Chapter 2
Ancient Statistics History in a Nutshell

Abstract This chapter is entirely devoted to the historical roots of human interest of chance, from gambling activities and theories to the role of insurance companies, the medical aspects of vaccination, the notion of legal evidence or even the existence and intervention of some divinity. Conjoined to several human practices, the philosophical aspects implied into the notion of hazard are explored which were found initially in gambling or predictive rituals, most of them from a religious context.

Keywords Dice · Insurance companies · Lottery · Determinism · Gambling · Astronomy · Luck · Destiny · Wager · Vaccines · Legal · Evidence

At the beginning of the eleventh century, Japanese Emperor Shirakawa[1] recited his own list of three unmanageable things: sōhei (armed monks), dice, and the water of the Kamo River. Those monks died centuries ago and Kamogawa was finally domesticated by modern disciplined engineers, but dice… are still a proof of daily surprise and the simplest example of the reality of chance. They are small toys for fun, sometimes portrayers of good or bad luck, but always a solid geometrical form with some signs painted or grabbed. Dice have fascinated us for several centuries, always changing sides without a reasonable way of knowing their next behavior. They will show us the pathway to statistics, be patient.

2.1 Dice in a Deterministic World

Five thousand years ago, dice were invented in India (David 1998). This fact implies that their users had at least a common sense approach to the idea of probability. Those dice were not the contemporary cubical standard dice, but fruit

Some of the data in this chapter has been extracted from my previous research (Vallverdú 2011a, b).

[1]白河天皇, Shirakawa-tennō, July 7, 1053—July 24, 1129, was the 72nd emperor of Japan.

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stones or animal bones (Dandoy 2006). They must surely have been used for fun and gambling as well as for fortunetelling practices. The worries about the future and the absurd idea that the world was causally guided by supernatural forces led those people to a belief in the explanatory power of rolling dice. In fact, cosmogonical answers were the first attempt to explain in a causal way the existence of things and beings. The Greek creation myth involved a game of dice between Zeus, Poseidon, and Hades. And in the classic Hindu book *Mahabharata* (section "Sabha-parva"), we can find the use of dice for gambling, where it is explained how the Pandavas were robbed of their kingdom by means of a game of dice, but in both cases, there is no theory regarding probability[2] in dice, just their use "for fun." At this point, it is nonetheless necessary to make a stop and take a tour into Indian culture.

According to Raju (Gabbay et al. 2011: 1174–1195), the permutational and combinational theories necessary to calculating probabilities in games of chance, such as dice or cards, were born in ancient India. First in Vedic metre conception and secondly in the Jain *Vyākhyāprajñapti*, commonly known as *Bhagavati sūtra*, the fifth of the 12 sacred āagam or janinist canonical texts. These texts were written by Mahavira's disciples who memorized and transmitted his ideas orally until they were fixed by writing them into books (specifically, *Sutras*) around 4th or 3rd BCE. There, permutations are called *vikalpa-ganita* (the calculus or alternatives) and combinations *bhanga*. While Greek and Roman mathematics were still slaves of a bad notational system, Indians had a perfect place value system and the number zero, something that made it possible to work with very big numbers. It is true that even before, in Egyptian mathematics of 12th Dynasty (ca 1990–1800 BCE) existed even a number for one million drawn as a god with his hands raised in adoration pictogram,[3] and we know that Egyptians worked with high numbers as a consequence of their big governmental necessities (food, prisoners, soldiers,…). Nonetheless even in that case their numbers' size was nothing compared to Indian ones: Jain literature typically runs into very large numbers as 10^{12}, 10^{53}, or even 10^{60}. Large numbers, beyond their magical meaning, demand a use of probability. On the other hand, dice games in India are popular and have attracted intellectual interest. For example, in the *Rigveda*, *Mandala* (book) 10, chapter 34,[4] we find several references to the gambling practices. Then, Indians have at the same time knowledge about dice (and that they are always loaded), and fair/deceitful gambling. This later introduces us to a notion of knowledge of large numbers where the notion of convergence can be found and, even, the foundations or probability theory.[5] We note this and leave the debate at this point, more interesting for

[2]For a very detailed history of probability and how the empire of chance emerged among several disciplines, see Gigerenzer, Gerd et al. (1989).

[3]Information obtained from Burton (2005).

[4]Although not written until fourth or sixth century of our era, *Rigveda* was much more ancient.

[5]As attempted to justify in the 1950s P.C. Mahalanobis, J.B.S. Haldane and D.S. Kothari. Raju (2011): 1191 from the 3-valued logic present in Jainism.

specialists in history of mathematics than us, who are following the path from natural numbers to human ways to design strategies to deal with them. Anyhow, numbers are not free from conceptual frameworks from which they emerge.

Again in Western territories, we can consider Aristotle as the strongest defender of the causal and empirical approach to reality (*Physics*, II, 4–6) although he considered the possibility of chance, especially the problem of the game of dice (*On Heavens*, II, 292a30) and probabilities implied in it. These ideas had nothing to do with those about atomistic chance by Leucippus, Democritus[6], or Lucrecius' controversial *clinamen*'s theory. Hald (1988, Sect. 3.2.) affirms the existence of mathematical rather than statistical thought in Classical Antiquity, surely due to the imperfection of the used randomizers (bones of hooved animals instead of regular dice), something that made an axiomatization of games of chance impossible; regardless, we can accept that some authors (like Aristotle) were worried about the idea of chance (as well as about the primordial emptiness and other types of conceptual *cul-de-sac*), but they made no formal analysis of it. Anyhow, trust in order and regularities was the aim of life and philosophy, as we can find in verse 490 of Book 2 of the *Georgics* (29 BC), by the Latin poet Virgil: "Felix qui potuit rerum cognoscere causam" (translated as "Can he happy who is able to know the causes of things").

