Chapter 2
Some Facts About Neutron and Reactor Physics

Abstract Chapter 2 describes some facts about neutron and reactor physics needed for the understanding of Chaps. 3–10. It starts with the radioactive decay and the definitions of the decay constant and the half-life. It continues with the explanation of the fission process for fissile nuclear isotopes, e.g. U-233, U-235, or Pu-239 and the fission energy release by creation of fission fragments (products), prompt fission neutrons and delayed neutrons and radiation (β-particles, γ-rays and antineutrinos). This is followed by the definition of reaction rates of neutrons with other atomic nuclei, the presentation of measured microscopic cross sections for absorption, capture and fission as well as the definition of the macroscopic cross section and the neutron flux.

In LWR cores the fuel is arranged heterogeneously in lattice cells together with a moderator (water) in order to slow down the fission neutrons with high kinetic energy to kinetic energies in the range of 0.025 eV (thermal energy). This is most effective if the enriched uranium fuel is put in cylindrical rods which are arranged in e.g. a square grid. The optimization of the geometrical distance between the fuel rods leads to important safety characteristics of LWR cores: the negative fuel Doppler coefficient and the negative coolant (moderator) coefficient.

The definition of the criticality factor or effective multiplication factor, \( k_{\text{eff}} \), allows a characterization whether the reactor core is operated in steady state condition or whether it is subcritical or even supercritical. The criticality or effective multiplication factor, \( k_{\text{eff}} \), can be changed by moving or by insertion or withdrawing of absorber material (boron, cadmium, gadolinium, indium, silver, hafnium, erbium) in the core. This allows control of the reactor. The reactor core is controlled always in a \( k_{\text{eff}} \) range where the delayed neutrons are dominating. The delayed neutrons are therefore of highest importance for the control of the reactor.

During reactor operation over months and years the initially loaded U-235 in the low enriched uranium fuel will be consumed, neutron absorbing fission products will build up or other heavy nuclei with masses above U-235 and Pu-239 will be created. This decreases the criticality of the effective multiplication factor \( k_{\text{eff}} \). This burnup effect on the criticality factor \( k_{\text{eff}} \) is accounted for by the design of the reactor core. The enrichment of the initially loaded fuel is increased such that \( k_{\text{eff}} \)...
becomes slightly $>1$. This is balanced by absorber materials (moveable absorber 
rods, burnable neutron poisons, e.g. gadolinium or boric acid) which keep the 
reactor core always at $k_{\text{eff}} \geq 1$.

After shutdown of the reactor the gradually decaying fission products and the 
radioactive decay of higher actinides creates afterheat in the reactor core. This 
afterheat (decay heat) must be transferred by the coolant water to outside coolant 
towers or to river or sea water.

Prior to the description of Light Water Reactor designs some basic characteristics of 
reactor physics and reactor safety will be presented. For a deeper understanding of 
these characteristics the literature given in the reference is recommended [1–8].

2.1 Radioactive Decay, Decay Constant and Half-Life

Radioactive decay changes the number of unstable (radioactive) isotopes, $N(t)$, 
existing per cm$^3$ as a function of time, $t$. This change can be described by the 
exponential law of

$$N(t) = N_0 \cdot \exp(-\lambda t)$$

where $\lambda$ is the decay constant and $N_0$ the number of atomic nuclei per cm$^3$ at 
the time $t=0$. Instead of the decay constant, $\lambda$, one can also use the half-life, 
$T_{1/2} = (\ln2)/\lambda$, which is the time by which half of the nuclei existing at $t=0$ have 
decayed. The decay rate, $\lambda \cdot N(t)$, is called the activity of a specimen of radioactive 
material. This activity is measured in units of Curie or Becquerel [1, 2].

One Becquerel, denoted Bq, is defined as one disintegration per second. One 
Curie, denoted Ci, is defined as $3.7 \times 10^{10}$ disintegrations per second, which is 
approximately the activity of 1 g of radium. Low activities are also measured in 
mCi = $10^{-3}$ Ci or $\mu$Ci = $10^{-6}$ Ci [1, 2].

