Chapter 2
Polarization Imaging

Advances in sensing technology enable acquisition of a large amount of information from the physical world. Vision techniques have played a key role in information sensing, but with a limitation that most vision systems can only perceive partial information beyond the visible spectrum. For instance, one cannot perceive the information carried by a polarized light since human vision systems are not sensitive to polarization. On the other hand, however, some marine and terrestrial animals and insects demonstrate their abilities to sense and utilize polarized lights to navigate, locate, and hunt for prey in their daily activities. Polarization is a unique characteristic of transverse wave, which is the asymmetry phenomenon of vibration direction and propagation direction. Comparing with conventional image techniques, the polarization imaging technique can detect the polarization information of targets, which will be beneficial to subsequent applications, such as target detection, recognition and identification. This chapter will focus on the principle of polarization imaging and its applications, then discuss the factors that affect polarization imaging and the method to reduce errors.

2.1 Electromagnetics and Polarization

2.1.1 Maxwell’s Equations

A light traveling in the free space is known to propagate in a form of a transversal electromagnetic wave, vibrating in the plane perpendicular to its direction of propagation. Polarization is a unique phenomenon of transversal waves. Most imaging systems sense an electrical field vector only, which has dominant effects, photographic and physical, on the interaction of the light and the material. This section discusses the characteristics of an electrical field vector in a polarized light. Maxwell’s equations describe the properties and interrelations between the basic
fields (electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$) and the derived fields (electric displacement field $\mathbf{D}$ and magnetizing field $\mathbf{H}$), in which the charge distribution $\rho$ and current density $\mathbf{J}$ are considered as well [1]. Derived field is the result of interaction of a field and the material. The differential form of Maxwell’s equations is given by

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

(2.1)

The interaction of the fields and the material can be described using complex physical equations. These equations will only involve a constant relationship among dielectric constant $\varepsilon$, magnetic permeability $\mu$, and electrical conductivity $\sigma$ if they are linear, static, and isotropic. The basic fields and derived fields satisfy a constant relationship related to the wavelength at a given spatial point.

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mathbf{J} = \sigma \mathbf{E}$$

(2.2)

Taking the curl of the curl equations, and using the curl identity, we obtain the wave equation:

$$\nabla^2 \mathbf{E} = \frac{\mu \varepsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu \sigma \frac{\partial \mathbf{E}}{\partial t}$$

(2.3)

Similarly, the same derivation steps can be applied to the magnetic field. The electrical conductivity $\sigma$ of a dielectric material is zero in optical systems, so (2.3) can be simplified as

$$\nabla^2 \mathbf{E} = \frac{\mu \varepsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

(2.4)

Equation (2.4) is a standard wave equation and the phase velocity of light in materials or the frequency is given by

$$v = \frac{1}{\sqrt{\mu \varepsilon}}$$

(2.5)

A monochromatic light with frequency $\nu$ or angular frequency $\omega = 2\pi\nu$ has an electric field $\mathbf{E}(\mathbf{r}, t) = E(\mathbf{r}) e^{\pm i\omega t}$ where $\mathbf{r}$ represents a vector distance from the
origin to the observation point. So the wave equation can be changed in the form of Helmholtz equation.

\[ \nabla^2 E + k^2 E = 0 \quad (2.6) \]

where \( \nabla^2 \) denotes the Laplacian, and \( k \) the wave number, \( k = \sqrt{\mu \varepsilon \omega} = 2\pi / \lambda \). Plane wave solution is used to describe the effect of polarization.

\[ \mathbf{E}(\mathbf{r},t) = E_0 \exp(\pm i(\omega t - \mathbf{k} \cdot \mathbf{r} + \delta)) \quad (2.7) \]

\( k = k_s \), where \( s \) denotes a unit vector in the direction of plane wave propagation, and \( \delta \) a constant phase reference. If (2.7) is applied to the Maxwell’s equations

\[ \mathbf{E} = -\sqrt{\frac{\mu}{\varepsilon}} \mathbf{s} \times \mathbf{H} \quad (2.8) \]

and

\[ \mathbf{H} = \sqrt{\frac{\varepsilon}{\mu}} \mathbf{s} \times \mathbf{E} \quad (2.9) \]

Then we use \( s \) to obtain the inner product

\[ \mathbf{H} \cdot s = \mathbf{E} \cdot s = 0 \quad (2.10) \]

Equation (2.10) proves that the field is transversal, which means the electrical and magnetic field vectors are located in the same plane and are perpendicular to the direction of propagation.

### 2.1.2 Polarization Ellipse

Since electrical field is perpendicular to the direction of propagation, the z-axis is chosen as the direction of light wave propagation in the coordinate system where electric field vector lies in the x-y plane. The relationship between the direction of electric field vector in the x-y plane and time should be investigated. The rectangular components of electric field can be seen from the general form of plane wave solution (2.7):

\[ \text{Re}\{a \exp(i(\omega t - \mathbf{k} \cdot \mathbf{r} + \delta))\} = a \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \delta) \quad (2.11) \]
where \( \text{Re}\{\} \) represents the real part of a complex value. So the \( x, y, z \) components of the electric field are given by

\[
E_x = a_x \cos(\omega t - k \cdot r + \delta_x) \quad (2.12)
\]
\[
E_y = a_y \sin(\omega t - k \cdot r + \delta_y) \quad (2.13)
\]
\[
E_z = 0 \quad (2.14)
\]

The electric field \( \mathbf{E} \) propagates through in \( x-y \) plane at any point of the space. Removing the \( \omega t - k \cdot r \) term in (2.12) and (2.13):

\[
\left(\frac{E_x}{a_x}\right)^2 + \left(\frac{E_y}{a_y}\right)^2 - 2 \frac{E_x E_y}{a_x a_y} \cos \delta = \sin^2 \delta \quad (2.15)
\]

where \( \delta = \delta_y - \delta_x \). The trace becomes an ellipse by rotating \( (E_x, E_y) \) with the angle \( \theta \),

\[
\tan 2\theta = \frac{2a_xa_y}{a_x^2 - a_y^2} \cos \delta \quad (2.16)
\]

which is called a polarization ellipse as shown in Fig. 2.1. The curve is drawn by the tip of electric field vector on the plane which is perpendicular to the direction of wave propagation. The principal axis has the same coordinate axis as \( (E_x, E_y) \) at \( \theta = 0 \) or at an odd integral multiples of \( 2\pi \). The ellipse can be rotated both in clockwise and counter-clockwise directions. Polarization direction is called R-polarization when the ellipse rotates in the clockwise rotation \( (\sin \delta > 0) \), otherwise it is L-polarization. The state of polarization is defined by the ratio of ellipse axis, rotation angle \( \theta \) and R-polarization or L-polarization of the ellipse. Two special cases in the polarization ellipse are:

**Fig. 2.1** Polarization ellipse
(1) If $\theta$ is integral multiple of $\pi$, (2.15) becomes a linear equation

$$E_y = \pm \frac{a_y}{a_x} E_x \quad (2.17)$$

(2) If $a_x = a_y = a$ and $\delta$ is an odd integral multiple of $2\pi$, (2.15) becomes a circle

$$(E_x)^2 + (E_y)^2 = a^2 \quad (2.18)$$

Then the light is circularly polarized whose ratio of ellipse axis is 1 for an arbitrary rotation angle $\theta$.

### 2.1.3 Fresnel’s Equations

To obtain polarization state of an optical system, the effect of electric field at the boundary of the media of different refractivity should be calculated. In general, a beam is refracted and the other portion is reflected when a plane wave comes across a boundary of different media. Suppose that the light will not be absorbed by the medium. Fresnel’s equations describe reflection and refraction. Laws of reflection and refraction prove that an incident light, reflected light, refracted light, and the normal of interface are in the same plane. We can get the same results as geometrical optics by solving Maxwell’s equations and adding necessary boundary conditions: incident and reflection angles are the same but in the opposite sign. The refraction angle is given by Snell’s Law. The plane consisting of incident light propagation vector $\mathbf{S}$, incident light, a reflected light, and the normal component is called incident plane as shown in Fig. 2.2.