Later, we can find traces of interest in the moral aspects of gambling with dice in Talmudic (*Babylonian Talmud, Book* 8*: Tract Sanhedrin, chap. 3, Mishnas* I *to* III) and Rabbinical texts, and we know that in 960, Bishop Wibolf of Cambrai calculated 56 diverse ways of playing with three dice. *De Vetula*, a Latin poem from the thirteenth century, tells us of 216 possibilities. But the first occurrence of combinatorics per se arose from Chinese interest in future prediction through the 64 hexagrams of the *I Ching* (previously eight trigrams derived from four binary combinations of two elemental forces, *yin* and *yang*). The idea of making combinations in order to obtain several results and find the best options was also described by the Catalan philosopher Raimon Llull in his *Ars Magna* [Ars Maior (1273–74), *Ars inventiva* (1289), and *Ars generalis* (1308)], later updated and improved by Wilhelm Leibniz in his *Dissertatio de arte combinatorial* (1666). What Llull tried to design was a method to convince Muslims about their fundamental error and to demonstrate the "evidence" of the Christian truth. With his conceptual wheels, Llull embraced as a real polymathist all the wisdom of his era: "arbor scientiae," "arbor elementalis," "arbor vegetalis," "arbor moralis," "arbor aspostocalis," "arbor coelestialis," "arbor christianalis," "arbor divinalis," "arbor naturalis et logicalis." It can look like the classic encyclopedism of the Middle Age, but Llull tried in fact to surpass this with a new heuristic of knowledge generation.

Despite the religious flavor of Llull's attempts, it was a Muslim who started the history of statistics, and beyond any religious framework. Instead of being worried

[6]As an exception of a whole Western paradigm, however, we find this point in one of the conserved fragments of Democritus: "Everything existing in the universe is the fruit of chance and necessity." A whole recompilation can be found in the classic Diels (1903). Demokritos. A very good paper on this topic is Edmunds (1972).

by proselytism, Abu Yūsuf Ya'qūb ibn' Isḥāq aṣ-Ṣabbāḥ al-Kindī, usually known by Western historians as Al-Kindi (801–873), gave a detailed description of how to use statistics and frequency analysis to decipher encrypted messages. With his book *Manuscript on Deciphering Cryptographic Messages*, he gave birth to both statistics and cryptanalysis. The truth is that Al-Kindi, while working at al-Ma'mun's House of Wisdom (together with al-Khwarizmi and the Banu Musa brothers!) was faced with religious confrontations among orthodox factions, although he was always a neutral philosopher and scientist more interested in general knowledge than religious discussions. Anyway, cryptography had connected politics and numbers or letters and now, statistical approaches had started to change the way by which secrets could be transmitted.

In 1494, Luca Paccioli defined the basic principles of algebra and multiplication tables up to 60 × 60 in his book *Summa de arithmetica, geometria, proportioni e proportionalita*. He posed the first serious statistical problem of two men playing a game called "balla," which is to end when one of them has won six rounds. However, when they stop playing A has only won five rounds and B three. How should they divide the wager? It would be another 200 years before this problem was solved. In 1545, Girolamo Cardano wrote the books *Ars magna* (the great art) and *Liber de ludo aleae* (the book on games of chance). This was the first attempt to use mathematics to describe statistics and probability and accurately describe the probabilities of throwing various numbers with dice. Galileo expanded on this by calculating probabilities using two dice, writing a small text in 1620, *Sopra le scoperte dei dadi* (*Concerning an Investigation on Dice*). At the same time, the measurement and quantification of all aspects of daily life (art, music, time, space) between the years 1250 and 1600 made possible the numerical analysis of nature and, consequently, the discovery of the distribution of events and their rules (Crosby 1996). It was finally Blaise Pascal who refined the theories of statistics and, later, Pierre de Fermat solved the "balla" problem of Paccioli (Devlin 2008). All these paved the way for modern statistics, which essentially began with the use of actuarial tables to determine insurance for merchant ships (Hacking 1984, 1990). Pascal was also the first to apply probability studies to the theory of decision, curiously, in the field of religious decisions. Despite the previous affirmation, and according to Bellhouse (1988: 63) the beginning of probability began in 1645, it was the time of the Pascal–Fermat correspondence in the middle of puritan casuistry. Puritans were faced with the problem of conciliate to the presence of divination and gambling in the Bible with the notion of God's will into a deterministic world. There is a long list of examples of such events in the Bible,[7] and crucial

[7]Bellhouse (1988): 66 quotes Acts 1:23–26; Luke 1:9–11; Matthew 27:35–37; Mark 15:22–24; Luke 23:35, John 19:23–24…but there is a long list of false prophets (Deut. 18:10, 14; Micah 3:6, 7, 11), of necromancers (1 Sam. 28:8), of the Philistine priests and diviners (1 Sam. 6:2), of Balaam (Josh. 13:22). Three kinds of divination are mentioned in Ezek. 21:21, by arrows, consulting with images (the teraphim), and by examining the entrails of animals sacrificed. The practice of this art seems to have been encouraged in ancient Egypt. Diviners also abounded among the aborigines of Canaan and the Philistines (Isa. 2:6; 1 Sam. 28). At a later period,

Christian theologists like Thomas Aquinas devoted part of their researches to battle against chance falls. In the case of Aquinas, he expressed in his *Summa Theologiae* that chance events were part of contingent events, and thus they were far from the true and definitive nature of divine necessary events. Providence is thus deterministic. Finally, in 1662, John Graunt published his mortality tables that produced what has been called "empirical statistics."

It is in this historical moment that the Latin term "probabilis" acquires its actual meaning evolving from "worthy of approbation" to "numerical assessment of likelihood on a determined scale" (Moussy 2005). In fact, Pascal introduced a new concept: the moral wager.