2.2 Fission Process

If a neutron of a certain velocity (kinetic energy) is absorbed by a fissile heavy 
nucleus, e.g. U-233, U-235 or Pu-239, the resulting compound nucleus can become 
unstable and split (fission) into two or even three fragments (Fig. 2.1). The fission 
fragments are created essentially according to a double humped yield distribution 
function with mass numbers between about 70 and 165. The mass yield distribution 
functions are similar for heavy nuclei fissioned by neutrons with kinetic energies of 
0.0253 eV (thermal spectrum reactors) up to about 0.2 MeV for Fast Breeder
Reactors\(^1\) (Fig. 2.2). They depend slightly on the kinetic energy of the incident neutrons causing fission and on the type of heavy nuclei (U-233, U-235, Pu-239).

In addition to the fission products (fragments), 2–3 prompt neutrons are emitted during the fission process. These prompt fission neutrons appear within some \(10^{-14}\) s. They are created with different kinetic energies following a certain distribution curve around an average neutron energy of about 2 MeV. In some heavy nuclei with even mass numbers, e.g. Th-232 and U-238, nuclear fission can only be initiated by incident neutrons with a certain, relatively high, threshold kinetic energy (Table 2.1), whereas the uneven heavy nuclei, e.g. U-233, U-235, Pu-239 etc. can be fissioned by neutrons with all kinetic energies >0 eV. However, the even-uneven rule is not a rigorous one, e.g. Am-242\(^m\) can also be fissioned by thermal neutrons.

Fig. 2.1 Fission of U-235 nucleus by a thermal neutron

The fission products can either be solid, volatile or gaseous. Many of the fission products decay further emitting so-called delayed neutrons, \(\beta\)-particles, \(\gamma\)-rays and antineutrinos. The delayed neutrons resulting from the decay of particular fission products—called precursors—represent less than 1 % of all released neutrons.

\(^1\) 1 eV = 1.602 \times 10^{-19} \text{ J} \) is the kinetic energy acquired by an electron passing through a potential gradient of 1 V. 1 keV is equal to \(10^3\) eV and 1 MeV is equal to \(10^6\) eV. The energy of 0.0253 eV corresponds to a neutron velocity of 2,200 m/s.
The fraction $\beta$ of delayed neutrons originating from fissioning by thermal neutrons (0.0253 eV) of U-235 is $\beta = 0.67\%$, and $\beta = 0.22\%$ from fissioning of Pu-239. They appear following decay constants of 0.01–3 s$^{-1}$ for U-235 and 0.01–2.6 s$^{-1}$ for Pu-239. These delayed neutrons are of absolute necessity for the safe control and operation of nuclear fission reactors [6, 7, 11].

The total energy release per fission, $Q_{\text{tot}}$, appears as kinetic energy of the fission products, $E_f$, of the prompt fission neutrons, $E_n$, as $\beta^-$-radiation, $E_\beta$, as $\gamma$-radiation, $E_\gamma$, or as neutrino radiation, $E_\nu$, (Table 2.2). The neutrino radiation does not produce heat in the reactor core due to the small interaction probability of neutrinos with matter. Table 2.2 also shows the total energy, $Q_{\text{tot}}$, and the thermal energy, $Q_{\text{th}}$, released during fission of a nucleus. Some of $\beta^-$-radiation and $\gamma$-radiation of the fission products is not released instantaneously, but delayed according to the decay of the different fission products.

On the average, about 194 MeV or $3.11 \times 10^{-11}$ J are released per fission of one U-235 atom. Most of the fission energy is released instantaneously.

Since 1 g of U-235 meal contains $2.56 \times 10^{21}$ atoms, the complete fission of 1 g of U-235 results in:
For other fissile materials like U-233 or Pu-239 the energy release per fission is similar. Also fission by neutrons with thermal energies (0.025 eV) or by energies of 0.5 MeV leads to almost equal energy releases.

Therefore, it is usually assumed that the fission of the mass of 1 g of fissile material, e.g. U-235 or Pu-239 produces roughly 1 MWd\text{th} and the measure of burnup in MWd\text{th} per tonne of fuel also corresponds roughly to the number of grams of, e.g. U-235 fissioned in 1 ton of spent fuel [12].

