

# Preface

This book discusses discretization of differential equations of continuous nonlinear systems and implicit mapping dynamics of periodic flows to chaos. In recent years, approximate analytical solutions for periodic motions to chaos in continuous nonlinear systems were developed by the author through finite Fourier series. However, for many nonlinear dynamical systems, it is difficult to achieve such approximate analytical solutions of periodic motions to chaos. With computer extensive applications in numerical computations, one has used the discrete forms of differential equations of nonlinear systems to obtain numerical solutions via recurrent iterations. The discrete forms in recurrent iterations will cause accumulated computational errors for numerical results. Once the iteration number increases, numerical results given by the discrete forms cannot approximately represent true solutions of nonlinear dynamical systems. To improve the computational accuracy, one has tried to adopt implicit maps as discrete forms to achieve numerical results. However, such implicit mapping forms cannot be iterated directly, which cause the difficulty to extensive applications of discrete implicit maps in continuous nonlinear systems. In this book, the author would like to systematically discuss implicit mapping dynamics of periodic motions to chaos in continuous dynamical systems, and discrete Fourier series based on the discrete nodes of periodic motions will be used to obtain the harmonic responses in frequency space, which can be measured from experiments.

This book includes six chapters. In Chap. 1, a brief literature survey is completed. Chapter 2 reviewed the nonlinear theory for stability and bifurcation of fixed points in discrete nonlinear systems. In Chap. 3, discretization of differential equations is discussed comprehensively. The explicit and implicit discrete schemes in nonlinear dynamical system are discussed through one-step and multi-step discretization of differential equations, and the corresponding stability and convergence of the explicit and implicit discrete maps are discussed. In Chap. 4, implicit mapping dynamics of period- $m$  fixed points in discrete dynamical systems are discussed with positive and negative discrete maps, and the complete solutions of Ying-Yang states of period- $m$  fixed points are presented. In Chap. 5, the methodology for the solutions of periodic motions in continuous dynamical systems

with/without time delay is presented through the mapping dynamics of discrete implicit mappings under specific truncated errors. The discrete Fourier series of periodic motions are discussed from discrete nodes of periodic motions, and the corresponding approximate analytical expression can be obtained. Harmonic amplitude quantity levels can be analyzed for periodic motions in continuous nonlinear systems. Chapter 6 discusses the bifurcation trees of periodic motions to chaos in the Duffing oscillator to demonstrate the implicit mapping dynamics of the discretized Duffing oscillator. Such semi-analytical results of periodic motions in the Duffing oscillator are compared with the approximate analytical solutions of periodic motions based on the finite Fourier series solutions.

Finally, I would like to appreciate my former student, Dr. Yu Guo, for completing all numerical computations. Herein, I thank my wife (Sherry X. Huang) and my children (Yanyi Luo, Robin Ruo-Bing Luo, and Robert Zong-Yuan Luo) for their understanding and infinite support.

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