2.2 From Dice to Moral Wagers and God in Mathematics

In 1669, seven years after his death, Blaise Pascal's book *Pensées* was published posthumously. At the beginning of the Third Section, aforism §233, it reads:

> "(…)Let us then examine this point, and say, "God is, or He is not." But to which side shall we incline? Reason can decide nothing here. There is an infinite chaos which separated us. A game is being played at the extremity of this infinite distance where heads or tails will turn up. What will you wager? According to reason, you can do neither the one thing nor the other; according to reason, you can defend neither of the propositions.
>
> Do not then reprove for error those who have made a choice; for you know nothing about it. "No, but I blame them for having made, not this choice, but a choice; for again both he who chooses heads and he who chooses tails are equally at fault, they are both in the wrong. The true course is not to wager at all."

(Footnote 7 continued)

multitudes of magicians poured from Chaldea and Arabia into the land of Israel and pursued their occupations (Isa. 8:19; 2 Kings 21:6; 2 Chr. 33:6). This superstition widely spread, and in the time of the apostles there were "vagabond Jews, exorcists" (Acts 19:13), and men like Simon Magus (Acts 8:9), Bar-jesus (13:6, 8), and other jugglers and impostors (19:19; 2 Tim. 3:13). Every species and degree of this superstition was strictly forbidden by the Law of Moses (Ex. 22:18; Lev. 19:26, 31; 20:27; Deut. 18:10, 11). But beyond these various forms of superstition, there are instances of divination on record in the Scriptures by which God was pleased to make known his will. (1) There was divination by lot, by which, when resorted to in matters of moment, and with solemnity, God intimated his will (Josh. 7:13). The land of Canaan was divided by Lot (Num. 26:55, 56); Achan's guilt was detected (Josh. 7:16–19), Saul was elected as king (1 Sam. 10:20, 21), and Matthias chosen to the apostleship, by the solemn Lot (Acts 1:26). It was thus also that the scape-goat was determined (Lev. 16:8–10). (2) There was divination by dreams (Gen. 20:6; Deut. 13:1, 3; Judg. 7:13, 15; Matt. 1:20; 2:12, 13, 19, 22). This is illustrated in the history of Joseph (Gen. 41:25–32) and of Daniel (2:27; 4:19–28). (3) By divine appointment, there was also divination by the Urim and Thummim (Num. 27:21), and by the ephod. (4) God was pleased sometimes to vouch-safe direct vocal communications to men (Deut. 34:10; Ex. 3:4; 4:3; Deut. 4:14, 15; 1 Kings 19:12). He also communed with men from above the mercy-seat (Ex. 25:22), and at the door of the tabernacle (Ex. 29:42, 43). (5) Through his prophets, God revealed himself and gave intimations of his will (2 Kings 13:17; Jer. 51:63, 64). From: Divination. (n.d.). Easton's 1897 *Bible Dictionary.*

> Yes; but you must wager. It is not optional. You are embarked. Which will you choose then? Let us see. Since you must choose, let us see which interests you least: You have two things to lose, the true and the good; and two things to stake your reason and your will, your knowledge and your happiness; and your nature has two things to shun, error and misery. Your reason is no more shocked in choosing one rather than the other, since you must of necessity choose. This is one point settled. But your happiness? Let us weigh the gain and the loss in wagering that God is. Let us estimate these two chances. If you gain, you gain all; if you lose, you lose nothing. Wager, then, without hesitation that He is.—"That is very fine. Yes, I must wager; but I may perhaps wager too much."—Let us see. Since there is an equal risk of gain and of loss, if you had only to gain two lives, instead of one, you might still wager."[8]

This colloquial style scandalized his contemporaries as well as posterior thinkers: faith could be rational or not (the classic debate of Middle age), but never be the result of a wager, because this act joined ignominiously the fields of religion and games. This idea will be destroyed by the powerful Kantian moral Metaphysics (*Grundlegung zur Metaphysik der Sitten*, 1785) and not will change until the radical works of Friedrich Nietzsche at the end of nineteenth century and the antifundamentalist ethical advances in the twentieth century. Although Kant also made an indirect reference to a bet as a way to understand whether the things in which we trust are solid enough,[9] he was far from the belief in the presence of chance into

[8]Quoted from http://www.gutenberg.org/files/18269/18269-h/18269-h.htm#SECTION_III, accessed May 28, 2013.

[9]"For the subjective grounds of a judgement, such as those that produce belief, cannot be admitted in speculative inquiries, inasmuch as they cannot stand without empirical support and are incapable of being communicated to others in equal measure. But it is only from the practical point of view that a theoretically insufficient judgement can be termed belief. Now the practical reference is either to skill or to morality; to the former, when the end proposed is arbitrary and accidental, to the latter, when it is absolutely necessary. If we propose to ourselves any end whatever, the conditions of its attainment are hypothetically necessary. The necessity is subjectively, but still only comparatively, sufficient, if I am acquainted with no other conditions under which the end can be attained. On the other hand, it is sufficient, absolutely and for every one, if I know for certain that no one can be acquainted with any other conditions under which the attainment of the proposed end would be possible. In the former case my supposition—my judgement with regard to certain conditions—is a merely accidental belief; in the latter it is a necessary belief. The physician must pursue some course in the case of a patient who is in danger, but is ignorant of the nature of the disease. He observes the symptoms, and concludes, according to the best of his judgement, that it is a case of phthisis. His belief is, even in his own judgement, only contingent: another man might, perhaps come nearer the truth. Such a belief, contingent indeed, but still forming the ground of the actual use of means for the attainment of certain ends, I term Pragmatical belief. The usual test, whether that which any one maintains is merely his persuasion, or his subjective conviction at least, that is, his firm belief, is a bet. It frequently happens that a man delivers his opinions with so much boldness and assurance, that he appears to be under no apprehension as to the possibility of his being in error. The offer of a bet startles him, and makes him pause. Sometimes it turns out that his persuasion may be valued at a ducat, but not at ten. For he does not hesitate, perhaps, to venture a ducat, but if it is proposed to stake ten, he immediately becomes aware of the possibility of his being mistaken—a possibility which has hitherto escaped his observation. If we imagine to ourselves that we have to stake the happiness of our whole life on the truth of any proposition, our judgement drops its air of triumph, we take the alarm, and discover the actual strength of our belief. Thus pragmatical belief has degrees, varying in proportion to the interests at stake."

moral sphere. Even in the natural domain, scientists like Einstein declared in the twentieth century that "God does not play dice." There is a cultural horror toward the idea of the chance in the universe, as a rule of the destiny. Nonetheless, the birth of moral statistics and the relationship between God and numbers must still to be explained. Let's go!