### Table 2.2 Different components of energy release per fission of some heavy nuclei in MeV by incident neutrons of different kinetic energy (in the eV or MeV range) [10]

<table>
<thead>
<tr>
<th>Heavy nucleus</th>
<th>Incident neutron energy</th>
<th>E_\gamma</th>
<th>E_\beta</th>
<th>E_\nu</th>
<th>E_\nu</th>
<th>Q_{\text{tot}}</th>
<th>Q_{\text{th}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-235</td>
<td>0.025 eV</td>
<td>169.75</td>
<td>4.79</td>
<td>6.41</td>
<td>13.19</td>
<td>8.62</td>
<td>202.76</td>
</tr>
<tr>
<td></td>
<td>0.5 MeV</td>
<td>169.85</td>
<td>4.8</td>
<td>6.38</td>
<td>13.17</td>
<td>8.58</td>
<td>202.28</td>
</tr>
<tr>
<td>U-238</td>
<td>3.10 MeV</td>
<td>170.29</td>
<td>5.51</td>
<td>8.21</td>
<td>14.29</td>
<td>11.04</td>
<td>206.24</td>
</tr>
<tr>
<td>Pu-239</td>
<td>0.025 eV</td>
<td>176.07</td>
<td>5.9</td>
<td>5.27</td>
<td>12.91</td>
<td>7.09</td>
<td>207.24</td>
</tr>
<tr>
<td></td>
<td>0.5 MeV</td>
<td>176.09</td>
<td>5.9</td>
<td>5.24</td>
<td>12.88</td>
<td>7.05</td>
<td>206.66</td>
</tr>
<tr>
<td>Pu-240</td>
<td>2.39 MeV</td>
<td>175.98</td>
<td>6.18</td>
<td>5.74</td>
<td>12.09</td>
<td>7.72</td>
<td>206.68</td>
</tr>
</tbody>
</table>

7.96 \times 10^{10} \text{J} or 2.21 \times 10^{4} \text{kWh} or 0.92 \text{MWd}_{\text{th}} \text{thermal energy}

### 2.3 Neutron Reactions

Neutrons produced in nuclear fission have a certain velocity or kinetic energy and direction of flight. In a fission reactor core, e.g. with U-235/U-238 fuel they may be scattered elastically or inelastically or absorbed by different atomic nuclei. In some cases the absorption of neutrons may induce nuclear fissions in heavy nuclei (U-235 etc.) so that successive generations of fission neutrons are produced and a fission chain reaction is established.

#### 2.3.1 Reaction Rates

If $n(\vec{r}, v, \Omega)$ is the number of neutrons at point $\vec{r}$, with velocity $v$ and the direction of flight $\Omega$, then these neutrons can react within a volume element $dV$ with $N \cdot dV$ atomic nuclei ($N$ being the number of atomic nuclei per $\text{cm}^3$ of reactor volume). The number of reactions per second e.g. scattering or absorption, is then proportional to
\[ v \cdot n \left( \vec{r}, v, \Omega \right) \] and to \( N \cdot dV \)

The proportionality factor \( \sigma(v) \) is a measure for the probability of the nuclear reactions and is called microscopic cross section of the nucleus for a specific type of reaction. The microscopic cross section \( \sigma(v) \) is measured in \( 10^{-24} \text{ cm}^2 \equiv 1 \text{ barn} \). It is a function of the velocity, \( v \), or kinetic energy, \( E \), of the neutron and of the type of reaction and differs for every type of atomic nucleus. As for an absorption reaction the neutron can either remain captured or lead to fission of a heavy nucleus the relation

\[ \sigma_a(v) = \sigma_c(v) + \sigma_f(v) \]

is valid with

- \( \sigma_a(v) \) microscopic absorption cross section
- \( \sigma_c(v) \) microscopic capture cross section
- \( \sigma_f(v) \) microscopic fission cross section

The reaction rate can be written

\[ R \left( \vec{r}, v \right) = \sigma(v) \cdot N \left( \vec{r} \right) \cdot v \cdot n \left( \vec{r}, v \right) = \Sigma \left( \vec{r}, v \right) \cdot \phi \left( \vec{r}, v \right) \]

The quantity \( \Sigma \left( \vec{r}, v \right) = N \cdot \sigma(v) \) is called macroscopic cross section.

The quantity \( \phi \left( \vec{r}, v \right) = v \cdot n \left( \vec{r}, v \right) \) is called the neutron flux.

Figure 2.3 shows the microscopic fission cross section as a function of the neutron kinetic energy for the heavy nuclei U-235, U-238 and Pu-239. The fission cross sections for U-235 and Pu-239 increase with decreasing kinetic energies. In the energy region of about 0.1–10³ eV this behavior is superposed by resonance cross sections [12–14].

