\varepsilon)R_3(-\Delta\psi)R_1(\varepsilon) \\ &= \begin{pmatrix} \cos\Delta\psi & -\sin\Delta\psi\cos\varepsilon & -\sin\Delta\psi\sin\varepsilon \\ \sin\Delta\psi\cos\varepsilon_t & \cos\Delta\psi\cos\varepsilon_t\cos\varepsilon + \sin\varepsilon_t\sin\varepsilon & \cos\Delta\psi\cos\varepsilon_t\sin\varepsilon - \sin\varepsilon_t\cos\varepsilon \\ \sin\Delta\psi\sin\varepsilon_t & \cos\Delta\psi\sin\varepsilon_t\cos\varepsilon - \cos\varepsilon_t\sin\varepsilon & \cos\Delta\psi\sin\varepsilon_t\sin\varepsilon + \cos\varepsilon_t\cos\varepsilon \end{pmatrix} \\ &\approx \begin{pmatrix} 1 & -\Delta\psi\cos\varepsilon & -\Delta\psi\sin\varepsilon \\ \Delta\psi\cos\varepsilon_t & 1 & -\Delta\varepsilon \\ \Delta\psi\sin\varepsilon_t & \Delta\varepsilon & 1 \end{pmatrix}, \end{aligned} \tag{2.17}$$

where ε is the mean obliquity of the ecliptic angle of date, $\Delta\psi$ and $\Delta\varepsilon$ are nutation angles in longitude and obliquity, $\varepsilon_t = \varepsilon + \Delta\varepsilon$, and

$$\varepsilon = 84381.''448 - 46.''8150T - 0.''00059T^2 + 0.''001813T^3. \tag{2.18}$$

The approximation is made by letting $\cos\Delta\psi = 1$ and $\sin\Delta\psi = \Delta\psi$ for very small $\Delta\psi$. For precise purposes, the exact rotation matrix shall be used. The nutation parameters $\Delta\psi$ and $\Delta\varepsilon$ can be computed by using the International Astronomical Union (IAU) theory or IERS theory:

$$\begin{aligned} \Delta\Psi &= \sum_{i=1}^{106} (A_i + A_i'T)\sin\beta, \\ \Delta\varepsilon &= \sum_{i=1}^{106} (B_i + B_i'T)\cos\beta \end{aligned}$$

or

$$\begin{aligned} \Delta\Psi &= \sum_{i=1}^{263} (A_i + A_i'T)\sin\beta + A_i''\cos\beta, \\ \Delta\varepsilon &= \sum_{i=1}^{263} (B_i + B_i'T)\cos\beta + B_i'\cos\beta, \end{aligned}$$

where argument

$$\beta = N_{1i}l + N_{2i}l' + N_{3i}F + N_{4i}D + N_{5i}\Omega,$$

where l is the mean anomaly of the moon, l' is the mean anomaly of the Sun, $F = L - \Omega$, D is the mean elongation of the moon from the sun, Ω is the mean longitude of the ascending node of the moon, and L is the mean longitude of the moon. The formulas of l, l', F, D, and Ω, are given in Sect. 11.2.8. The coefficient values of N_{1i}, N_{2i}, N_{3i}, N_{4i}, N_{5i}, A_i, B_i, A', B_i', A_i'', and B_i'' can be found in, e.g., McCarthy (1996). The updated formulas and tables can be found in updated IERS conventions. For convenience, the coefficients of the IAU 1980 nutation model are given in Appendix 1.

Earth Rotation

The earth rotation matrix can be represented as

$$R_S = R_3(\mathrm{GAST}), \tag{2.19}$$

where GAST is Greenwich Apparent Sidereal Time and

$$\mathrm{GAST} = \mathrm{GMST} + \Delta\Psi\cos\varepsilon + 0.''00264\sin\Omega + 0.''000063\sin 2\Omega, \tag{2.20}$$

where GMST is Greenwich Mean Sidereal Time. Ω is the mean longitude of the ascending node of the Moon; the second term on the right-hand side is the nutation of the equinox. Furthermore,

$$\mathrm{GMST} = \mathrm{GMST}_0 + \alpha\mathrm{UT1}, \tag{2.21}$$

$$\begin{aligned}\mathrm{GMST}_0 = {} & 6 \times 3600.''0 + 41 \times 60.''0 + 50.''54841 \\ & + 8640184.''812866 T_0 + 0.''093104 T_0^2 - 6.''2 \times 10^{-6} T_0^3,\end{aligned}$$

$$\alpha = 1.002737909350795 + 5.9006 \times 10^{-11} T_0 - 5.9 \times 10^{-15} T_0^2,$$

where GMST_0 is Greenwich Mean Sidereal Time at midnight on the day of interest. α is the rate of change. UT1 is the polar motion corrected Universal Time (cf. Sect. 2.6). T_0 is the measuring time in Julian centuries (36,525 days) counted from J2000.0 to 0^{h}UT1 of the measuring day. By computing GMST, UT1 is used (cf. Sect. 2.6).