In the year of 1612, a big first prize was obtained by the winner of the London Lottery. This Lottery was organized by King James I to obtain funds to help to the colonies established in Virginia.[10] At the same time, he granted to the Virginia Company of London the right to raise money to help establish those settlers in the first permanent English colony at Jamestown (Virginia). Lotteries were created in China in the third to second century BC, in order to obtain funds to run politic activities (financial aid to State projects, pay armies…). 46 years had passed after the first lottery in England, authorized by Queen Elizabeth I, when there was a debate on some social problems on chance games, so it is normal that intellectuals devoted themselves to its study. The French Calvinist Lambert Daneau was the first (in 1566, *Deux traittez de S.c. Cyprian. L'un, contre les ieux ei iouers de cartes & de dez. Le tout mis en francois par L. Daneau*) to write about gambling and religion and suggested which games should be allowed or forbidden to Christians: to the first section belong games of pure chance, while in the second corresponded games of mixed chance and skill. The first English Puritan to write about this topic was Northbrooke (1577).

Some decades later, in 1619, Thomas Gataker published *Of the Nature and Use of Lots*, offering a historical review on games in which hazard is involved, as well as a religious interpretation of Chance. From his own words, in Chap. 2, §1, 6, 7[11]:

> Now because Chance or Casualty bears much sway in Lottery, Casual Events being the subject matter of Lots, the due consideration thereof will help not a little to the clearing of the nature of Lots and Lottery, and those Questions that are moved concerning the same. Concerning Chance therefore or Casualty we will consider four things: (1) the name of it; (2) the nature of the thing so named; (3) two distinct Acts concurring in it, and (4) and lastly, certain conclusions or aphorisms concerning it. (…)By the means whereof it comes oft to pass, the same events are casual to some that foresaw them not, and yet not casual to others that foresaw them before. And so it is true, that Casualty depended upon our ignorance; which therefore the more we know, the less we are subject unto. §7. And hence follows the fourth and last Conclusion: *That there is no casualty with God, because there is no ignorance in God.* There is nothing, I say, casual unto Him; nothing comes contingently, but all things are necessarily in regard of Him and His decree."

(Footnote 9 continued)

Critique of Pure Reason, A825/B853. Quoted from http://www.gutenberg.org/files/4280/4280-h/4280-h.htm, accessed in May 28th 2013. In 20th Century Bruno de Finetti will offer a more sophisticated version of Kant's approach to the confidence evaluation of own opinions.

[10]Only in 1612, the benefits of this lottery amounted to nearly of £30.000, according to Holmes (1826).

[11]The full edited text can be found at http://www.conallboyle.com/lottery/GatakerNature_UseofLots.pdf, accessed in May 29, 2013.

Here, and italics in the previous text are mine, we find one of the most current ideas of ancient thinkers: casualty (or chance or hazard) is nothing but the result of human ignorance; God knows everything and, then, for him (yes it is a *he*), there is no casualty. Chance is a consequence of ignorance, not a real dimension of the reality, just a mistake emerged from human fuzzy cognition.[12] Consequently, childish bibliomantic practices were banned by early Christians, although not very successfully: among the Christians remained some fortune-telling practices, as Bible lottery or *sortes Biblicae*, a method consisting by taking random passages from the Bible and to interpret them as signs of fortune. This was an inherited practice from Greek and Roman cultures (*Sortes Homericae*—usually from *Iliad, sortes Virgilianae*—using *Aeneid* fragments or verses). In France, the Gallican synods of Vannes (465 CE), Agde (506), Orleans (511), and Auxerre (570–590) passed ordinances vowing to excommunicate any Christian who "should be detected in the practice of this art, either as consulting or teaching it'" (Metzger 1993). What is most surprising is that the most well-known instance of *sortes biblicae* was by St. Augustine of Hippo who in the year 386 was prompted by a childlike voice he heard telling him to "take up and read" (in Latin: *tolle, lege)*. Augustine opened a Bible at random, selecting from the two sides the verses of Romans 13:13–14 ("Not in rioting and drunkenness, not in chambering and impurities, not in strife and envying; but put you on the Lord Jesus Christ, and make not provision for the flesh in its concupiscences."), and later wrote that "as if before a peaceful light streaming into my heart, all the dark shadows of doubt fled away" (*Confessions*, Bk. 8, Chap. 29). Augustine was then converted, calling the experience a direct work of God, but a few centuries later this would have been considered just a blaspheme and sinful behavior (some millennia later, it could be easily typified as "schizophrenic").

So, Gataker studied lotteries and hazard games because he wanted to clear the darkness inside them, and at the same time to reflect the paucity of the human mind, which always needed the divine omniscient guidance (Rescher 1995).