The capture cross section for U-238 (Fig. 2.4) shows distinct narrow resonance peaks above about 5 eV. At medium neutron energies (keV range) the resonance peaks become smaller and above about 10 keV—in the so-called unresolved resonance energy range—they cannot be resolved any more by experiments because of resonance overlapping. These resolved and unresolved resonance peaks broaden if the temperature of the U-238 fuel increases. This phenomenon...
is very important for the fuel-Doppler-temperature coefficient, which determines—among other temperature coefficients—the safety characteristics of nuclear power reactors [12, 18].

The microscopic fission cross sections of U-235 or Pu-239 (Fig. 2.3) become higher—with the exception of the energy range where the resonances occur—at low neutron energies (about <0.1 eV).

![Fig. 2.3](image1.png)

**Fig. 2.3** Microscopic fission cross sections (measured in barn) for U-235, U-238 and Pu-239 [13]

![Fig. 2.4](image2.png)

**Fig. 2.4** Microscopic capture cross section for U-238 [13]
Therefore, the U-235/U-238 fuel in Light Water Reactor cores is mixed with materials of low mass number (moderator) in order to slow down the fission neutrons having high kinetic energy by a number of elastic and inelastic collisions to kinetic energies in the range < 1 eV (Fig. 2.3). This is most effective if the enriched uranium fuel is put in cylindrical rods which are arranged in e.g. a square grid. This lattice of fuel rods arranged with certain distances must be cooled by a flowing coolant which can be identical with the moderator as in case of water in Light Water Reactors. The fission neutrons after a few collisions with the fuel atoms then fly with high velocity into the surrounding water/moderator. They are slowed down by collisions within a short distance to so-called thermal energies of 0.025 eV (Fig. 2.5). The neutrons are then in thermal equilibrium with the relatively low kinetic energies of the water molecules. Another advantage of the deceleration of the neutrons in the surrounding water is given by the fact that the probability is lower that the neutrons can be captured in the resonance region of U-238 inside the fuel rods (Fig. 2.4) [1–5].

After the neutrons are thermalized within the moderator region they migrate back by diffusion processes into the fuel rods. As they have lower kinetic energies now also the microscopic fission cross sections are much higher (in the 0.025 eV energy range) than those for fission neutrons. The consequences are more fission reactions. Also the ratio between fission and capture reactions becomes more favorable in the fuel.

An optimum volume ratio between moderator and fuel for the grid of fuel rods is found around 2 for light water (H_2O). This optimal volume ratio can be achieved by adaption of the distance between the fuel rods. For heavy water (D_2O) as a moderator this optimal volume ratio is about 20 and for graphite as a moderator it is found to be around 54 [1–5].

Light water (H_2O) has a higher microscopic capture cross section than heavy water or pure graphite. Consequently it is possible to build and operate nuclear power reactors with natural uranium (0.72 % U-235 enrichment) if heavy water or graphite are used as moderator. In fact the first reactor used natural uranium as fuel and graphite as moderator. With light water as moderator in Light Water Reactors the uranium fuel must be enriched to 3–5 % in U-235 (depending on the burnup of the uranium fuel). Structural materials which must be used for the design of the reactor core should also have low microscopic capture cross sections. Light Water Reactors, therefore, use an alloy of zirconium and aluminum (Zircaloy) for the cladding of the fuel rods and grid spacers of the fuel elements [1–4].

Uranium dioxide (UO_2) with its high melting point (2,865 °C) and its good irradiation properties in the neutron field of the reactor core is used as fuel in LWR cores.
2.4 Criticality or Effective Multiplication Factor $k_{\text{eff}}$

The ratio between the number of newly generated neutrons by fission and the number of neutrons absorbed in the reactor core or escaping from the reactor is called the criticality factor or effective multiplication factor, $k_{\text{eff}}$.

When $k_{\text{eff}} = 1$, the reactor core is critical and can be operated in steady state. At $k_{\text{eff}} < 1$ the reactor core is subcritical, e.g. with control or absorber rods fully inserted in the core.

Boron, Cadmium or Gadolinium etc. can be used as absorber materials, either as metallic alloys in control rods or as burnable poisons in ceramic form in fuel rods and special poison rods or as a fluid, e.g. boric acid in the coolant of an LWR [1–4].

For a $k_{\text{eff}} > 1$ the reactor core is supercritical. More neutrons are produced than are absorbed in the reactor core or do escape from the core. The neutron chain reaction is ascending (reaction rates and the number of neutrons and, thus, reactor power increase as a function of time).