The electric field can be decomposed into two components, parallel and perpendicular to the incident plane. For incident angle $\theta_i$, refraction angle $\theta_r$, amplitude

![Fig. 2.2 Reflected light, refracted light, and the normal component in the incident plane](image-url)
ratio \( r \) of reflection light and incident light, and amplitude ratio \( t \) of refraction light and incident light, the Fresnel’s equations are given by:

\[
\begin{align*}
    r_\perp &= \frac{n \cos \theta_i - n' \cos \theta_i}{n \cos \theta_i + n' \cos \theta_i} \\
    r_\parallel &= \frac{n' \cos \theta_i - n \cos \theta_i}{n' \cos \theta_i + n \cos \theta_i} \\
    t_\perp &= \frac{2n \cos \theta_i}{n \cos \theta_i + n' \cos \theta_i} \\
    t_\parallel &= \frac{2n' \cos \theta_i}{n' \cos \theta_i + n \cos \theta_i}
\end{align*}
\] (2.19) (2.20) (2.21) (2.22)

where \( n \) and \( n' \) denote refractive indices of incident medium and refractive medium, respectively. Suppose that the magnetic conductivities of the media are same as that in vacuum \((\mu = \mu' = \mu_0)\). Then the reflectivity \( R \) is determined by the square of amplitude reflection coefficient:

\[
R_\perp = r_\perp^2, \quad R_\parallel = r_\parallel^2
\] (2.23)

and

\[
|r_\perp|^2 + |t_\perp|^2 = |r_\parallel|^2 + |t_\parallel|^2 = 1
\] (2.24)

Equation (2.24) proves that the energy of the light is conserved at the boundary of two dielectric materials.

### 2.2 Principles of Polarization Imaging

Polarization characteristics of a light refer to change of polarization state during transmission or reflection, which can generally be expressed by the Jones vector, Stokes vector, or Muller matrix [1, 2]. The Stokes vector was proposed when Stokes studied partially polarized light in 1852. Stokes vector represents completely as well as partially polarized light being used in polarization imaging detection. The four parameters of a Stokes vector \( S = [s_0, s_1, s_2, s_3] \) describe polarization information of target which can be obtained by various imaging equipment both directly and indirectly.

We commonly use electro-optic, magneto-optic, photoelastic modulation and the four detector amplitude segmentation methods to measure the Stokes vector. These
modulation methods are usually adopted as follows: Incident light gets through a series of detecting elements (e.g. the polarizer and the phase retarder) and changes the state of detecting elements (e.g. the polarizing angle or the phase delay angle) by mechanical rotating or continuous periodic modulation, then get a group of intensity values of output light and use Fourier analysis to obtain four Stokes images, so the degree of polarization, the angle of polarization and the ellipticity of polarization are acquired by further analysis. In 1999, Azzam [20] proposed a beam polarization measurement system without mechanical rotation and modulation, which can approximately get four real-time Stokes parameters by amplitude division. Then we take the single detector modulation polarization imager system as an example to introduce the acquisition of Stokes images. Figure 2.3 illustrates the principle of a polarization imaging detection system based on modulation. The light is assumed monochromatic for the sake of simplicity.

The intensity of the partially polarized light is denoted as $I_{total}$ which propagates along the direction of $z$ axis. The average frequency of $I_{total}$ is denoted as $v$. And the electric field $E$ can be divided into two orthogonal plane waves $E_x(t)$ and $E_y(t)$ in the direction of $x$- and $y$-axis

\[
E_x(t) = a_x(t)e^{i[\phi_x(t) - \omega t]} \quad (2.25)
\]

\[
E_y(t) = a_y(t)e^{i[\phi_y(t) - \omega t]} \quad (2.26)
\]

where $a_x(t)$ and $a_y(t)$ denote the amplitudes of electric field components in $x$- and $y$-direction while $\phi_x(t)$ and $\phi_y(t)$ are the phases. $E_x(t)$ and $E_y(t)$ go through a retarder with angle $\phi$ and polarizer of angle $\theta$, then the electrical field component of emergent light is

\[
E(t; \theta, \phi) = E_x \cos \theta + E_y e^{i\phi} \sin \theta \quad (2.27)
\]

So the intensity of emergent light is $I(\theta, \phi)$:

\[
I(\theta, \phi) = \langle E(t; \theta, \phi), E^*(t; \theta, \phi) \rangle = J_{xx} \cos^2 \theta + J_{yy} \sin^2 \theta + J_{xy} e^{-i\phi} \cos \theta \sin \theta + J_{yx} e^{i\phi} \sin \theta \cos \theta \quad (2.28)
\]
where the coherence matrix $J$ is given by

$$J = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = \begin{bmatrix} \langle E_x, E_x^* \rangle & \langle E_x, E_y^* \rangle \\ \langle E_y, E_x^* \rangle & \langle E_y, E_y^* \rangle \end{bmatrix}$$

(2.29)

$$= \begin{bmatrix} a_1^2 & a_1a_2e^{i(\phi_1-\phi_2)} \\ a_1a_2e^{-i(\phi_1-\phi_2)} & a_2^2 \end{bmatrix}$$

Off-diagonal elements of the coherence matrix $J$ are complex-valued, so the intensity value $I(\theta, \varepsilon)$ can be expressed as:

$$I(\theta, \varepsilon) = J_{xx} \cos^2 \theta + J_{yy} \sin^2 \theta + 2 \sqrt{J_{xx}J_{yy}} \cos \theta \sin \theta |J_{xy}| \cos(\beta_{xy} - \varepsilon) \quad (2.30)$$

where $\beta_{xy}$ denotes the phase angle for $J_{xy}$. The four elements $J_{xx}$, $J_{yy}$, $J_{xy}$, and $J_{yx}$ in coherence matrix $J$ are determined by taking different angles of $\{\theta, \varepsilon\}$ for retarder and polarizer during image acquisition process. For example, if the angles of $\{0^\circ, 0\}, \{45^\circ, 0\}, \{90^\circ, 0\}, \{135^\circ, 0\}, \{45^\circ, \pi/2\}$, and $\{135^\circ, \pi/2\}$ are substituted for $\{\theta, \varepsilon\}$, the four elements are determined by

$$J_{xx} = I(0^\circ, 0) \quad (2.31)$$

$$J_{yy} = I(90^\circ, 0) \quad (2.32)$$

$$J_{xy} = \frac{1}{2} \{I(45^\circ, 0) - I(135^\circ, 0)\} + i\frac{1}{2} \{I(45^\circ, \pi/2) - I(135^\circ, \pi/2)\} \quad (2.33)$$

$$J_{yx} = \frac{1}{2} \{I(45^\circ, 0) - I(135^\circ, 0)\} + i\frac{1}{2} \{I(45^\circ, \pi/2) - I(135^\circ, \pi/2)\} \quad (2.34)$$

Equations (2.31)–(2.34) show that the light wave can be described by the four elements of the coherent matrix $J$. Stokes proposed the four parameters to describe polarization properties of a light. According to the definition of Stokes parameters and (2.29) along with the expression in (2.31)–(2.34), Stokes parameter expressions are:

$$s_0 = \langle a_1^2 \rangle + \langle a_2^2 \rangle = J_{xx} + J_{yy} = I(0^\circ, 0) + I(90^\circ, 0) \quad (2.35)$$

$$s_1 = \langle a_1^2 \rangle - \langle a_2^2 \rangle = J_{xx} - J_{yy} = I(0^\circ, 0) - I(90^\circ, 0) \quad (2.36)$$

$$s_2 = 2\langle a_1a_2 \cos \delta \rangle = J_{xy} + J_{yx} = I(45^\circ, 0) - I(135^\circ, 0) \quad (2.37)$$

$$s_3 = 2\langle a_1a_2 \sin \delta \rangle = i(J_{yx} - J_{xy}) = I(45^\circ, \pi/2) - I(135^\circ, \pi/2) \quad (2.38)$$
where $\delta = \phi_1 - \phi_2$. Partially polarized light can be decomposed into independent, not polarized and complete, polarized light. The Stokes parameters of partially polarized light can be expressed using the cumulative Stokes parameter of independent not polarized and complete polarized light. So the Stokes vector $S$ of partially polarized light is

$$S = S^{(1)} + S^{(2)}$$  \hspace{1cm} (2.39)

where $S^{(1)}$ and $S^{(2)}$ denote the Stokes vector of non-polarized and polarized light.