The next attempt to join theology and statistics was the demographic theology of Johann Peter Süssmilch. In 1741, this German priest with interests in demography published *Die göttliche Ordnung in den Veränderungen des menschlichen Gesglechts, aus der Grut, dem Tode, un der Fortpflanzung* (*The Divine order in the changes in the human sex from birth, death and reproduction of the same*), a very curious work full of still more curious theses, all about the invisible guidance of God through the hand of His Providence. According to Süssmilch, if somebody analyzes long rungs of birth registers, it can be found that approximately the 50 % ratio of males and females is stable. For him, this was a logic consequence of the evident hand of God, and this text becomes one of the first attempts to talk about

[12]We will find in one of the leading founders of modern statistics, Laplace, a similar idea: "(probability) is relative, in part to our ignorance, and in part to our knowledge", Laplace (1814: 8).

intelligent design. In fact, Süssmilch had not been the first to point to this fact, but the Scottish John Arbuthnot. In 1710, he published *An argument for Divine Providence, taken from the constant regularity observed in the births of both sexes* in the Royal Society's *Philosophical Transactions*, where he analyzed birth data and demonstrated that males were born at a greater rate than females. He considered that this fact was against the 50 % equal odds and that the only explanation was the active influence of divine providence into this process, in order to correct the early deaths of males who die young more often than females. This problem of sex ratio was attacked but not solved one century later by Charles Darwin and was necessary to reach the twentieth century to find an answer: Ronald A. Fisher established it in 1930 with the book *The Genetical Theory of Natural Selection* the so-called Fischer principle, a ratio of 1:1 between sexes as an evolutionary stable strategy. But cultural incidence is changing this ratio, as has been noted by Hvistendahl (2011).

2.3 Fortuna, Destiny, Luck, Chance, or Probability …

Until this moment we have seen that before seventeenth century, a specific vocabulary to deal with probability did not exist, basically because cultural paradigms cannot allow it inside them. Nevertheless, these different cultures had the necessity to express several notions of non-deterministic events. That is, not controlled events. I will make a short journey across these words and their meanings.

Greek Goddess *Ananké* (*Necessitas* for Roman mythology), the mother of the Moiri and Adrasteia, was considered the Goddess of destiny, necessity, and fate. Supranatural or divine rules guided human lives secretly, who should discover it and embrace their destiny. At a certain level, there was no free will for them just a terrible divinity will. *Tyche* was a different goddess (worshipped in Rome under the name of *Fortuna*) who was considered as the governor of the prosperity or decline of a city as well as the source of all unexpected events in human life, whether good or evil. So in a certain way, she was related to hazard or luck. Temples were built to Tyche asking for a better life and Romans considered her as the *fors*, the luck, fortuity, accident, and chance and sometimes painted her as a woman who spins a wheel, the *Rota Fortunae* (or *wheel of fortune*). Fortuna was also christianized and forms part of the history of medieval art and minds. Curiously, there was still a third goddess, Ananke, also called "Necessity," the strongest force in the realm of gods who was also paired with Fortuna. Something similar exists in Hinduist tradition under the name of *karma* (a cosmic regulatory law of cause–effect, along with *samsara* (reincarnation cycle) *and moksha* (liberation from *samsara*). In our days, the presence in human life of the notion of chance is overwhelmingly present. As the poet tells us, luck is everything.[13] Close to the notion of "fortuna" in the

[13]Childish (1988). Poem 'h.m. prison maidstone'. A wonderful poem written by a different poet.

twelfth and thirteenth centuries in Europe appearing in the oriental Mediterranean area, the notion of *risicum* or risk emerged, probably from the Arab word *riszq* (Piron 2004). This *risicum* was related to the games practices as well as to the perils of economic procedures, most of them analyzed by Franciscan monks (Ceccarelli 1999). These theologians were mainly interested in the notion of contract and how random elements present in that contract should be considered (Meusnier and Piron 2007). This was very close to the first maritime insurances invented in Tuscany in the first half of the fourteenth century were the risk of a commercial operation, accepted by the insurer against the payment of a *premium*. For a psychological analysis of luck, see Pritchard and Smith (2004).

Finally, the term "probability" can be found in Classic Rome in Latin as the word "probabilis," translated as "credible," but in 1660 when it turned the meaning toward the modern use (Hacking 1984). It was in 1657 that Hyugens published *De Ratiociniis in Ludo Aleae* (1714 English version published as "The VALUE of all CHANCES IN Games of Fortune; CARDS, DICE, WAGERS, LOTTERIES, &c. Mathematically Demonstrated"). Very soon *La logique, ou l'art de penser*, in 1662 also appeared by Antoine Arnauld and Pierre Nicole, commonly quoted as Port-Royal Logic. They introduced the idea of the necessary quantification of probability.

2.4 *Pay Me Again, Sam*…From New Gods and Taxes to Statistics

Approximately 6/7 years before year 1[14] of our Era and during the reign of Emperor Augustus, Publius Sulpicius Qurinius was appointed governor of Syria. One of his first actions was to improve his taxes recollection performing a new census of the Jewish population. The *Gospel of Luke* explains that was then when Joseph and the pregnant Maria travelled to Jerusalem to notify their data, but because of the advanced situation of her pregnancy, they gave birth to the child in Bethlehem: he was Jesus, the founder of the biggest and most widespread religion existing to this day. Don't be lost by this shell game with words: here the important thing is the census, not the religion.

If we look at the Oxford English Dictionary, as a simple source to the topic and look for the entry "statistic" this is found:

> The earliest known occurrence of the word seems to be in the title of the satirical work *Microscopium Statisticum*, by 'Helenus Politanus', Frankfort (?), 1672. Here the sense is prob. 'pertaining to statists or to statecraft' (cf. statistical a. 1). The earliest use of the adj. in anything resembling its present meaning is found in mod.L. *statisticum collegium*, said to have been used by Martin Schmeizel (professor at Jena, died 1747) for a course of lectures

[14]See Gould (1997) for the debate on when exactly it is supposed that Jesus was born and how it should be considered numerically the first year of Jesus's life.

> on the constitutions, resources, and policy of the various States of the world. The G. *statistik* was used as a name for this department of knowledge by G. Achenwall in his *Vorbereitung zur Staatswissenschaft* (1748); the context shows that he did not regard the term as novel. The F. statistique n. is cited by Littré from Bachaumont (died 1771); Fr. writers of the eighteenth century refer to Achenwall as having brought the word into use. The sense-development of the word may have been influenced by the notion that it was a direct derivative of L. status.