The criticality or effective multiplication factor $k_{\text{eff}}$ of a reactor core is determined by the proper choice of its geometrical dimensions (diameter and spacing of the fuel rods, diameter and height of the reactor core), by the choice of the moderator and coolant as well as by the choice of the fuel and structural materials. The choice of the U-235 enrichment of the fuel is of decisive importance for LWRs.

2.5 Neutron Density and Power Distribution

Figure 2.6 displays the spatial distribution of the neutron density in the range of thermal energies for a Pressurized Water Reactor (PWR). The distribution of fission reaction rates or of the power generated by fissions is essentially proportional to the
neutron density distribution. The absorber- or control-rods are partially inserted in axial direction in the PWR core. The control rods absorb neutrons and are responsible for the spatial distortions of the thermal neutron flux. They influence the criticality level and the spatial power distribution.

The spatial distribution of the neutrons with a certain velocity or kinetic energy and flight direction can be described by the Boltzmann neutron transport equation or by Monte Carlo methods [1–4]. For both cases, numerical methods in one-, two- or three-dimensional geometry were developed. Computer program packages (deterministic codes for the solution of the Boltzmann transport equation and Monte Carlo codes using stochastic solution methods) are available for various applications [1, 3–5, 18, 38].
For many practical applications it is sufficient to solve the neutron diffusion equation which is an approximation to the Boltzmann neutron transport equation.

The microscopic cross sections shown in Figs. 2.3, 2.4 and 2.7 are collected in a special format in cross section libraries, e.g. JEFF [15], ENDF/B [16], JENDL [17]. Their continuous energy range can be approximated and divided into a number of energy groups with specifically defined microscopic group cross sections applying codes, e.g. NJOY [19] or MC2-3 [20, 21]. The heterogeneous cell geometry of the reactor core (Fig. 2.5) can be accounted for by codes, e.g. WIMS [22] or MC2-3 [20, 21]. These computational methods are summarized in [23, 24].

Whole-core calculations can be done in diffusion theory by codes, e.g. DIF3D [25] or SIMULATE-4 [26]. Whole-core (Fig. 2.6) codes applying Boltzmann neutron transport theory were developed for two- and three-dimensional geometries. Examples for such computer codes are, e.g. DANTSYS [27] or PARTISN [28]. Monte Carlo Codes are available for both lattice (Fig. 2.5) and whole-core (Fig. 2.6) geometries. Such codes include, e.g. MCNP5 [29] or VIM [30].

The number of neutrons in the reactor core can be controlled by moving or adding, e.g. absorber materials (neutron poisons). This is done in a $k_{eff}$-range, where the delayed neutrons are dominating the transient behavior of the neutron flux. The delayed neutrons come into being in a time range of seconds. Therefore, the number of neutrons or the power in reactor cores can also be controlled safely by moving absorber materials in the time range of seconds [1, 2, 6, 11].

![Fig. 2.7 Microscopic capture cross sections of Cadmium and Gadolinium isotopes and B-10 (n,α) cross sections [13]](image-url)
2.6 Neutron Poisons for the Control of the Reactor Power

Neutron poisons are used to control the number of neutrons and the power in the reactor core. Such neutron poisons have high microscopic absorption cross sections for neutrons. Neutron poisons are, e.g. Boron, Cadmium, Gadolinium etc. Figure 2.7 displays microscopic absorption cross sections for Boron, Cadmium and Gadolinium isotopes.

The neutron poisons are inserted into the reactor core as, e.g. axially moveable cylindrical control rods (Pressurized Water Reactors) or axially moveable cruciform control elements (Boiling Water Reactors). Another possibility is to add boric acid ($\text{H}_2\text{BO}_3$) to the cooling water, or to extract it from its solution in the cooling water.

Withdrawing the, e.g. Cadmium or the Boron carbide control elements from the reactor core changes the effective multiplication factor from $k_{\text{eff}} < 1$ to $k_{\text{eff}} > 1$. Inserting the control elements changes $k_{\text{eff}}$ from 1.0 to $k_{\text{eff}} < 1$. This action changes the number of fission reactions and the power of the reactor, correspondingly. Similarly, the reactor can be controlled by the variation of the concentration of the boric acid in the cooling water. However, all variations of the effective multiplication factor $k_{\text{eff}}$ are limited by design such that they remain below about half of the fraction of the delayed neutrons (see Sect. 2.10).