Polar Motion

As shown in Fig. 2.7, the polar motion is defined as the angles between the pole of date and the CIO pole. The polar motion coordinate system is defined by *xy*-plane coordinates, whose *x*-axis is pointed to the south and is coincided to the mean Greenwich meridian, and whose *y*-axis is pointed to the west. x_p and y_p are the angles of the pole of date, so the rotation matrix of polar motion can be represented as

$$\begin{aligned} R_M = R_2(-x_p)R_1(-y_p) &= \begin{pmatrix} \cos x_p & \sin x_p \sin y_p & \sin x_p \cos y_p \\ 0 & \cos y_p & -\sin y_p \\ -\sin x_p & \cos x_p \sin y_p & \cos x_p \cos y_p \end{pmatrix} \\ &\approx \begin{pmatrix} 1 & 0 & x_p \\ 0 & 1 & -y_p \\ -x_p & y_p & 1 \end{pmatrix}. \end{aligned} \tag{2.22}$$

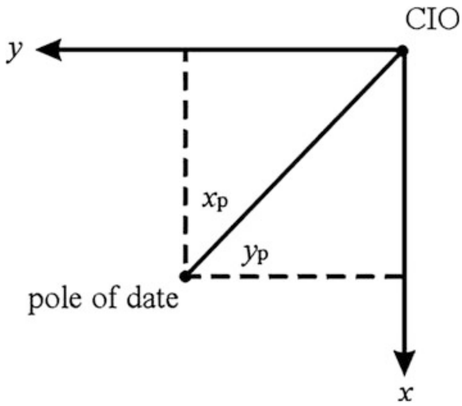


Fig. 2.7 Polar motion

The IERS determined x_p and y_p can be obtained from the home pages of IERS.

2.5 IAU 2000 Framework

At its 2000 General Assembly, the International Astronomical Union (IAU) adopted a set of resolutions that provide a consistent framework for defining barycentric and geocentric celestial reference systems (Petit 2002). The consequence of the resolution is that coordinate transformation from celestial reference system (CRS, i.e. the ECI system) to the terrestrial reference system (TRS, i.e. the ECEF system) has the form

$$X_{ECEF} = R_M R_S R_{NP} X_{ECI}, \tag{2.23}$$

where R_{NP} is the precession-nutation matrix, R_S is the earth rotation matrix, R_M is the polar motion matrix, X is the coordinate vector, and indices ECEF and ECI denote the related coordinate systems. The rotation matrices are functions of time T, which is defined (see McCarthy and Petit 2003) by

$$T = (\mathrm{TT} - 2000\ \mathrm{January}\ 1\mathrm{d}\ 12\mathrm{h}\ \mathrm{TT})\ \mathrm{in\ days}/36{,}525, \tag{2.24}$$

where TT is the Terrestrial Time (for details see Sect. 2.7) and

$$\begin{aligned} R_M &= R_2(-x_p)R_1(-y_p)R_3(s'), \\ R_S &= R_3(\vartheta),\ \text{and} \\ R_{NP} &= R_3(-s)R_3(-E)R_2(d)R_3(E), \end{aligned} \tag{2.25}$$

where x_p and y_p are the angles of the pole of date (or polar coordinates of the Celestial Intermediate Pole (CIP) in TRS), and s' is a function of x_p and y_p:

$$s' = \frac{1}{2}\int_{T0}^{T} (x_p \dot{y}_p - \dot{x}_p y_p)\mathrm{d}t \text{ or}$$

approximately (see McCarthy and Capitaine 2002)

$$s' = (-47\mu as)\,T, \tag{2.26}$$

where T is time in Julian century counted from J2000.0 and

$$\vartheta = 2\pi(0.7790572732640 + 1.00273781191135448\,T_u), \tag{2.27}$$

where T_u = (Julian UT1 date - 2,451,545.0) and UT1 = UTC + (UT1-UTC). (UT1-UTC) is published by the IERS.