And by "statistics":

> c.B.1.c Statistics. Any of the numerical characteristics of a sample (as opposed to one of the population from which it is drawn). Cf. parameter 2f.

So, statistics has a direct relationship with *census,* the registration of citizens and their property for purposes of taxation. From this close tie between social numbers and government, the etymological trace of the work "statistics" can be understood. The German jurist and philosopher Gottfried Achenwall coined the word "Statistik" in his 1752 work *Staatsverfassung der Europäischen Reiche im Grundrisse* (Constitution of the Present Leading European States), when he related mathematical calculations of country activities like commerce or agriculture. He also gave currency to the word "Staatswissenschaft" (science of politics), the knowledge necessary to understand and run a modern State. At the beginning of the section "Vorbereitung von der Statistik überhaupt," he identifies some authors who in the past talked about things close to his notion of statistics, a concept that he defined in section §5: "Staatsverfassung eines oder mehrerer einzelnen Staaten ist die Statistik" (The constitution of one ore more individual states is the statistics), and in §6 he added "Durch die Statistik erlangt man die Staatskenntniß" (Thanks to statistics somebody can achieve knowledge about the State). Achenwall makes a qualitative approach to numbers and the affairs of the State (in this sense he also talks of Staatslehre, Staatswissenschaft, Staatrecht…), not merely one quantitative as we can infer in our days from the notion of "statistics." Going to the core of his ideas, we find a very charming notion of State, §2: "Staat ist eine Gesellschaft von Familien, welche zu Beförderung ihrer gemeinsamen Glückseelichkeit unter einem Oberhaupte mit einander vereiniget leben" (The State is a society of families who live together under the guidance of a superior power for the conveyance of a common happiness).

The *Bills of Mortality* (1662, the complete title is *Natural and Political Observations Made upon the Bills of Mortality*), published by the haberdasher John Graunt included the first life table and turned his author into one of the first demographers and epidemiologists. He made statistical analysis of the population of London and his impressive results appointed him to the election as member of The Royal Society despite the class reluctances. One of the 12 who were at Gresham College in November 28,1660, who proposed a new institution that would be the Royal Society, William Petty is considered, together with Graunt, the founder of the modern census statistics, basically due to his interests in what he called "political

arithmetic." He made estimations and used simple averages, always as part of his duties working alongside Oliver Cromwell and serving as parliamentarian.

From the "Herodes Census" (Quirinius), to the first colonial census made in Peru by the Spanish Don Pedro de La Gasca at Perú of 1548, the interest in such tables of data was mainly due to economics as well as military. It is not strange that mathematicians, even leading experts like Leibniz, were attracted by governmental forces to this research field, which became secret for national security. At the same time, these huge lists of data required from new ways to be easily understandable, that is, visually mapping. And, finally, with so many objective data, the idea of "normality," emerged that introduced into human studies the notion of "average man" (*home moyen*, according to Quetelet, the astronomer who first applied statistical analyses to human biological domains). This also made possible a mathematization of the whole human sphere, allowing the birth of the social mathematics, by Condorcet, as well as the consequent positivist view of Auguste Comte.

2.5 From Dice to Vaccines and Assurance Companies: The Birth of Probability

1660 is the year, if we follow Hacking (1984, 1990), of the birth of probability. But several things concurred in order to generate the complex and extended notion of probability, which we will analyze in this section.

2.5.1 First of All, Vaccines

The dispute between Daniel Bernoulli and Jean Le Rond D'Alembert on the efficacy and utility of smallpox vaccination was a different context in which the probability issues were discussed, far from previous recreational, hypothetical, or mathematical discussions. It was the year 1760, in the middle of an intense controversy on the benefits of inoculation that had started with the works of Pierre Louis Moreau de Maupertius (1698–1759) and Charles Marie de la Condamine (1701–1774). The latter, especially, has written several memoranda favoring the introduction of inoculation into France, then a very young technique (Dietz and Heesterbeek 2002). Daniel Bernoulli wrote a paper modeling smallpox, using Halley's life table and some data concerning smallpox to show that inoculation was advantageous if the associated risk of dying was less than 11 %. Inoculation could increase life expectancy at birth to up to three years (Bacaër 2011). This was the *first* mathematical model employed in epidemiology, a discipline that we will

discuss in later chapters because of its close links with statistics and causality debates. Immediately, D'Alembert criticized Bernoulli's work from a seminal presentation at *Académie royale des sciences* to its several publications.[15] Bernoulli's model was probably the first compartmental model and described the age-specific prevalence of immunes for an endemic infection which is potentially lethal. In a letter to the mathematician Euler, Bernoulli showed himself as sad at D'alembert's criticisms and considered his work as "c'etoit, si j'ose le dire, comme une *nouvelle province* incorporée au corps des mathematiques" (translated as "it was, I dare say, like incorporating a *new province* into the body of mathematics"; cursives are mine). This debate aroused an enthusiasm in France for the social uses of probability (Zabell 2011: 1153). The star-system philosopher Voltaire joined the general debate on probability and its uses incorporated into the list of friendly statistics authors. He wrote a book in 1772 entitles *Essai sur les probabilités en fait de justice*. There he explained (p. 371) that "Presque toute la vie humaine roule sur des probabilité" (Almost all human life is based on "probability"), a curious declaration in a deterministic era, sign of the changes that were happening in his time.