The degree of polarization (DoP) is defined as:

$$\text{DoP} = \frac{s_1^2 + s_2^2 + s_3^2}{s_0}$$  \hspace{1cm} (2.40)

And the polarization ellipticity is defined as

$$\beta = \tan^{-1} \frac{s_3}{\sqrt{s_1^2 + s_2^2}}$$  \hspace{1cm} (2.41)

A vast majority of circularly polarized components in the range of instrument detection can be ignored, therefore, in most polarization imagers, $s_3 \approx 0$ [2, 3]. For linearly polarized light, Stokes vectors can be changed to the degree of linear polarization (DoLP), polarization angle (Orient) and other parameters:

$$\text{DoLP} = \frac{s_1^2 + s_2^2}{s_0}$$  \hspace{1cm} (2.42)

$$\text{Orient} = \frac{1}{2} \tan^{-1} \left( \frac{s_2}{s_1} \right)$$  \hspace{1cm} (2.43)

### 2.3 Polarization Imaging for Object Recognition

Polarization of the light provides a way to characterize physical properties of an object of different materials. A target produces polarization characteristics in the process of reflection and radiation on earth’s surface as well as in the atmosphere. Therefore, polarization characteristics provide useful information for detection, tracking, and recognition of a target. Polarization gives a great benefit to improve the signal-to-clutter ratio, speed and precision of edge detection, accuracy and reliability of target recognition in the process of target detection [4, 5]. In 1980s, the researchers began research on polarization imaging in military applications, which involved detection and tracking of military vehicles, detection and recognition of
concealed and camouflaged targets, detection of shallow underground and underwater mines, submarine, and reef.

2.3.1 Object Detection and Tracking

The surface of an artificial target, on the ground or in the atmosphere, always reflects and radiates some electromagnetic waves. We may receive electromagnetic waves with a large degree of linear polarization of reflection and radiation in a certain point of view. Comparing with that of artificial objects, the surface of natural objects tends to be rough, so the degree of linear polarization is nearly zero. From the observation of such differences, polarization imagers can improve the contrast ratio of a target and the background. Artificial targets are often brighter than natural objects in DoLP and Orient images, which provide a great clue for target detection. Filippidis et al. [6] used polarization imagers in the application of preventing antipersonnel and vehicle mine, shallow ground or underwater, and achieved good results. Katkovaky et al. [7] applied polarization images in the detection of man-made targets in the natural scenes, which greatly reduced required time for detection and improved detection accuracy. Military targets under the cover of natural objects can be detected using different polarization properties of electromagnetic waves that natural and man-made objects have. Polarization imagers are also applied for the detection of underwater reef, surface or underwater naval vessels, and to track man-made targets such as automobiles or aircrafts [8].

2.3.2 Edge Detection

According to the Fresnel’s polarized reflection model [3, 5] (Figs. 2.4 and 2.5), unpolarized lights have equal amplitude components in an arbitrary direction. After the specular reflection of insulator, the electromagnetic wave perpendicular to the incident plane $F_\perp$ (the dotted line plane in Fig. 2.5) has a much higher amplitude of polarization component $I_\perp$ than $I_\parallel$, while $I_\parallel$ is the amplitude of polarization

![Fig. 2.4](image-url) Light reflection on the surface of material
component, produced by electromagnetic wave which is parallel to the incident plane $F_{||}$. The reason for this difference is that the reflection coefficient on plane $F_{\perp}$ is higher. While the polarization components of electromagnetic wave have the relationship that $I_{||} > I_{\perp}$, which is reflected as a $90^\circ$ difference of the phase angle between the surface of specular reflection and diffuse reflection in the polarization angle images. According to this difference, we can detect the edge of the surface of specular reflection and diffuse reflection. The maximum amplitude of electromagnetic wave $I_{\max}$ can be observed in the direction which is perpendicular to the plane of incidence after specular reflection, and the maximum intensity of electromagnetic wave $I_{\max}$ can be observed in the direction which is parallel to the incidence plane after diffuse reflection. If $I_d$ and $I_s$ represent the intensity of diffuse reflection and specular reflection components, $I_{||}$ and $I_{\perp}$ can be expressed in (2.44) and (2.45).

$$\frac{I_d}{2} + \left[ \frac{F_{\perp}}{F_{||} + F_{\perp}} \right] I_s = I_{\perp} \quad (2.44)$$

$$\frac{I_d}{2} + \left[ \frac{F_{||}}{F_{||} + F_{\perp}} \right] I_s = I_{||} \quad (2.45)$$

For a polarized light parallel to incident plane, the refractive index is given by

$$r_{||}(n_i, \eta, \theta_i, \theta_r) = \frac{E_{r||}(n_i, \eta, \theta_i, \theta_r)}{E_{i||}} = \frac{\eta \cos \theta_i - n_i \cos \theta_r}{\eta \cos \theta_i + n_i \cos \theta_r} \quad (2.46)$$

A polarized light is perpendicular to the incidence plane:

$$r_{\perp}(n_i, \eta, \theta_i, \theta_r) = \frac{E_{r\perp}(n_i, \eta, \theta_i, \theta_r)}{E_{i\perp}} = \frac{n_i \cos \theta_i - \eta \cos \theta_r}{n_i \cos \theta_i + \eta \cos \theta_r} \quad (2.47)$$

**Fig. 2.5** Specular reflection.
where $\theta_i$ denotes the incident angle and $\theta_r$ is the output angle. $E_\perp$ is the polarization component perpendicular to the incident plane and $E_\parallel$ is parallel with the incident plane. The relationship between $\theta_i$ and $\theta_r$ is determined by Snell’s law.

$$n_i \sin \theta_i = c \sqrt{\left( \varepsilon - i \frac{\sigma}{\omega} \right)} \mu \sin \theta_r = (n - ik) \sin \theta_r = \eta \sin \theta_r \quad (2.48)$$

The refraction coefficient of the air is $n_i = 1$, $\varepsilon$ is the dielectric constant of the object, $\omega$ is the frequency of light, $\sigma$ is the conductivity on the surface of object, $\mu$ is the magnetic conductivity, $n$ is the single refractive coefficient and $k$ is the extinction coefficient. The total intensity of incident light is defined as

$$I_{total}(\lambda, \eta, \theta_i) = I_{min}(\lambda, \eta, \theta_i) + I_{max}(\lambda, \eta, \theta_i) \quad (2.49)$$

where

$$I_{max}(\lambda, \eta, \theta_i) = \frac{1}{2} I_d + \frac{r_1^2(\lambda, \eta, \theta_i)}{r_1^2(\lambda, \eta, \theta_i) + r_\parallel^2(\lambda, \eta, \theta_i)} I_s \quad (2.50)$$

$$I_{min}(\lambda, \eta, \theta_i) = \frac{1}{2} I_d + \frac{r_\parallel^2(\lambda, \eta, \theta_i)}{r_1^2(\lambda, \eta, \theta_i) + r_\parallel^2(\lambda, \eta, \theta_i)} I_s \quad (2.51)$$

According to this rule, we can acquire good results by detecting and extracting the surface of different insulator in the scene [3], and the polarization characteristics of the edge detection method can also be used in the scene with multiple targets.