2.7 Fuel Burnup and Transmutation During Reactor Operation

During reactor operation over months and years the initially loaded U-235 in the low enriched uranium fuel will be consumed due to neutron fission and capture processes. As a consequence also the initial criticality or effective multiplication factor $k_{\text{eff}}$ decreases. Neutron capture in U-235 leads to U-236. Subsequent neutron capture in U-236 leads to Np-237. Neutron capture in the fertile isotope U-238 leads to U-239 and after decay to Np-239 and further decay to the buildup of the new fissile isotope Pu-239. Subsequent neutron captures in Pu-239 lead to the higher Pu-Isotopes Pu-240, Pu-241, Pu-242. After $\beta^-$-decay of Pu-241 americium is created. Neutron capture in americium leads to curium. This increases somewhat the criticality or effective multiplication factor $k_{\text{eff}}$. Fission products originating from the fission of fissile isotopes decrease the criticality or effective multiplication factor $k_{\text{eff}}$ due to their absorption cross sections. The combination of these three effects results in a time dependent change—usually a decrease—of the criticality factor, $k_{\text{eff}}$, during reactor operation.

This burnup effect on $k_{\text{eff}}$ is accounted for by design of the reactor core. The enrichment of the initially loaded fuel is increased such that $k_{\text{eff}}$ becomes slightly $>1$. As the $k_{\text{eff}}$ shall be equal 1 during the whole reactor operation cycle, this is balanced by absorber materials in the core (moveable absorber rods or special rods
with burnable absorber material or burnable absorber materials dissolved in the coolant or mixed with the fuel). The accumulating fission products and the decreasing $k_{\text{eff}}$ are counteracted, e.g. by moving absorber rods slowly out of the core during reactor operation. At the end of the operation cycle the absorber rods are almost withdrawn out of the core and spent fuel must be unloaded and replaced by new fuel elements.

### 2.7.1 Prediction of the Burnup Effects

The calculation of the change in concentration of all isotopes, actinides and fission products requires besides the power history or the equivalent time-dependent neutron flux distribution, the knowledge of the microscopic cross sections and decay constants of all isotopes as well as the yields of fission products during the reactor operation [18, 23, 31–34, 38]. The solution of a coupled system of ordinary differential equations with these data as coefficients and given initial concentrations at $t = 0$ results in the concentration of each isotope at the time $t$ during reactor operation. Figure 2.8 shows the masses of the most important isotopes for 1 ton of initial reactor fuel after a reactor operation time of 6 years and a fuel burnup of 60,000 MWd/t.

### 2.8 Reactor Control and Temperature Effects

A start up of the reactor by withdrawing control elements leads to a rise in reactor power and temperatures. Temperature changes provoke changes in material densities and microscopic cross sections by the Doppler broadening of resonances (see Sect. 2.2). Also the neutron energy spectrum can be shifted by moderation of the neutrons [11, 23]. All these effects together result in changes of the criticality or effective multiplication factor, $k_{\text{eff}}$.

The design parameters of the reactor core are selected such that increases of power and temperatures always lead to a smaller criticality or effective multiplication factor $k_{\text{eff}}$. Therefore, a power increase is only possible by withdrawing absorber control elements or decreasing the concentration of boric acid in the cooling water. In Boiling Water Reactors the power can also be increased by increasing the coolant flow, which leads to smaller concentrations of steam bubbles in the reactor core.

The most important safety design requirements for LWRs are, therefore

- a negative fuel-Doppler-temperature coefficient
- a negative coolant/moderator-temperature coefficient
2.9 Afterheat of the Fuel Elements After Reactor Shut Down

For reactor shut down the absorber/control elements are inserted into the reactor core and the coolant flow is drastically reduced. After reactor shut down the fuel elements—having reached their maximum burnup—can be unloaded. However, although the power is shut down, the gradually decaying fission products generate heat in the reactor core, even if the neutron fission chain reaction has been interrupted (after shut down of the reactor core). This afterheat, or decay heat, is composed of the contributions by the decay chains of the fission products and of contributions of radioactive decay by U-239, Np-239, and the higher actinides, which are unstable. It is a function of the power history of the reactor core before shutdown and is thus strongly influenced by the burnup of the fuel. Figure 2.9 shows the relationship between the power of the fuel elements in the reactor core of a PWR after shut down, $P(t)$, and the power during operation, $P_0$. The afterheat, $P(t)$, drops very sharply as a function of time. Shortly after shut down it is about 6%, after 6 h it is still about 1%, after 1 week 0.3%, after 3 months about 0.1%, and after 1 year it is 0.04% of the nominal reactor power, $P_0$, during operation.