2.5.2 Secondly, Insurance Companies

As a second domain encapsulating an attraction toward the use of numbers to explain and predict future outcomes is the assurances. Yes, a pragmatic use, as usually happens with most new ideas of humanity. Babylonian merchant land traffics and later Phoenician merchant sea traffic were the first situations in which a rude and basic insurance idea was applied. Following Trenerry (2009: 6), the essentials of that Bottomry were reinforced by law for the first time in the Code of Hammurabi (2250 B.C.). Later, Achaemenids (Persians), Greeks, or Romans evolved this simple version and started a transformation (e.g., introducing life insurances by collegia funeraticia in Roman culture[16]) that led to the origin of modern insurance companies in seventeenth century. A first step toward this

[15]For the very strange reasons of life, D'Alembert immediately wrote a criticism which he presented on November 12, 1760, to the Royal Academy of Sciences and which he published his collected works in the following year. This means that his critique of Daniel Bernoulli appeared five years before Bernoulli's contribution was eventually published by the Academy in 1766. Bernoulli was very annoyed about the critique by d'Alembert, which can be seen from his letter to Euler in April 1768 (Dietz and Heesterbeek 2002: 12).

[16]A look at an old but still fascinating book like *Die römischen Collegia Funeraticia nach den Inschriften* (1888), by Traugott Schiess, is very informative in this topic. An online version is available at: http://archive.org/stream/diermischencoll00schigoog#page/n5/mode/2up. These societies allowed poor people to cover the expenses of their burial, as well as some other assistance during their life.

process was made in Genoa in 1347, when the first known insurance contract was created (Franklin 2001). The first book on insurances was written in 1557 by the Portuguese lawyer Pedro de Santarem (Petrus Santerna): *Tractatus de assecurationibus et sponsionibus mercatorum ad praxim quotidianam utilissimus & omnibus in foro presertim mercatorum versantibus quotidianus*. In this book, sea trade protection (*Mercatores maris*) was the basic target of the insurance system protecting merchandise as well as the crew or the vessel. The creation of quantitative lists trying to evaluate the possible risks from the destiny, boat, captain, crew, or merchandise required from mathematical tools from statistical nature. A coffee house led by Edward Lloyd in London in 1688 was a common place for sailors and shipping industry investors to share last notices about the field, and it was a source of information for insurance experts. It was in this conceptual arena in which ideas like "normal curves" or "normal man" (Adolphe Quetelet's l'homme moyen) emerged from statistical data to enter into political, medical, artistic, and even the anthropological arena. Fire protection was another important speciality of insurance companies, and curiously, Benjamin Franklin founded in 1752 America's oldest, continuously active insurance company: *Philadelphia Contributorship for the Insurance of Houses from Loss by Fire*. The Contributorship, as is now its common reference, was a proactive insurance carrier refusing to provide coverage to houses and other structures that were not constructed according to strict building standards.

2.5.3 Third, Legal Issues and the Notion of "Evidence"

In Sect. 2.5.1, it was mentioned that Voltaire wrote a book on probability and evidence in the legal context. This work inspired Minister Turgot in the reform of the French legal system and prepared the field for fruitful research across the time. In 1837, for example, the mathematician Siméon-Denis Poisson wrote *Recherches sur la probabilité des judgements*, where he made an interesting distinction between subjective and objective senses of probability (Zabell 2011: 1153). At a certain level, he followed some previous but not well-defined concepts of Hume. The British philosopher wrote in 1739, *A Treatise on Human Nature* (T 1.3.11.3, SBN 124–125): "Probability or reasoning from conjecture may be divided into two kinds, viz. that which is founded on *chance*, and that which arises from *causes*. We shall consider each of these in order." For Hume, chance was merely the negation of a cause and causes themselves were not real just mental habits, and even more "chance is nothing real in itself" (*ibid.* p. 125). Without causality, all the beliefs of Enlightenment religious scientists, who looked at nature to find the justification of the existence of God and his rules, disappeared. Then, the study of causal events became a priority for theologians. Probability was then the result of an imperfect

experience[17] and consequently chances were equal and indifferent (no place for statistical multicausality). Poisson tried to make a new objective field of research, avoiding previous errors and misunderstandings or fatidic double senses, basically those concerning the meaning of words, into a philosophical problem: "Fundamentally, the theory of chance and mathematical probability applies to two kinds of questions that are quite distinct: to questions of *possibility* which have an objective existence, as has been explained, and to questions of *probability* which are relative, in part to our knowledge, in part to our ignorance" (quoted from Zabell 2011: 1156). This philosophical debate continued with Cournot in his 1843 *Exposition* and 1851 *Essai,* where he affirmed the existence of two kinds of probability (and types of studies): philosophical and mathematical. First one was not reducible to a calculus of chance, while the second was. Philosophical probability was closer to natural phenomena, and out of the realm of mathematical analysis, and hence, of absolute (frequentist) truth. This distinction between *chance* (philosophical) and *probability* (mathematical) was also proposed in a certain way by Jakob Fries in Germany (1842, *Versuch einer Kritik der Prinzipien der Wahrscheinlichkeitsrechnung*, Braunschweig: Vieweg),[18] Richard Leslie Ellis (1843, "On the Foundations of the Theory of Probabilities", *Transactions of the Cambridge Philosophical Society* vol 8) and John Stuart Mill (1843, *A System of Logic, Ratiocinative and Inductive*) in England. It is interesting to note that Mill was hostile to statistical thinking and related thinking (like Quetelet's social physics). Auguste Comte, another influential thinker of that century, was an opponent of social statistics as well as of other, for him, biased uses of statistics into scientific realms, despite his defense of scientific quantification. In the case of Mill, his approach to the statistical debate came from the analysis of coincidences: how to distinguish coincidences that are casual from those that come from natural laws or processes. Mill argued against Laplace's (initial) subjective interpretation of probability and made him affirm that statistics misuses were "the real opprobrium of

[17]Specifically he wrote (*ibid.* T 1.3.12.25, SBN 142): "BUT beside these two species of probability, which are deriv'd from an *imperfect* experience and from *contrary* causes, there is a third arising from ANALOGY, which differs from them in some material circumstances. According to the hypothesis above explain'd all kinds of reasoning from causes or effects are founded on two particulars, viz. the constant conjunction of any two objects in all past experience, and the resemblance of a present object to any one of them. The effect of these two particulars is, that the present object invigorates and in livens the imagination; and the resemblance, along with the constant union, conveys this force and vivacity to the related idea; which we are therefore said to believe, or assent to." The complete work of Hume, very professionally digitalized, can be accessed from http://www.davidhume.org.