E and d are such that the coordinates of the CIP in the CRS are

$$\begin{aligned} X &= \sin d \cos E \\ Y &= \sin d \sin E \\ Z &= \cos d \end{aligned} \tag{2.28}$$

Equivalently, R_{NP} can be given by

$$R_{NP} = R_3(-s)\cdot\begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{pmatrix}^{-1} \tag{2.29}$$

where

$$a = \frac{1}{1 + \cos d} \approx \frac{1}{2} + \frac{1}{8}(X^2 + Y^2). \tag{2.30}$$

The developments of X and Y can be found on the website of the IERS Conventions and have the following form (in mas: microarcseconds) (Capitaine 2002):

$$\begin{aligned} X = {} & -16616.99'' + 2004191742.88''T - 427219.05''T^2 \\ & - 198620.54''T^3 - 46.05''T^4 + 5.98''T^5 \\ & + \sum_i \left[(a_{s,0})_i \sin\beta + (a_{c,0})_i \cos\beta\right] \\ & + \sum_i \left[(a_{s,1})_i T \sin\beta + (a_{c,1})_i T \cos\beta\right] \\ & + \sum_i \left[(a_{s,2})_i T^2 \sin\beta + (a_{c,2})_i T^2 \cos\beta\right] + \cdots \end{aligned} \tag{2.31}$$

$$
\begin{aligned}
Y = {} & -6950.78'' - 25381.99''T - 22407250.99''T^2 \\
& + 1842.28''T^3 - 1113.06''T^4 + 0.99''T^5 \\
& + \sum_i \left[(b_{s,0})_i \sin\beta + (b_{c,0})_i \cos\beta\right] \\
& + \sum_i \left[(b_{s,1})_i T \sin\beta + (b_{c,1})_i T \cos\beta\right] \\
& + \sum_i \left[(b_{s,2})_i T^2 \sin\beta + (b_{c,2})_i T^2 \cos\beta\right] + \cdots
\end{aligned}
\tag{2.32}
$$

In Eq. 2.29, s is the accumulated rotation between the reference epoch and the date T of CEO on the true equator due to the celestial motion of CIP and can be expressed as

$$
s(T) = -\frac{1}{2}[X(T)Y(T) - X(T_0)Y(T_0)] + \int_{T_0}^{T} \dot{X}Y \mathrm{d}t - \left(\sigma_0 N_0 - \sum\nolimits_0 N_0\right)
$$

where σ_0 and $\sum_0$ are the positions of CEO at J2000.0 and the x-origin of CRS, respectively, and N_0 is the ascending node at J2000.0 in the equator of CRS. In above equation, term $s(T) + \frac{1}{2}[X(T)Y(T)]$ can be expressed as (in mas)

$$
\begin{aligned}
s + XY/2 = {} & 94.0 + 3808.35T - 119.94T^2 \\
& - 72574.09T^3 + 27.70T^4 + 15.61T^5 \\
& + \sum_i \left[(c_{s,0})_i \sin\beta + (c_{c,0})_i \cos\beta\right] \\
& + \sum_i \left[(c_{s,1})_i T \sin\beta + (c_{c,1})_i T \cos\beta\right] \\
& + \sum_i \left[(c_{s,2})_i T^2 \sin\beta + (c_{c,2})_i T^2 \cos\beta\right] + \cdots
\end{aligned}
\tag{2.33}
$$

In Eqs. 2.31–2.33, coefficients $(a_{s,j})_i, (a_{c,j})_i$, $(b_{s,j})_i, (b_{c,j})_i$ and $(c_{s,j})_i, (c_{c,j})_i$ can be extracted from Table 5.2a–c (available at ftp://tai.bipm.org/iers/conv2003/chapter5/). β is the combination of the fundamental arguments of nutation theory:

$$
\beta = \sum_{j=1}^{14} N_j F_j \tag{2.34}
$$

The first five F_j are the Delaunay variables l, l', F, D, Ω (given in Sect. 11.2.8); the amplitudes of sines and cosines β can be derived from the amplitudes of the precession and nutation series (see McCarthy and Petit 2003); F_6 to F_{13} are the mean longitudes of the planets (Mercury to Neptune), including the earth; F_{14} is the general precession in longitude. They are given in radians and T in Julian centuries of TDB (see Sect. 2.7). The coefficients N_j are functions of index i and can be found in the IERS website:

$$
\begin{aligned}
F_6 &= l_{Me} = 4.402608842 + 2608.7903141574T \\
F_7 &= l_{Ve} = 3.176146697 + 1021.3285546211T \\
F_8 &= l_E = 1.753470314 + 628.3075849991T \\
F_9 &= l_{Ma} = 6.203480913 + 334.0612426700T \\
F_{10} &= l_{Ju} = 0.599546497 + 52.9690962641T \\
F_{11} &= l_{Sa} = 0.874016757 + 21.3299104960T \\
F_{12} &= l_{Ur} = 5.481293872 + 7.4781598567T \\
F_{13} &= l_{Ne} = 5.311886287 + 3.8133035638T \\
F_{14} &= P_a = 0.024381750T + 0.00000538691T^2
\end{aligned}
\tag{2.35}
$$

