2.6 Non-parametric Optimization

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A typical optimization problem in structural design is to find the optimal layout or shape of a structure within a specific region. Often the only known quantities are loads, supports, and constraints such as mass or material restrictions. With this starting point, the problem is not easily represented by design parameters. The purpose of non-parametric optimization is to give engineers a method to define a design space in regions or whole components without the process of defining the problem in design parameters. Frequently used non-parametric structural optimization methods are Topological and Local Growth Optimization (Bendsoe and Sigmund 2003; Haftka and Gürdal 1992; Rozvany 1997; Vanderplaats 1984). In the early stage of concept generation, Topological Optimization can be used to develop an efficient structural layout. In a later process of product development, Local Growth Optimization is an efficient tool to fine-tune the optimized structural proposal.

In the process of these optimization methods, the software implementation automatically does a parameterization of the design space. Standard mathematical optimization techniques are often not suitable for these problem formulations due to the high number internal design variables.

Fig. 2.18  Traveling salesman problem: find shortest route to visit all locations
2.6.1 Topological Optimization

Topological Optimization is a method for optimizing material layout within a defined design space with respect to loads and constraints. The process of Topological Optimization is similar to the process of bone mineralization in living creatures (Fig. 2.19). This biological process leads to a stiffness-optimized structure with minimum stresses and minimum weight by modifying the material distribution towards highly loaded areas.

In general the implementation consists of a Finite Element Analysis combined with an optimization technique for iterative updates to the material distribution. The design space is divided into small regions of varying density. Here we often use Finite Elements to define these regions. To find an optimized design, the density of each element of the FE-meshed design space is adjusted by an optimizer to match desired objective and constraints. No shifting of nodes is performed. In practice, the material density has been adjusted by modifying the stiffness of the corresponding Finite Element.

The result of a Topological Optimization run is a density field of the design space, which needs to be interpreted in most cases. Domains with high stiffness shape the structural design proposal of the component; domains with low stiffness form void areas.

In general the resulting design proposal needs to be re-designed and fine-tuned to satisfy manufacturing requirements. The main steps of Topological Optimization are represented in Fig. 2.20. The corresponding pseudocode is shown in Table 2.4.

Structural proposals by Topological Optimization often are infeasible to manufacture. Because of this, most commercial CAE-Software for Topological Optimization is commonly extended with features for considering manufacturing
Fig. 2.20 Topological Optimization: from problem definition to structural design proposal.
(a) Problem definition.
(b) Result of Topological Optimization.
(c) Re-designed proposal to satisfy manufacturing requirements

Table 2.4 Pseudocode: Topological Optimization

<table>
<thead>
<tr>
<th>Initial: Define problem and parameters:</th>
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<tbody>
<tr>
<td>– Define design space</td>
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<tr>
<td>– Define boundary conditions</td>
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<tr>
<td>Start \textit{ngenerations} loop:</td>
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<tr>
<td>– Evaluate FE-model</td>
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<tr>
<td>– Adjust density(and stiffness) of each FE-element</td>
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<tr>
<td>End loop</td>
</tr>
<tr>
<td>Stop</td>
</tr>
<tr>
<td>Derive structural design from proposal</td>
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constraints, such as casting constraints (Fig. 2.21), symmetry and pattern constraints, or member sizing directly in the formulation of the optimization problem. The fact that the user does not need to define complex design variables makes Topological Optimization an excellent tool in the early stages of design processes to find a first design proposal or to understand basic load paths.

Fig. 2.21 History of Topological Optimization of a trailer coupling, taking into account the direction in which the casting tools are removed. (a) Given design space at $t_0$. (b) Design proposal at $t_1$. (c) Design proposal at $t_2$. (d) Design proposal at $t_3$. (e) Design proposal at $t_4$. (f) Final design proposal at $t_{end}$. 
2.6.2 Local Growth

Biological structures such as tree stems change their own shape by growth and shrink their surfaces to adapt to external loads (c.f. Fig. 1.3b). Stress peaks are reduced by adding material at high stressed surface areas. The volume in low stressed areas is reduced in size by shrinking the surface. This biological growth rule creates lightweight structures with minimized notch stresses and maximized stiffness. Since this is often the preferred objective in structural optimization problems, the local surface growth process is adapted as an optimization tool in many CAE-Systems. There are various different approaches to solving these problems. With simple optimality criteria methods or growth rules, one can get good results.

The process of simulated growth is based on an iterative process of FE-simulations and an optimization technique that updates the surface to change the shape of the structure to meet with objectives and constraints. The shape perturbations are either manually defined by the user or automatically determined by the CAE-System. A common way to describe the shape changes of the Finite Element model is to define some shapes as a perturbation $b$ of nodal coordinates $r$.

$$ r = r_0 + b $$ \hspace{1cm} (2.11)

The new design can be generated by doing a linear combination of these shape vectors. The design variable is defined as the weighting factors $w_i$ of the shape vector,

$$ r = r_0 + \sum_{i=0}^{n} w_i b_i $$ \hspace{1cm} (2.12)

where $n$ is the number of shapes and design variables.

Figure 2.22 shows an example for optimizing notch stresses. We start with an initial geometry. The defined region we want to optimize is given by a simple shape.

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Fig. 2.22 Notch stress optimization results in smooth change of shape. (a) Problem definition. (b) Optimized shape
representation. After some iterations, we get a smooth shape with minimized notch stress that takes into consideration the applied loads and constraints.

### 2.7 Meta Models

Rolf Steinbuch

When we do any optimization or sensitivity study, we produce sets of data of goals, and restrictions and other information in a certain region of the space of possible or acceptable parameters. As the computation of each of these variants may be costly in terms of time and computing power, we generally should restrict the number of variants to the absolute minimum. From this motivation came the idea of meta-models. Surrogate models such as quadratic Response Surfaces (RS), which we will discuss in more detail later, and polynomial chaos expansions or Kriging (which are built from a limited number of runs of the original model) have been introduced as substitutes for the original time consuming FE-job model to reduce the total computational cost.

As designers in mechanical engineering, we are dealing with variants of a basic design of a macroscopic component. As the changes of the parameters are limited, the change of the systems response is limited and more or less smooth as well, when examined as functions of their parameters. So why don’t we use a sufficient number of data in an interesting region and approximate the responses by simple and smooth functions? If there are no catastrophic changes in the results of the computation, their representation, often called Response Surface (RS), should be a fairly regular function. Such functions often may be approximated by smooth functions. There are many other ways to define meta-models see, e.g. McKay et al. (1979), Au and Beck (1999), Das and Zheng (2000), Matthies et al. (2013), Dubourg et al. (2013), Bourinet et al. (2011).

Often we use low-level polynomials to do the approximation, e.g. parabolic interpolations. We restrict our presentation to these simple polynomial RS, as they help to explain everything we want to demonstrate.

**Example 2.3** For a 1D Problem, the second order RS is nothing more than the polynomial found by the classical least square method. So if we take a set of, e.g. five points \( \{(p, s)\} \)

\[
p = \{-2, -1, 0, 1, 2\}; s = \{2, 3, 4, 4, 2\}
\]

we may approximate them by
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