Scaling Laws

If a cube is to be separated into smaller ones, in such a way, that each edge is cut into two equal pieces (cf. Fig. 1), three cuts through the cube need to be made. Each cut generates a surface area equal to the surfaces of the original cube parallel to the cut. All together the overall surface is duplicated while the volume remains to be the same. If each edge is cut in n pieces, the surface is increased by a factor n. Thus, the ratio of surface to volume is changing by the inverse of the factor s_f by which the dimensions of a cube (and in general any object) is scaled up or down.

This is the reason why ships are built as large as possible. The building costs are proportional to the hull—the surface of the ship; the driving costs are proportional to the friction with the water—which is proportional to the surface again. But the benefit is proportional to the amount of goods transported—the volume of the ship. Therefore, the costs grow like the surface with the square of the scaling factor s_f of a ship while the benefit increases like the volume with the third power of s_f, i.e., the larger a ship is, the more economic it will be. This means also, that micro ships are not a good idea if large amounts of goods are to be transported. However, there are other problems where miniaturization is an advantage.

The simple geometrical fact that the ratio of surface to volume scales with s_f^{-1} has a lot of consequences in micro technique in general, and especially for the design of micro components and systems. E.g., noble metals are widely used in micro technique because corrosion acts on the large area of micro components while the small volume is used up very quick. If 100 μm of the supporting pillar of a macroscopic bridge are corroded away, nobody will care about that, but if 100 μm of a micro structure are corroded away, it may have vanished. On the other hand, the cost of the small volume of noble metal needed is of no matter.

The increase of chemical reactions is a disadvantage with respect to corrosion, but it is an advantage when a chemical analysis is to be made in a small volume. This is the basis for chemical micro reactors and micro analysis chips for DNA recognition and other biological or chemical assays.

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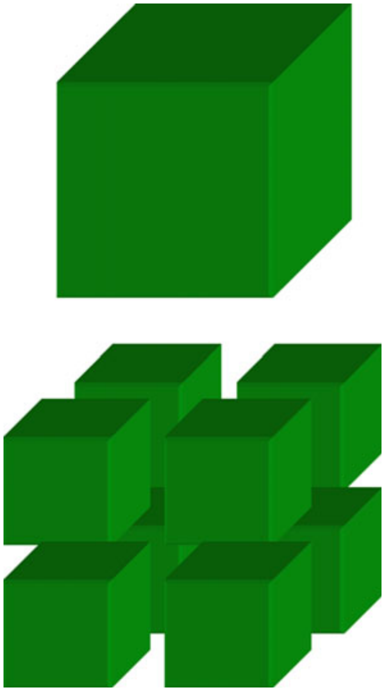

Fig. 1 Surface increase as a result of the separation of a cube

The surface to volume ratio is the reason also for a quick exchange of heat for micro components. The large surface facilitates heat exchange while the small volume results in a small heat capacity. The flow of heat, fluids, and electrical current is scaled down when the dimensions are reduced, because the cross-section is diminished with s_f^{-2} while the length is reduced by s_f^{-1} only. This shows that micro technique is more than just a reduction of dimensions but new concepts and new design principles are required.

This becomes especially clear when the scaling down of forces is investigated. All forces which act on the surface of objects become larger in comparison to forces acting on the volume or mass. Therefore, electrostatic force, piezo-electric force, capillary force, and friction are of more importance in micro technique than gravity, inertia, and magnetism which govern the macroscopic world.

The principal interrelationships which change the nature of things when their overall size is altered are called the scaling laws. They apply not only to effects and forces but to entire sensors and actuators as described in this book. The scaling laws are the reason why it was necessary to write this book which describes the distinguished designs required in micro technique.

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