After reactor shut down this afterheat must be transferred to the cooling towers or a river by the normal cooling system. After unloading from the reactor core the spent fuel elements are cooled in intermediate spent fuel storage pools [11].
Power reactors are generally operated at constant criticality or steady power. Exemptions are: startup conditions and power rise, transition from partial to full load power, reactor shut down and accidental conditions. Accidental conditions must be analyzed and presented to licensing authorities prior to begin of reactor operation. Such accidental conditions are, e.g. inadvertent or faulty withdrawal of absorber/control elements (increase of the criticality or effective multiplication factor, $k_{\text{eff}}$, above 1.0) or coolant loss as a consequence of pipe rupture or a faulty opening of valves followed by primary coolant pressure loss. An increase of the criticality or effective multiplication factor, $k_{\text{eff}}$, provokes an increase of fission reactions and a rise of power as a function of time. The increase of power results in an increase of fuel temperature and by thermal conduction—with a certain time delay—also to an increase of the coolant temperature. As already mentioned this results in important negative feedback effects which counteract the power increase. These negative feedback effects and the delayed neutrons allow the safe control of nuclear reactors. These important negative feedback effects—already mentioned above—will be explained now in more detail.
2.10.1 The Fuel-Doppler-Temperature Coefficient

The fuel-Doppler-temperature coefficient is due to the fact that the microscopic resonance cross sections depend on the temperature of the fuel and the relative velocities, respectively, of neutrons and atomic nuclei [8, 35].

The resonance cross sections for U-238, U-235, Pu-239, etc. show very pronounced peaks at certain neutron kinetic energies (Figs. 2.3 and 2.4). An increase in fuel temperature broadens this shape of the resonance curve and lowers its peak which, in turn, results in a change in the fine structure of the neutron energy spectrum. The neutron reaction rates for capture and fission are changed as a consequence. Above all, the resonance absorption for U-238 increases as a result of rising fuel temperatures, while the effect of a temperature change in the resonance cross sections of the fissile materials, U-235 and Pu-239, is so small that it can generally be neglected if the fuel enrichment is not extremely high. For these reasons, temperature increases in the fuel result in a negative temperature feedback effect (Doppler effect) brought about by the increase in neutron absorption in U-238. The Doppler effect is somewhat less pronounced at very high fuel temperatures because adjacent resonances will overlap more and more. The resonance structure then is no longer as pronounced as at low temperatures, which leads to a reduction of the negative Doppler effect.

As a consequence of the negative fuel-Doppler-temperature coefficient the criticality or effective multiplication factor, \( k_{\text{eff}} \), decreases at higher fuel temperatures. Typical values for the fuel-Doppler-temperature coefficient are

- for PWRs \(-2.5 \times 10^{-5}\) change in \( k_{\text{eff}} \) per degree of fuel temperature increase
- for BWRs \(-2 \times 10^{-5}\) change in \( k_{\text{eff}} \) per degree of fuel temperature increase

As the fuel-Doppler-temperature coefficient is coupled to the fuel temperature it is acting practically instantaneously.

2.10.2 The Moderator/Coolant-Temperature Coefficient of LWRs

The main contribution to the coefficients of moderator or coolant temperatures stem from changes in the densities of the moderator or coolant and from resultant shifts in the neutron energy spectrum. Temperature rises decrease the density of the coolant and accordingly reduce the moderation of neutrons. The neutron spectrum is shifted towards higher energies. As a result of the lower moderator density and the correspondingly higher transparency to neutrons of the core it is also possible that appreciably more neutrons will escape from the reactor core and neutron losses due to leakage rate will increase [8, 34, 36].
For the present line of PWRs, the sum total of the individual contributions to changes in various energy ranges finally leads to a negative coefficient of the moderator temperature which, however, also depends on the concentration of boric acid dissolved in the coolant and the burnup condition of the reactor core.

Figure 2.10 shows the criticality or effective multiplication factor, $k_{\text{eff}}$, as a function of the volume ratio $V_{\text{H}_{2}\text{O}}/V_{\text{UO}_{2}}$ in a lattice cell for two examples (3.2 % U-235 enriched U-235/U-238 fuel and 3.0 % Pu-239/Pu-241 enriched plutonium/uranium dioxide fuel [37]).