[18]Fries, besides of his studies on Kantian psychology and ethics, wrote in 1816 a text *How the Welfare and Character of the Germans are Endangered by the Jews* (Über die Gefährdung des Wohlstandes und Charakters der Deutschen durch die Juden. Eine aus den Heidelberger Jahrbüchern der Litteratur besonders abgedruckte Recension der Schrift des Professors Rühs in Berlin: Ueber die Ansprüche der Juden an das deutsche Bürgerrecht, Heidelberg: Mohr und Winter). It is very curious that statistics was so close related (or used) in a future with eugenics and racist theories, as we will see with the use of Darwin's work by Galton. *L'esprit du siècle* was a bad spirit…directing European culture to the worst and darkest future.

mathematics." (íbid. p. 382).[19] Under this accusation, we must identify a debate on the procurance of inferences. One of the theoreticians involved in this debate was Charles S. Peirce, who wrote on the topic on *Illustrations of the Logic of Science* (1877–1878) and *A Theory of Probable Inference* (1883). He conducted research on regression models and defended a propensity theory of probability (later continued by eminent philosopher Karl Popper together with his ideas on falsifiability, close to the null hypothesis testing ideas, as suggested by Meehl in 1967, hinting at achieving the Popperian principle of representing theories as null hypotheses and subjecting them to challenge).

Close to the Ellis probability ideas, in 1866 the British Philosopher and logician John Venn wrote *The Logic of Chance: An Essay on the Foundations and Province of the Theory of Probability*, which is considered one of the first texts on frequentist statistics paradigm, something we have still not explained and that we will analyze in the following chapters. Venn was also the inventor of the very important *Venn diagrams*, and one of these can be seen as peacefully mingling colors from the sun light at a stained glass window in the dining hall of Gonville and Caius College, in Cambridge (UK). In the preface to the first edition, Venn explained:

> This supposed want of harmony between Probability and other branches of Philosophy is perfectly erroneous. It arises from the belief that Probability is a branch of mathematics trying to intrude itself on to ground which does not altogether belong to it. I shall endeavour to show that this belief is unfounded. To answer correctly the sort of questions to which the science introduces us does generally demand some knowledge of mathematics, often a great knowledge, but the discussion of the fundamental principles on which the rules are based does not necessarily require any such qualification. (…) The opinion that Probability, instead of being a branch of the general science of evidence which happens to make much use of mathematics, is a portion of mathematics, erroneous as it is, has yet been very disadvantageous to the science in several ways. Students of Philosophy in general have thence conceived a prejudice against Probability, which has for the most part deterred them from examining it.

Venn considered that the foundations of probability needed to be explained as well as the same probability. That mathematicians were not interested in entering into this philosophical quicksand was not an excuse about the necessity of the project. The philosophical debate surrounding the statistical analysis of nature has disappointed, hassled, abashed, or disgusted those researchers with a more mathematical training and pragmatic spirit, although at the end the philosophical debate appeared again to justify the "best" statistical approach. The debate on statistics is not only

[19]The exact and full quote is as follows: "It is obvious, too, that even when the probabilities are derived from observation and experiment, a very slight improvement in the data, by better observations, or by taking into fuller consideration the special circumstances of the case, is of more use than the most elaborate application of the calculus to probabilities founded on the data in their previous state of inferiority. The neglect of this obvious reflection has given rise to misapplications of the calculus of probabilities which have made it the real opprobrium of mathematics. It is sufficient to refer to the applications made of it to the credibility of witnesses, and to the correctness of the verdicts of juries". To provide justice to his words, Mill did not have a negative attitude towards statistics, just to some (for him) misuses of statistical tools.

ontological, but also epistemological: the idea of method and truth. Venn even included some exceptions to this philosophical disinterest, like de Morgan's *Formal Logic* and Boole's *Laws of Thought*. He also considered Mill and Whewell (1840, *Philosophy of the Inductive Sciences*) as trend inducers to an aversion toward interest on probability, something wrong and misleading. Herschel, a leading thinker at that time, considered probability in his *preliminary Discourse* (1830) only as something related to measurement techniques, but not as a basic characteristic of normal science.

Finally, the introduction of statistics into the judicial arena was seen as a great mistake for several authors although Joseph Louis François Bertrand, for example, wrote against this common view in his 1889 *Calcul des probabilités*, p. 43, 5: "L'application du calcul aux decisions judiciaires est, dit Stuart Mill, le scandale des Mathematiques.[20] L'accusation est injuste. On peut peser du cuivre et le donner pour or, la balance reste sans reproche. Dans leurs travaux sur la theorie des jugements, Condorcet, Laplace et Poisson n'ont pese que du cuivre."

After all these seminal ideas on probability, we are now prepared to introduce the reader to Prof. Bayes and his revolutionary ideas on numbers and events.

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[20]Bertrand refers to Mill's notion of "the real opprobrium of mathematics", in his *A System of Logic Ratiocinative and Inductive., Chapter 18. Of The Calculation Of Chances.* The full section says" It is obvious, too, that even when the probabilities are derived from observation and experiment, a very slight improvement in the data, by better observations, or by taking into fuller consideration the special circumstances of the case, is of more use than the most elaborate application of the calculus to probabilities founded on the data in their previous state of inferiority. The neglect of this obvious reflection has given rise to misapplications of the calculus of probabilities which have made it *the real opprobrium of mathematics*. It is sufficient to refer to the applications made of it to the credibility of witnesses, and to the correctness of the verdicts of juries."

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