### 2.3.3 Object Classification and Recognition

In general, a matter can be classified as either conductor or insulator according to its conductive properties. Following the Fresnel reflection model, the incident light is assumed to be unpolarized and its diffuse reflectance and specular reflection components are related to Fresnel reflection coefficient after the reflection on the surface of metal or insulator. While there is a direct relationship between the Fresnel reflection coefficient and the properties of a matter like the dielectric constant and magnetic constant, therefore, the conductive property of object can be indirectly determined through the Fresnel reflection coefficient, and then it can be classified as conductor or insulator [3–5]. The majority of specular reflection angles represent strong partial polarization characteristics after the reflection on the surface of insulators. With the enhancement of conductivity on the surface of material, the partial polarization characteristics will be weakened after specular reflection. For the insulator, there is only certain wavelengths of light can be absorbed after diffuse reflection, while as for the metal, nearly all of the light is absorbed and only a very
small amount reflects out of surface. So the specular reflection mostly happens on metal surface that explains metal surface appears to be shiny. According to (2.24) and (2.25), the degree of linear polarization is defined as

$$\text{DoLP} = \frac{I_{\text{max}}(\lambda, \eta, \theta_i) - I_{\text{min}}(\lambda, \eta, \theta_i)}{I_{\text{max}}(\lambda, \eta, \theta_i) + I_{\text{min}}(\lambda, \eta, \theta_i)}$$

$$= \frac{r_{\parallel}^2(\lambda, \eta, \theta_i) - r_{\perp}^2(\lambda, \eta, \theta_i)}{r_{\parallel}^2(\lambda, \eta, \theta_i) + r_{\perp}^2(\lambda, \eta, \theta_i)} \cdot \frac{1}{1 + I_d/I_s}$$  \hspace{1cm} (2.52)

And the polarization angle is

$$\text{Orient} = \frac{1}{2} \tan^{-1} \left( \cos(\delta) \tan(2\alpha) \right)$$ \hspace{1cm} (2.53)

where $\tan \alpha = E_{oy}/E_{ox}$, $\delta$ is the relative phase. $E_{oy}$ and $E_{ox}$ are the maximum amplitude in the x- and y-axis. From (2.52) and (2.53), we know that the degree of linear polarization and polarization angle are closely related with material dielectric constant and incident angle. Figures 2.6 and 2.7 show that how the degree of polarization and polarization angle change with the incident angle on the surface of three types of materials, glass ($n = 1.89$), iron ($n - ik = 1.51 - i1.63$, $\lambda = 0.589 \mu m)$ and copper ($n - ik = 0.82 - i5.99$, $\lambda = 0.65 \mu m$). DoLP and Orient changes with incident angle, but when the incident angle is fixed, they are shown obvious differences on the surface of metal and insulator [5].

According to the Fresnel’s reflection model, approximate Fresnel coefficient is calculated using the maximum value $I_{\text{max}}$ and the minimum radiation value $I_{\text{min}}$ of

![Fig. 2.6 The degree of linear polarization changes with incident angle](image-url)
the reflected light which comes out of a polarizer to obtain the information that the object surface is insulator or metal [3–5]. A sharp decrease (or increase) of the material conductivity will lead to an increase (or decrease) of $I_{\text{max}}/I_{\text{min}}$ at a certain angle range of specular reflection, so metal or insulator can easily be determined. However, the material classification method based on the Fresnel reflection model requires mainly specular reflection and a perfect light source, otherwise it may cause a large error [5]. Since metal produces phase delay of light waves, but not insulators, another classification method uses reflected light’s phase delay to metal and insulator. This method is more robust and accurate than the method using the ratio $I_{\text{max}}/I_{\text{min}}$, and easy to classify the objects. The polarization angle image is seriously affected by the noise. Classification of materials based on polarization characteristics has good classification performance to be widely applied to the recognition of concealed military targets, mineral substance hidden in the shallow surface or under the water, and cultural relics.

### 2.4 Factors Affecting Polarization Imaging

In polarization imaging detection systems, polarization parameter images usually have large errors due to the noise, non-ideal characteristics of polarization devices, and partial polarization characteristics of incident light. This section simplifies the analysis on the influence of non-ideal polarization properties and different weather conditions on the acquired polarization images.

Fig. 2.7 The polarization angle changes with incident angle
2.4.1 Transmission of Polarized Light

Equations (2.35)–(2.37) are used to calculate Stokes parameters $s_0$, $s_1$, $s_2$, and $s_3$. $I(0^\circ, 0)$, $I(45^\circ, 0)$, $I(90^\circ, 0)$, and $I(135^\circ, 0)$ are the quantity of photoelectron, which is obtained by the beam going through the polarizer in the directions of $0^\circ$, $45^\circ$, $90^\circ$, and $135^\circ$. Figure 2.8 shows that an incident light can be decomposed into two components, parallel (P-polarized light) and perpendicular (S-polarized light) to the incident plane. P-polarized light is incident linearly polarized light with polarization direction lying in the plane of incidence. S-polarized light has polarization perpendicular to the plane of incidence. The strength of reflection from the surface is determined by the Fresnel Eqs. (2.19)–(2.22), which are different for S- and P-polarized light. Any light striking a surface at a special angle of incidence known as Brewster’s angle, where the reflection coefficient for P-polarization is zero, will be reflected with only the S-polarization remaining. This principle is employed in the so-called “pile of plates polarizer” in which part of the S-polarization is removed by reflection at each Brewster angle surface, leaving only the P-polarization after transmission through many such surfaces.

The polarizer is considered having ideal transfer characteristics, $t_p = 0$ and $t_s = 1$. Since $t_p \neq 0$ and $t_s \neq 1$ in reality, however, the Stokes parameters are represented by

$$s'_0 = \frac{I'(0^\circ, 0) + I'(90^\circ, 0)}{t_s + t_p}$$  \hspace{1cm} (2.54) \\
$$s'_1 = \frac{I'(0^\circ, 0) - I'(90^\circ, 0)}{t_s - t_p}$$  \hspace{1cm} (2.55) \\
$$s'_2 = \frac{I'(45^\circ, 0) - I'(135^\circ, 0)}{t_s - t_p}$$  \hspace{1cm} (2.56)

Fig. 2.8 Two components of incident light
The degree of linear polarization $\text{DoLP}'$ and the polarization azimuth angle $\text{Orient}'$ are given by:

\[
\text{DoLP}' = \sqrt{s'_1^2 + s'_2^2} / s'_0
\]  
(2.57)

\[
\text{Orient}' = \frac{1}{2} \tan^{-1} \left( \frac{s'_2}{s'_1} \right)
\]  
(2.58)

To make a reasonable evaluation of the effect on polarization parameter image caused by polarizer’s non-ideal characteristics, acquired polarization parameter images without the effect of $t_p$ and $t_s$ are assumed to be a noisy image. On the other hand, polarization parameter images ($s'_0, s'_1, s'_2, \text{DoLP}', \text{and Orient}'$) with the effect of $t_p$ and $t_s$ are signal images. The noise-to-signal ratio (NSR) is defined as

\[
\text{NSR} = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (I(i,j) - \hat{I}(i,j))^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} I^2(i,j)}
\]  
(2.59)

where $M$ and $N$ are the size of the image. In the simulation, the parameters were set to $t_p = 96\%$, $t_s = 1\%$, $M = 260$, and $N = 280$ (Fig. 2.9).

Comparing Figs. 2.10 and 2.11, the Stokes parameter images, the DoLP’ image and the Orient’ image are almost of no change. At the same time, we can find that the NSR value of these three images is fixed in any scenes by analyzing (2.59).

![Fig. 2.9 Images collected in different polarization angles. a $I'(0^\circ, 0)$, b $I'(45^\circ, 0)$, c $I'(90^\circ, 0)$, d $I'(135^\circ, 0)$](image)

![Fig. 2.10 Parameter images computed using (2.35)–(2.37), (2.42), and (2.43). a $s_0$, b $s_1$, c $s_2$, d DoLP, e Orient](image)
The NSR value of parameter images is only related to the polarizer’s own characteristics and has nothing to do with the scene information. Table 2.1 shows that the NSR value of Orient image is zero and the values of \( s_0 \), \( s_1 \), \( s_2 \), and DoLP' are small, while the corresponding NSR value decreases with the improvement of permeability. Consequently, the calculation of Stokes parameter images, the DoLP' and Orient' can be done using (2.35)–(2.42), which indicates that the polarizer has ideal characteristics, and the affection of \( t_p \) and \( t_s \) should be ignored unless high accurate polarization images are needed.