Using the new paradigm, the complete procedure of transforming GCRS to ITRS, which is compatible with the IAU2000 precession-nutation, is based on the expressions of 2.31–2.33.

An equivalent way to realise the transformation between TRS and CRS under the definition of IAU 2000 can be implemented in a classical way by adding IAU2000 corrections to the corresponding rotating angles. This is done by using the transformation formula 2.14, where the three precession rotating angles (see McCarthy and Petit 2003) are

$$
\begin{aligned}
z = &- 2.5976176'' + 2\,306.0803226'' T + 1.0947790'' T \\
&+ 0.0182273'' T + 0.0000470'' T^4 - 0.0000003'' T^5 \\
\theta = &\, 2\,004.1917476'' T - 0.4269353'' T - 0.0418251'' T \\
&- 0.0000601'' T^4 - 0.0000001'' T^5 \\
\zeta = &\, 2.5976176'' + 2\,306.0809506'' T + 0.3019015'' T \\
&+ 0.0179663'' T - 0.0000327'' T^4 - 0.0000002'' T^5
\end{aligned}
\tag{2.36}
$$

The IAU 2000 nutation model is given by two series for nutation in longitude $\Delta\psi$ and obliquity $\Delta\varepsilon$, referred to the mean equator and equinox of date, with T measured in Julian centuries from epoch J2000.0:

$$
\Delta\psi = \sum_{i=1}^{N} (A_i + A_i' T) \cos\beta + (A_i'' + A_i''' T) \cos\beta, \tag{2.37}
$$

$$
\Delta\varepsilon = \sum_{i=1}^{N} (B_i + B_i' T) \cos\beta + (B_i'' + B_i''' T) \cos\beta,
$$

where argument β can be found on the IERS website. For these two formulas, rate and bias corrections are necessary because of the new definition of the Celestial Intermediate Pole and the Celestial and Terrestrial Ephemeris Origin:

$$d\Delta\psi = (-0.0166170 \pm 0.0000100)'' + (-0.29965 \pm 0.00040)''T,$$
$$d\Delta\varepsilon = (-0.0068192 \pm 0.0000100)'' + (-0.02524 \pm 0.00010)''T. \tag{2.38}$$

The earth's rotation angle (i.e. the apparent Greenwich Sidereal Time GST or GAST) can be computed by adding a correction *EO* to the GMST in Eq. 2.27 (in mas):

$$\begin{aligned} \mathrm{EO} = {} & 14,506 + 4,612,157,399.66T + 1,396,677.21T^2 - 93.44T^3 + 18.82T^4 \\ & + \Delta\psi\cos\varepsilon + \sum_i \left[(d_{s,0})_i \sin\beta + (d_{c,0})_i \cos\beta\right] \\ & + \sum_i \left[(d_{s,1})_i T \sin\beta + (d_{c,1})_i T \cos\beta\right] + \cdots \end{aligned} \tag{2.39}$$

where coefficients $(d_{s,j})_i, (d_{c,j})_i$ can be extracted from Table 5.4 (available at ftp://tai.bipm.org/iers/conv2003/chapter5/). $\Delta\psi$ is defined in Eq. 2.37 and ε is defined in Eq. 2.18.

Similarly, the rotation matrix of polar motion shall be represented as the first formula of 2.25 and 2.26.

2.6 Geocentric Ecliptic Inertial Coordinate System

As discussed above, ECI used the CIO pole in space as the z-axis (through consideration of the polar motion, nutation, and precession). If the ecliptic pole is used as the z-axis, then an ecliptic coordinate system is defined, and it may be called the Earth-Centred Ecliptic Inertial (ECEI) coordinate system. ECEI places the origin at the mass centre of the earth, its z-axis is directed to ecliptic pole (or the xy-plane is the mean ecliptic), and its x-axis points to the vernal equinox of date. The coordinate transformation between the ECI and ECEI systems can be represented as

$$X_{\mathrm{ECEI}} = R_1(-\varepsilon)X_{\mathrm{ECI}}, \tag{2.40}$$

where ε is the ecliptic angle (mean obliquity) of the ecliptic plane related to the equatorial plane. The formula for ε is given in Sect. 2.4. Usually, coordinates of the sun and the moon as well as planets are given in the ECEI system.