2.4.2 Lighting

Under normal circumstances, the incident light is assumed to be non-polarized and will be changed to partially polarized light after reflection from the target. Non-polarized light changes into linearly polarized light after the reflection of objects with smooth surface but it will remain non-polarized when reflected from objects with rough surface. This property is used to improve the image contrast between artificial targets and natural targets in polarization images. From Sect. 2.3.2, it is easy to obtain the edge information of artificial targets in Orient images. Natural light is considered non-polarized, artificial targets can be detected using polarization imaging. In reality the sunlight becomes partially polarized after scattering through the atmosphere because of the suspended particles and water molecules in the air, which will have noise interference on the polarization images, especially Orient images, and bring more difficulty to the calibration of target and edge detection based on the polarization image. On the other hand, the polarization images may have a certain difference in different weather and light conditions because the suspended particles and water in the air vary with the weather. The degree of polarization is the strongest in clear day while there is almost no polarization in cloudy day.
2.5 Reduction of Measurement Errors

From (2.35)–(2.42), the measurement error and noise in image \( I(\theta, 0) \) will transfer to the Stokes parameter images, the DoLP and Orient images and bring more errors to the subsequent target detection and recognition. How to reduce measurement error and noise of Stokes parameter images, DoLP and Orient, is the key to the success in the application of polarization imaging. Previous study demonstrated that we can combine multiple angle measurements, image pre-filtering and image fusion to reduce the effect on parameter images caused by the measurement error and noise in the image \( I(\theta, 0) \) [5].

### 2.5.1 Multi-angle Measurement

A detailed description of Stokes parameters can be found by changing polarization angles continuously. The error caused by beam migration and the error caused by transparent inhomogeneity of light decrease as polarization angle changes. From (2.30)–(2.38),

\[
I(\theta, 0) = s_0 + s_1 \cos(2\theta) + s_2 \sin(2\theta)
\]

Collect \( N \) images in \( N \) polarization angles, \( \theta_j, j = 0, \ldots, N - 1 \) from 0° to 360°, then calculate the Stokes-Muller polarization parameters \( \tilde{s}_0, \tilde{s}_1, \tilde{s}_2 \):

\[
\tilde{s}_0 = \frac{1}{N} \sum_{j=0}^{N-1} I(\theta_j, 0)
\]

\[
\tilde{s}_1 = \frac{2}{N} \sum_{j=0}^{N-1} I(\theta_j, 0) \cos(2\theta_j)
\]

\[
\tilde{s}_2 = \frac{2}{N} \sum_{j=0}^{N-1} I(\theta_j, 0) \sin(2\theta_j)
\]

where \( \theta_j = \frac{2\pi j}{N} \). The degree of polarization and polarization phase are:

\[
\text{DoLP} = \frac{\sqrt{\tilde{s}_1^2 + \tilde{s}_2^2}}{\tilde{s}_0}
\]

\[
\text{Orient} = \frac{1}{2} \tan^{-1} \left( \frac{\tilde{s}_2}{\tilde{s}_1} \right)
\]
Comparing \((2.61) - (2.63)\) to \((2.35) - (2.37)\), the Stokes parameters are only a difference of a certain constant coefficient when \(\theta_j = 0^\circ, 45^\circ, 90^\circ, 135^\circ\). Figures 2.12 and 2.13 show that the Stokes parameter images, DoLP, and Orient, using \((2.61) - (2.63)\) and \((2.35) - (2.37)\) and twelve \(I(\theta, 0)\) images calculated at every 30° from 0° to 360° (Table 2.2).

Comparing the maximum DoLP and maximum Orient calculated by the above two methods, we can find that the DoLP and Orient are maximum when \(N = 12\). Analysis can be seen in Table 2.3 that the entropy of Stokes parameter images, DoLP image, and Orient image increase with multi-angle measurement. The information of every polarization parameter image increases.

### Table 2.2 Maximum values of DoLP and Orient

<table>
<thead>
<tr>
<th></th>
<th>DoLP (%)</th>
<th>Orient (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Angle measurement</td>
<td>78.85</td>
<td>2.000</td>
</tr>
<tr>
<td>12-Angle measurement</td>
<td>81</td>
<td>2.1078</td>
</tr>
</tbody>
</table>

### Table 2.3 The entropy of Stokes parameters, DoLP, and Orient images obtained

<table>
<thead>
<tr>
<th></th>
<th>(s_0)</th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>DoLP</th>
<th>Orient</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Angle measurement</td>
<td>0.0218</td>
<td>0.0490</td>
<td>0.4794</td>
<td>0.9645</td>
<td>0.6619</td>
</tr>
<tr>
<td>12-Angle measurement</td>
<td>0.0311</td>
<td>1.3889</td>
<td>1.0470</td>
<td>1.5272</td>
<td>0.8314</td>
</tr>
</tbody>
</table>
2.5.2 Image Filtering

Multi-angle measurements reduce the error caused by non-ideal characteristics of polarizers and other optical devices. However, the noise in image \( I(\theta, 0) \) caused by sensitive components and external environment may not be completely eliminated using (2.61)–(2.63). From the equations for the DoLP and Orient, small noise in image \( I(\theta, 0) \) will be transferred to the Stokes parameter images using (2.35)–(2.37). The noise will be present in the DoLP and Orient images. Therefore it is necessary to filter out image \( I(\theta, 0) \) before calculating the Stokes images. The relative Stokes image and polarization parameter images should be calculated to prevent the amplification of noise in \( I(\theta, 0) \) and the effect on the later application of polarization image.

A pre-filtering method in the process of calculating polarization parameter image is:

1. Use a rotating polarizer to collect image \( I(\theta, 0) \)
2. Filter the collected image to remove background interference. The choice of specific image filtering algorithm is determined by many factors such as the corresponding characteristics of imaging devices and specific features of external interferences.
3. Calculate Stokes parameter images using the images \( I(\theta, 0) \) after filtering

2.5.3 Image Fusion

Calculation of Stokes parameter images, DoLP, and Orient images is a fusion process. We can calculate Stokes parameter image, DoLP, and Orient images using the frame based on image fusion to improve the performance. Image filtering can also be combined with image fusion. We have proposed a calculation method of Stokes parameter images based on the adaptive non-subsampled improvement frame, which can keep abundant detailed information and make calculation process insensitive to a tiny translation of the scene as well as rapid [9–11]. Here is the step of this method.

1. Pre-filter the image \( I(\theta, 0) \) which has been collect before.
2. Confirm the wavelet decomposition layers. The number of layers is determined according to the rotation speed of polarizer, the stability of acquisition platform and the relative movement of scene. If the rotation speed of polarizer is high (such as the use of electrically tunable polarizer, etc.), the acquisition platform is relatively stable and the relative movement of scene is slow, the decomposition layers of image \( I(\theta, 0) \) can be less; otherwise, the decomposition layers of image \( I(\theta, 0) \) will be more to satisfy the high-demanding Stokes parameter image.
(3) Decomposed image $I(\theta, 0)$ using the improvement framework without down-sampling. The wavelet decomposition coefficient at position $(x, y)$ in the $k$th layer and $l$th direction is $I(\theta, 0, x, y, k, l)$.

$$I(\theta_j, 0, k, l) = \text{DWT}\{I(\theta_j, 0)\} \quad (2.66)$$

(4) Perform (2.61)–(2.63) to get the corresponding fusion coefficient in each layer on the wavelet coefficients of the image, here $N$ represents the polarization angles:

$$\tilde{s}_0 = (x, y, k, l) = \frac{1}{N} \sum_{j=1}^{N} I(\theta_j, 0, x, y, k, l) \quad (2.67)$$

$$\tilde{s}_1 = (x, y, k, l) = \frac{2}{N} \sum_{j=1}^{N} I(\theta_j, 0, x, y, k, l) \cos(2\theta_j) \quad (2.68)$$

$$\tilde{s}_2(x, y, k, l) = \frac{2}{N} \sum_{j=1}^{N} I(\theta_j, 0, x, y, k, l) \sin(2\theta_j) \quad (2.69)$$

Reconstruct the fusion coefficients $\tilde{s}_0(x, y, k, l), \tilde{s}_1(x, y, k, l), \tilde{s}_2(x, y, k, l)$ to obtain corresponding Stokes parameter images using the inverse discrete wavelet transforms (IDWT),

$$\tilde{s}_i = \text{IDWT}\{\tilde{s}_i(x, y, k, l)\}, \quad i = 0, 1, 2 \quad (2.70)$$

Calculate the appropriate evaluation to confirm the performance of fusion results. If the evaluation satisfies with the requirements, stop and exit. Otherwise, return to step (2). We assume that there is only a certain degree of translation caused by device vibration and scene motion among the image $I_0, I_{45}, I_{90}$ and $I_{135}$ while the rotation is ignored, then transfer the image $I_0, I_{45}, I_{90}$ and $I_{135}$ for 3–4 pixels both in horizontal and vertical direction, the result is used to calculate Stokes parameter image, linear polarization and polarization phase images by (2.35)–(2.37). From Figs. 2.14, 2.15, 2.16, 2.17 and 2.18, we can find that the edge of images obtained by fusion is clearer than those calculated directly by using (2.61)–(2.63), which indicate that the fusion algorithm have better performance in preserving image details and tolerating any small translation.

In order to analyze the results in Figs. 2.14, 2.15, 2.16, 2.17 and 2.18 both qualitatively and quantitatively, we compare the difference of Stokes parameter image, the DoLP and Orient images between the two methods above, whose result is shown in Table 2.4 using four parameters: average, variance, entropy and boundary energy. Comparison can be seen in Table 2.4 that the values of entropy and boundary energy in the images obtained by fusion are higher than others.
The former shows that fusion increases the amount of information and the obvious change of latter proves that images which are obtained by fusion have more salient edge information and better effect. Meanwhile, the fusion algorithm also achieves

**Fig. 2.14** Parameter image $s_0$, a obtained by fusion and b obtained by directly calculation

**Fig. 2.15** Parameter image $s_1$, a obtained by fusion and b obtained by directly calculation

**Fig. 2.16** Parameter image $s_2$, a obtained by fusion and b obtained by directly calculation
better visual performance. Therefore, the fusion algorithm is proved to provide great convenience on calculating high-quality Stokes parameter, the degree of linear polarization and polarization angle images.

Table 2.4  Statistical comparisons of translating both horizontally and vertically

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Variance</th>
<th>Entropy</th>
<th>Average boundary energy</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct calculation</td>
<td>101.0211</td>
<td>87.7710</td>
<td>4.9826</td>
<td>29.6</td>
<td>0.0042</td>
</tr>
<tr>
<td>Image fusion</td>
<td>97.6721</td>
<td>80.6574</td>
<td>5.7120</td>
<td>40.1</td>
<td>0.0142</td>
</tr>
</tbody>
</table>

Fig. 2.17  Parameter image DoLP, a obtained by fusion and b obtained by directly calculation

Fig. 2.18  Parameter image Orient, a obtained by fusion and b obtained by directly calculation
2.6 Micro-polarizer Array Imaging

Polarization imaging devices described in the previous sections require multi-angle polarization by mechanical rotation or electronic tuning. However, loss of geometric features on a moving platform is serious. Addition of polarizers to imaging sensor reduces the energy to reach the focal plane so that obtained images demonstrate serious noise interference. Comparing with the spectral characteristics, the polarization characteristics of the surface features are not obvious. High sensitivity of polarization detectors is required to take advantage of polarization characteristics effectively. These factors also bring difficulties in the application of polarization imaging detection. On the other hand, they also contribute to the emergence of new and more efficient polarization imaging devices to improve the lack of polarization imaging devices and make a wide use of polarization imaging technology.

In recent years, more practical strategies have been developed, including division of focal plane (DoFP) devices [1]. DoFP devices, also called microgrid polarimeters, provide several advantages over traditional polarimeters. Microgrid polarimeters capture images by incorporating a pixel-wise micro-polarizer array (MPA) aligned to a focal plane array (FPA), as shown in Fig. 2.19. Such imagers are akin to the Bayer pattern of color imaging sensors, except that the pattern has different polarization states, not the primary colors. This allows an MPA image to contain a full set of polarization intensity measurements to reconstruct the Stokes vector at each region of the image. Acquired images in this way are called polarization mosaic images. As additional advantages, microgrids are rugged and compact, making them ideal for real-time video imaging applications.

2.6.1 Micro-polarizer Array

Both the design and performance characteristics of the MPA are essentially determined by the calculation method of the Stokes vector calculation ways and the
arrangements of the polarization filters in the MPA. These two basic MPA features specify the construction requirements and cost.

(A) Basic Theory of MPA

Stokes vector has been efficient in the representation of polarization states of a light. Most polarization imaging sensors are designed to acquire the Stokes vector images indirectly. As shown in Fig. 2.20a–c, four micro-polarizers of $0^\circ$, $45^\circ$, $90^\circ$ and $135^\circ$ are placed in a repeated $2 \times 2$ sub-array in three MPA models [12–14]. And the fourth MPA model in Fig. 2.20d consists of three micro-polarizers and one full pass cell placed in a repeated $2 \times 2$ sub-array [15]. With one full pass cell added, this model shows high sensitivity of light with small polarization artifacts. Gruev et al. [12] has given the formula to eliminate Stokes parameters using $I(0)$, $I(45)$ and $I(t)$ according to the principles of optical polarization:

\[
S_0 = I(t) \tag{2.71}
\]
\[
S_1 = 2I(0) - I(t) \tag{2.72}
\]
\[
S_2 = 2I(45) - I(t) \tag{2.73}
\]

where $I(t)$ denotes measured intensity through isotropic filter that passes through all the states equally. Based on (2.71)–(2.73), MPA model 5 is obtained with two micro-polarizers and two full pass filter placed in a repeated $2 \times 2$ sub-array, which is shown in Fig. 2.20e. With two full pass filters added, this model has higher light sensitivity and more polarization artifacts. Besides the MPA models, a repeated $2 \times 4$ sub-array is designed to provide better special resolution. While the MPA models designed by authors in [16] is shown in Fig. 2.20g, h.
(B) Matrix Representation of MPA in the Spatial Domain

Let $h_{\text{MPA}}(x, y)$ represent the output of an MPA sensor, sampled on a rectangular sampling lattice $D$. The horizontal and vertical sample spacing are equal, and this sample spacing is used as the unit length, called the pixel height. Assume that there exist underlying $I(0), I(45), I(90)$ and $I(135)$ component signals that need to be estimated. The MPA signal is obtained by subsampling and adding the four subsampled signals. The subsampling can be represented as a multiplication by subsampling functions that take the value 1 on and zero otherwise. A MPA $h_{\text{MPA}}(x, y)$ is usually a periodic tiling of a much smaller array (a red region in Fig. 2.20), called the MPA pattern $h_p(x, y)$. Using the model 1 in Fig. 2.20, a MPA pattern can be decomposed into four primary MPA patterns $h^d_p(x, y)$, each accounting for one polarization direction $d$. Then symbolically we can write:

$$h_p = \sum_D h^d_p(D)$$  \hspace{1cm} (2.74)

To ensure the same dynamic range of the sensed image at all pixels, the sum of all primary MPA patterns should be a matrix with all 1’s:

$$\sum_D h^d_p(x, y) = 1 \quad \forall x, y$$  \hspace{1cm} (2.75)

For the pattern $[I(0)I(45); I(135)I(90)]$ in Fig. 2.19a can be represented as:

$$h^0_p = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad h^{45}_p = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$h^{90}_p = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad h^{135}_p = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$  \hspace{1cm} (2.76)