2.7 Time Systems

Three time systems are used in satellite surveying. They are sidereal time, dynamic time, and atomic time (cf., e.g., Hofman-Wellenhof et al. 1997; Leick 1995; McCarthy 1996; King et al. 1987).

Sidereal time is a measure of the earth's rotation and is defined as the hour angle of the vernal equinox. If the measure is counted from the Greenwich meridian, the sidereal time is called Greenwich Sidereal Time. Universal Time (UT) is the Greenwich hour angle of the apparent sun, which orbits uniformly in the equatorial plane. Because the angular velocity of the earth's rotation is not a constant, sidereal time is not a uniformly scaled time. The oscillation of UT is also partly caused by the polar motion of the earth. The universal time corrected for the polar motion is denoted by UT1.

Dynamic time is a uniformly scaled time series used to describe the motion of bodies in a gravitational field. Barycentric Dynamic Time (TDB) is applied in an inertial coordinate system (its origin is located at the centre-of-mass, or barycentre). Terrestrial Dynamic Time (TDT) is used in a quasi-inertial coordinate system (such as ECI). Because of the motion of the earth around the sun (or in the sun's gravitational field), TDT will vary with respect to TDB. However, both the satellite and the earth are subject to almost the same gravitational perturbations. TDT may be used for describing the satellite motion without taking into account the influence of the gravitational field of the sun. TDT is also called Terrestrial Time (TT).

Atomic time is a system based on the output of atomic clocks, such as that used in the International Atomic Time (TAI) scale. It is uniformly scaled and is used in the ECEF coordinate system. In practice, TDT is realised by TAI with a constant offset (32.184 s). To take into account the slowing of the earth's rotation with respect to the sun, Coordinated Universal Time (UTC) was introduced in order to maintain the synchronisation of TAI to the solar day (by inserting leap seconds). GPS Time (GPST) is also an atomic time scale.

The relationships between different time systems are given as follows:

$$\begin{aligned} \text{TAI} &= \text{GPST} + 19.0\,\text{s} \\ \text{TAI} &= \text{TDT} - 32.184\,\text{s} \\ \text{TAI} &= \text{UTC} + n\,\text{s} \\ \text{UT1} &= \text{UTC} + \text{dUT1} \end{aligned}, \tag{2.41}$$

where dUT1 can be obtained by IERS, (dUT1 < 0.7 s, cf. Zhu et al. 1996), (dUT1 is also broadcasted with the navigation data), n is the number of leap seconds of date and is inserted into UTC on 1 January and 1 July for the years. The actual n can be found in the IERS report.

Time argument T (Julian centuries) is used in the formulas given in Sect. 2.4. For convenience, T is denoted by TJD, and TJD can be computed from the civil date (Year, Month, Day, and Hour) as follows:

$$\mathrm{JD} = \mathrm{INT}(365.25Y) + \mathrm{INT}(30.6001\,(M+1)) + \mathrm{Day} + \mathrm{Hour}/24 + 1,720,981.5$$

and

$$\mathrm{TJD} = \mathrm{JD}/36525, \tag{2.42}$$

where

$$\begin{array}{llll} Y = \mathrm{Year}-1, & M = \mathrm{Month}+12, & \text{if} & \mathrm{Month} \le 2, \\ Y = \mathrm{Year}, & M = \mathrm{Month}, & \text{if} & \mathrm{Month} > 2, \end{array}$$

where JD is the Julian date (JD), Hour is the time of UT, and INT denotes the integer part of a real number. The JD counted from JD2000.0 is then JD2000 = JD − JD2000.0, where JD2000.0 is the JD for 1 January 2000 at 12 h, and has a value of 2,451,545.0 days. One Julian century is 36,525 days.