(C) Frequency Structure Matrix of MPA

Inspired by the observed patterns of the spectra of MPA filtered images, we propose to represent the spectra by faithfully recording the frequency components: their frequency details and their positions. Such information can naturally be arranged in a matrix form. Inspired by the color filter array’s frequency structure matrix [17], the polarization frequency structure matrix is given. Accordingly, the MPA filtered image $f(x, y)$ is composed of $f^\theta(x, y)$ images, where $\theta \in \{0^\circ, 45^\circ, 90^\circ, 135^\circ\}$. Then it can be represented as:

$$f(x, y) = \sum_\theta f^\theta(x, y)h^\theta_{\text{MPA}}(x, y)$$  \hspace{1cm} (2.77)
where \( h_{\text{MPA}}^\theta \) is the corresponding MPA of direction \( \theta \) defined as the periodic replica of primary MPA pattern \( h_p^\theta : h_{\text{MPA}}^\theta(x,y) = h_p^\theta(x \mod n_x, y \mod n_y) \). With straightforward deduction, the DFT of \( f(x,y) \) can be computed by:

\[
F_{\text{MPA}}(\omega_x, \omega_y) = \sum_{k_x=0}^{n_x-1} \sum_{k_y=0}^{n_y-1} \left\{ \sum_\theta H_p^\theta \left( \frac{n_x}{k_x}, \frac{n_y}{k_y} \right) \cdot F^\theta \left( \omega_x - \frac{n_x}{k_x}, \omega_y - \frac{n_y}{k_y} \right) \right\}
\] (2.78)

where \( H_p^\theta(\omega_x, \omega_y) = \text{DFT}(h_p^\theta(x,y)) \), \( F^\theta(\omega_x, \omega_y) = \text{DFT}(f^\theta(x,y)) \) and \( F^\theta(\omega_x - \frac{n_x}{k_x}, \omega_y - \frac{n_y}{k_y}) \) has been circularly shifted. Equation (2.76) implies that in the frequency domain the spectrum \( F_{\text{MPA}} \) is a multiplexing of \( n_x n_y \) frequency components centered at \( \left( \frac{n_x}{k_x}, \frac{n_y}{k_y} \right) \), and each component is the sum of the original spectra \( F^\theta \) weighted by the value of the spectrum of the MPA pattern at the corresponding frequency \( H_p^\theta \left( \frac{n_x}{k_x}, \frac{n_y}{k_y} \right) \). Based on definition of symbolic DFT, the one period matrix representation in the spatial domain can be represented as:

\[
\text{DFT} \begin{bmatrix} I_0 & I_{45} \\ I_{135} & I_{90} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} I_0 + I_{45} + I_{90} + I_{135} & I_0 - I_{45} - I_{90} - I_{135} \\ I_0 + I_{45} - I_{90} - I_{135} & I_0 - I_{45} + I_{90} - I_{135} \end{bmatrix} = \begin{bmatrix} FS'_0 & FS'_1 \\ FS'_2 & FS'_3 \end{bmatrix}
\] (2.79)

Similarly with the frequency spectrum of a CFA-filtered image, the baseband \((I_0 + I_{45} + I_{90} + I_{135})/4\) in MPA model 1 is called polarization intensity, and other three are named polarization chroma. The frequency spectrum of the parameter in (2.79) is shown in Fig. 2.21.

With a frequency structure, we can easily find a linear relationship between the polarization intensity and polarization chrominance components and the images of the primary polarization images as:

\[
\begin{bmatrix} FS_0 \\ FS'_1 \\ FS'_2 \\ FS'_3 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & -0.25 & 0.25 & -0.25 \\ 0.25 & 0.25 & -0.25 & -0.25 \\ 0.25 & -0.25 & 0.25 & -0.25 \end{bmatrix} \begin{bmatrix} I_0 \\ I_{45} \\ I_{90} \\ I_{135} \end{bmatrix} = T \begin{bmatrix} I_0 \\ I_{45} \\ I_{90} \\ I_{135} \end{bmatrix}
\] (2.80)

where \( T \) is called the multiplexing matrix, which gives the specific composition of each component multiplexed from the image spectra of the primary polarization images, \( I_0, I_{45}, I_{90}, \) and \( I_{135} \). The demosaicking matrix can be found from the inverse or pseudo-inverse of the multiplexing matrix: \( D = T^{-1} \) if \( T \) is invertible, or \( D = T^+ \) if \( T \) is not square. Then the polarization images can be obtained from the multiplexed components for demosaicking in (2.80):
Frequency structure offers a simple universal frequency domain demosaicking algorithm for all rectangular MPAs. Aiming at minimizing the demosaicking error, some desired characteristics of an optimal MPA can be obtained, which are the principles that guide our MPA design. With the help of frequency structure, we can easily follow the design principles and turn the MPA design into an optimization problem.

(D) MPA Design Guidelines

Observation of the frequency structures matrix and general image spectra shows that:

1. The fewer polarization components, the less frequency aliasing;
2. The farther the distances between polarization intensity and polarization chroma and between polarization chroma, the less frequency aliasing, the polarization chroma should not be located in the horizontal or vertical axes of the intensity;
3. Another factor that influences the accuracy of demosaicked images of the primary polarization angle is from the inverse transform from multiplexed components to the primary polarization angle: the less the norm of demosaicking matrix $D$, the better the demosaicking performance.

\[
\begin{bmatrix}
I_0 \\
I_{45} \\
I_{90} \\
I_{135}
\end{bmatrix}
= T^{-1}
\begin{bmatrix}
FS_0 \\
FS_1' \\
FS_2' \\
FS_3'
\end{bmatrix}
= D
\begin{bmatrix}
FS_0 \\
FS_1' \\
FS_2' \\
FS_3'
\end{bmatrix}
\]

Fig. 2.21 Magnitude spectra of parameter in (2.77): a Parameter $s_0'$. b Parameter $s_1'$. c Parameter $s_2'$. d Parameter $s_3'$.

40 2 Polarization Imaging
Besides the three requirements, for polarization infrared imaging system, some special requirement should be added:

1. Cost-effective image reconstruction;
2. Immunity to polarization artifacts;
3. High sensitivity to light

Based on the requirements listed above, we choose a good frequency structure for less aliasing. Especially, we determine the locations of all the polarization chroma for less aliasing among polarization intensity and polarization chroma and specify some relations between the chroma, some of which may be conjugate to each other. Balancing the design expectations, we choose a new $5 \times 5$ frequency structure and a $2 \times 4$ frequency structure for our MPA design, as shown in (2.82) and (2.83):

$$S_{\text{NewMPA1}} = \begin{bmatrix} F_{s_0} & 0 & 0 & 0 & 0 \\ 0 & 0 & F_{s_1} & 0 & 0 \\ 0 & 0 & 0 & F_{s_2} & 0 \\ 0 & F_{s_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{s_1} & 0 \end{bmatrix} \quad (2.82)$$

$$S_{\text{NewMPA2}} = \begin{bmatrix} F_{s_0} & 0 & F_{s_1} & 0 \\ 0 & F_{s_2} & 0 & F_{s_2} \end{bmatrix} \quad (2.83)$$

According to the above frequency structures, we propose two new MPAs, which denoted as Model 7 and Model 8, shown in Fig. 2.20g, h. To test the performance of designed MPA for small target detection, mosaic and demosaicked images of MPA Models 7 and 8. The small target is visible in the demosaicked images, as shown in Fig. 2.22.

2.6.2 Experiment Results

To determine the performance of the MPAs listed in Fig. 2.20, a number of test images have been utilized. The images used in the experiments are $320 \times 240$ images of a man on chair (Fig. 2.23a), cars (Fig. 2.23b), trees and a building (Fig. 2.23c), and water and a building (Fig. 2.23d). The test polarization images, which vary in both the complexity of the structural content (edges, fine details) and the polarization appearance, have been captured using polarization filter wheel and FLIR infrared imaging sensor. Following the evaluation procedure depicted in Fig. 2.24, tests were performed by sampling the original images with each of the MPAs shown in Fig. 2.20 to obtain a MPA image.

To evaluate the structural content performance of the considered MPAs (Fig. 2.20), image quality was measured by comparing the original infrared polarization image to the demosaicked image. To facilitate the objective
Fig. 2.22 Small target detection by using MPA model 7 and 8. a Infrared Intensity Image, b Mosaic image by model 7, c $S_0$ image calculated by using (b), d Mosaic image by model 8, e $S_0$ image calculated by using (d).

Fig. 2.23 Test images: a a man and chair, b cars, c trees and a building, d water and a building.

Fig. 2.24 The evaluation procedure.
comparisons, the polarization space based peak-signal-to-noise-ratio (PSNR) and polarization structural similarity (PSSIM) index, and the polarization feature-similarity (PFSIM) index criterion are used. Polarization PSNR can be used to evaluate the demosaicking quality. Structural information is to show that pixels with strong inter-dependencies are spatially close. These dependencies carry important information about the structure of the objects in the scene.

\[ PSNR_p = 20 \log_{10} \left( \frac{S_{\text{peak,p}}}{\sqrt{MSE_p}} \right) \]  

(2.84)

where \( S_{\text{peak,p}} \) is the average peak signal value at all polarization images. \( MSE_p \) is the average mean square error between the ground truth and the demosaicked images:

\[ MSE_p = \frac{1}{MN} \| IG - ID \|^2 \]  

(2.85)

where \( M \) and \( N \) represent the number of rows and columns in the super-resolved image, \( ID \) is the estimated demosaicked images and \( IG \) is the ground truth polarization images.

By considering image degradation as perceived changes in structural information, PSSIM is proposed to measure the similarity between the ground truth and the demosaicked images. The PSSIM metric is calculated on various windows of an image. The measure between \( ID \) and \( IG \) of a common size \( N \times N \) is [18]:

\[
SSIM(IG, ID) = \frac{(2\mu_G^T\mu_D + C_1)(2\sigma_{GD} + C_2)}{(\mu_G^T\mu_G + \mu_D^T\mu_D + C_1)(\sigma_G^2 + \sigma_D^2 + C_2)}
\]  

(2.86)

where \( \mu_G \) is the average of \( IG \), \( \mu_D \) is the average of \( ID \), \( \sigma_G \) is the variance of \( IG \), \( \sigma_D \) the variance of \( ID \), \( \sigma_{GD} \) the covariance of \( IG \) and \( ID \), \( C_1 = (k_1L)^2 \) and \( C_2 = (k_2L)^2 \) are two variables to stabilize the division with weak denominator, \( L \) is the dynamic range of the pixel-values, \( k_1 = 0.01 \) and \( k_2 = 0.03 \) by default. The PFSIM metric between \( ID \) and \( IG \) is defined as [19]:

\[
FSIM(IG, ID) = \frac{\sum_{z \in \Omega} S_L(z) \cdot PC_m(z)}{\sum_{z \in \Omega} PC_m(z)}
\]  

(2.87)

where \( \Omega \) means the whole image spatial domain. \( PC_m(z) = \max\{PC_G(z), PC_D(z)\} \), where \( PC_G(z) \) is phase congruency for a given position \( z \) of image \( IG \). \( S_L(z) \) is the gradient magnitude for a given position \( z \).

Demosaicking results are shown in Tables 2.5, 2.6, and 2.7. Visual inspection of the demosaicked images reveals that the performance highly depends on the orientation and size of the edges and fine details in the area under consideration, and that the choice of the MPA plays an important role in obtaining the required visual quality.
Comparing the results in Tables 2.5, 2.6, 2.7, 2.8 and 2.9, the proposed Model 7 and 8 showed excellent results in terms of traditional evaluation criteria such as PSNR, SSIM, and FSIM, which are also effective on small targets as well as DOLP and Orient similarity.

Table 2.5 PSNR evaluation of images in Fig. 2.22

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image a</td>
<td>36.7775</td>
<td>36.2677</td>
<td>36.3900</td>
<td>34.8273</td>
<td>34.0855</td>
<td>27.4510</td>
<td><strong>43.2773</strong></td>
<td><strong>43.1621</strong></td>
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<tr>
<td>Image b</td>
<td>33.1984</td>
<td>34.0372</td>
<td>33.8113</td>
<td>32.3503</td>
<td>31.7563</td>
<td>30.7389</td>
<td><strong>35.7789</strong></td>
<td><strong>35.3486</strong></td>
</tr>
<tr>
<td>Image c</td>
<td>33.7686</td>
<td>35.0067</td>
<td>35.2823</td>
<td>34.4840</td>
<td>34.2392</td>
<td>32.0621</td>
<td><strong>41.6126</strong></td>
<td><strong>41.2439</strong></td>
</tr>
<tr>
<td>Image d</td>
<td>35.1157</td>
<td>34.5188</td>
<td>34.2403</td>
<td>32.5251</td>
<td>32.0481</td>
<td>29.4623</td>
<td><strong>38.6929</strong></td>
<td><strong>37.7799</strong></td>
</tr>
</tbody>
</table>

Table 2.6 SSIM evaluation of images in Fig. 2.22

<table>
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<tr>
<th></th>
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<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image a</td>
<td>0.9292</td>
<td>0.9237</td>
<td>0.9254</td>
<td>0.8652</td>
<td>0.8317</td>
<td>0.8877</td>
<td><strong>0.9765</strong></td>
<td><strong>0.9744</strong></td>
</tr>
<tr>
<td>Image b</td>
<td>0.8903</td>
<td>0.9056</td>
<td>0.9004</td>
<td>0.8438</td>
<td>0.8079</td>
<td>0.8497</td>
<td><strong>0.9428</strong></td>
<td><strong>0.9344</strong></td>
</tr>
<tr>
<td>Image c</td>
<td>0.8384</td>
<td>0.8987</td>
<td>0.8996</td>
<td>0.8645</td>
<td>0.8487</td>
<td>0.7404</td>
<td><strong>0.9622</strong></td>
<td><strong>0.9582</strong></td>
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<tr>
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<td>0.8797</td>
<td><strong>0.9428</strong></td>
<td><strong>0.9295</strong></td>
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</table>

Table 2.7 FSIM evaluation of images in Fig. 2.22

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<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image a</td>
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<td>0.9324</td>
<td>0.9337</td>
<td>0.9065</td>
<td>0.8874</td>
<td>0.9021</td>
<td><strong>0.9782</strong></td>
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</tr>
<tr>
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<td>0.9383</td>
<td>0.9362</td>
<td>0.9096</td>
<td>0.8934</td>
<td>0.9144</td>
<td><strong>0.9628</strong></td>
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</tr>
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<td>0.9381</td>
<td>0.9224</td>
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<tr>
<td>Image d</td>
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<td>0.9282</td>
<td>0.8916</td>
<td>0.8797</td>
<td>0.9191</td>
<td><strong>0.9627</strong></td>
<td><strong>0.9523</strong></td>
</tr>
</tbody>
</table>

Table 2.8 DoLP difference evaluation of images in Fig. 2.22

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<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image a</td>
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<td>−11.2548</td>
<td>−7.7196</td>
<td>12.3060</td>
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<td>−9.8222</td>
<td>−0.6995</td>
<td>−0.7486</td>
</tr>
<tr>
<td>Image b</td>
<td>−2.2497</td>
<td>−0.0522</td>
<td>−3.1469</td>
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<td>Image c</td>
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<td>Image d</td>
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Table 2.9  Orient difference evaluation of images in Fig. 2.22

<table>
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<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image b</td>
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<td>43.5596</td>
<td>−1.3936</td>
<td>−1.1737</td>
</tr>
</tbody>
</table>

References

Multi-band Polarization Imaging and Applications
Zhao, Y.; Yi, C.; Kong, S.G.; Pan, Q.; Cheng, Y.
2016, X, 194 p. 112 illus., 35 illus. in color., Hardcover
ISBN: 978-3-662-49371-7