Inversely, the civil date (Year, Month, Day and Hour) can be computed from the JD as follows:

$$\begin{aligned} b &= \mathrm{INT}(\mathrm{JD}+0.5)+1537, \\ c &= \mathrm{INT}\left(\frac{b-122.1}{365.25}\right), \\ d &= \mathrm{INT}(365.25c), \\ e &= \mathrm{INT}\left(\frac{b-d}{30.6001}\right), \\ \mathrm{Hour} &= \mathrm{JD}+0.5-\mathrm{INT}(\mathrm{JD}+0.5), \\ \mathrm{Day} &= b-d-\mathrm{INT}(30.6001\,e), \\ \mathrm{Month} &= e-1-12\,\mathrm{INT}\left(\frac{e}{14}\right), \text{ and} \end{aligned}$$

$$\mathrm{Year} = c - 4\,715 - \mathrm{INT}\left(\frac{7+\mathrm{Month}}{10}\right), \tag{2.43}$$

where b, c, d, and e are auxiliary numbers.

Because the GPS standard epoch is defined as JD = 2,444,244.5 (6 January 1980, 0 h), GPS week and the day of week (denoted by Week and N) can be computed by

$$N = \mathrm{modulo}(\mathrm{INT}(\mathrm{JD}+1.5), 7) \text{ and}$$

$$\mathrm{Week} = \mathrm{INT}\left(\frac{\mathrm{JD}-2,444,244.5}{7}\right), \tag{2.44}$$

where N is the day of week (N = 0 for Monday, N = 1 for Tuesday, and so on).

For saving digits and counting the date from midnight instead of noon, the modified Julian date (MJD) is defined as

$$\mathrm{MJD} = (\mathrm{JD}{-}2,400,000.5). \tag{2.45}$$

GLONASS time (GLOT) is defined by Moscow time $\mathrm{UTC_{SU}}$, which equals UTC plus three hours (corresponding to the offset of Moscow time to Greenwich time), theoretically. GLOT is permanently monitored and adjusted by the GLONASS Central Synchroniser (cf. Roßbach 2000). UTC and GLOT then has a simple relation

$$\mathrm{UTC} = \mathrm{GLOT} + t_{\mathrm{c}} - 3\mathrm{h}, \tag{2.46}$$

where τ_{c} is the system time correction with respect to $\mathrm{UTC_{SU}}$, which is broadcasted by the GLONASS ephemerides and is less than one microsecond. Therefore, there is approximately

$$\mathrm{GPST} = \mathrm{GLOT} + m - 3\mathrm{h}, \tag{2.47}$$

where m refers to the number of "leap seconds" between GPS and GLONASS (UTC) time and is given in the GLONASS ephemerides. m is indeed the leap seconds since GPS standard epoch (6 January 1980, 0 h).

Galileo system time (GST) will be maintained by a number of UTC laboratory clocks. GST and GPST are time systems of various UTC laboratories. After the offset of GST and GPST is made available to the user, the interoperability will be ensured. GST is apart from small differences (tens of nanoseconds), nearly identical to GPS time. The Galileo week starts at midnight Saturday/Sunday at the same second as the GPS week; The GST week as transmitted by the satellites is a 12-bit value with a roll-over after week 4095. The GST week started at zero at the first roll-over of the broadcast GPS week after 1023, i.e. on Sunday, 22 August 1999, at 00:00:00 GPS time.

The BDS Time (BDT) system is a continuous timekeeping system, with length of seconds being an SI second. BDT zero time started at 00:00:00 UTC on 1 January 2006 (GPS week 1356); therefore, BDT is 14 s behind GPS time. BDT is synchronized with UTC within 100 ns (modulo 1 s). The BDT week starts at midnight Saturday/Sunday. The BDT week is transmitted by the satellites as a 13-bit number. It has a roll-over after week 8191.

Apart from the small errors in the realizations of the different time systems, the relations between systems are:

$$\mathrm{GLOT} = \mathrm{UTC} = \mathrm{GPST} - \Delta\mathrm{tLS} \tag{2.48}$$

$$\mathrm{GST} = \mathrm{GPST} = \mathrm{UTC} + \Delta\mathrm{tLS} \tag{2.49}$$

$$\mathrm{BDT} = \mathrm{UTC} + \Delta\mathrm{tLS_{BDS}} \tag{2.50}$$

where ΔtLS is the delta time between GPST and UTC due to leap seconds, such that (2005: $\Delta tLS = 13$; 2006: $\Delta tLS = 14$; 2008: $\Delta tLS = 15$; 2012: $\Delta tLS = 16$ and 2015: $\Delta tLS = 17$). ΔtLS_{BDS} is the delta time between BDT and UTC due to leap seconds, such that (2006: $\Delta tLS_{BDS} = 0$; 2008: $\Delta tLS_{BDS} = 1$; 2012: $\Delta tLS_{BDS} = 2$ and 2015: $\Delta tLS_{BDS} = 3$).

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