The three curves of Fig. 2.10 are valid for fuel with 3.2 % U-235 enriched uranium (U-235/U-238) dioxide fuel and for 3.0 % (Pu-239/Pu-241) enriched so-called mixed plutonium/uranium dioxide fuel as well as for two fuel rod diameters in case of the plutonium/uranium fuel.

LWRs are always designed with an undermoderated lattice cell in the fuel element (left side of the $k_{\text{eff}}$ curves in Fig. 2.10). In this case a temperature increase followed by a decrease of the moderator (H$_2$O) density and of the effective water volume $V_{\text{H}_{2}\text{O}}$ as well as of the ratio $V_{\text{H}_{2}\text{O}}/V_{\text{UO}_{2}}$ results in a decrease of the criticality or effective multiplication factor $k_{\text{eff}}$ (shift to the left). For LWRs the moderator/coolant-temperature coefficient is:
for PWRs $-2 \times 10^{-4}$ change in $k_{\text{eff}}$ per degree of temperature increase of the water
for BWRs $-1.3 \times 10^{-3}$ change in $k_{\text{eff}}$ per\% increase of steam volume (void coefficient)

The negative moderator temperature- or void-coefficient determine the safety behavior of LWRs during coolant loss accidents [11, 36]. Contrary to the fuel-Doppler-temperature coefficient which acts in case of power increases instantaneously, the coolant/moderator-temperature coefficient can react only with a certain time delay during power transients. This is due to the fact that in the case of a fuel temperature increase the moderator temperature or vapor production increase only after a certain time delay (thermal conductivity in the fuel and cladding). However, in case of moderator/coolant depressurization (pipe break or faulty opening of a valve in the primary coolant system) the void coefficient also reacts instantaneously.

Figure 2.10 shows the criticality or effective multiplication factor for low enriched uranium fuel and for low enriched plutonium/uranium fuel. The curves for these two fuel types are shifted against each other. Usually, for uranium fuel a design value of $V_{\text{H}2\text{O}}/V_{\text{UO}2} = 2.1$ is selected for PWRs to obtain a sufficiently negative moderator/coolant-temperature coefficient. For plutonium/uranium fuel a ratio $V_{\text{H}2\text{O}}/V_{\text{UO}2} = 3.0$ is chosen.

Figure 2.10 shows another important result for the case of a molten core as a result of severe core melt accidents. In case of a molten core, the water in LWRs is evaporated, the lattice structure (Fig. 2.5) is destroyed and the fuel is arranged in form of a core melt. In this case $V_{\text{H}2\text{O}}/V_{\text{fuel}} \rightarrow 0$ and $k_{\text{eff}} < 0.9$, i.e. the molten rearranged core material is subcritical (see Chap. 9).

2.11 Behavior of the Reactor in Non-steady State Conditions

As has been explained above, $k_{\text{eff}} = 1$ corresponds to the steady state condition of the reactor core, in which case the production of fission neutrons is in a state of equilibrium with the number of neutron absorbed and the number of neutrons escaping from the reactor core.

For $k_{\text{eff}} \neq 1$, either the production or the loss term become dominant, i.e., the number of neutrons $n(t)$ and $k_{\text{eff}}(t)$ vary as a function of time.

Axial movements of the absorber rods in the core change the loss term of neutrons and influence $k_{\text{eff}}(t)$. The relative change as a function of time $k_{\text{eff}}(t)$ is called reactivity $\rho(t)$.
Reactivity \( \rho(t) = \frac{k_{\text{eff}}(t) - 1}{k_{\text{eff}}(0)} = \frac{\Delta k_{\text{eff}}(t)}{k_{\text{eff}}(0)} \)

\( \rho(t) \) is measured in units of the fraction of the delayed neutrons \( \beta \approx 0.0064 \) for U-235. Historically it was defined that \( \beta = 1 \) dollar (1 $) = 100 cents.

The behavior of the reactor power, temperatures or other changes of the steady state conditions, e.g. variations of the system pressure or coolant velocity can be described by a system of differential equations [8, 11, 18, 35, 36]. These are:

- The differential equations for the instationary neutron kinetics (space- and time-dependent prompt neutron flux distribution and concentrations of the delayed neutrons and their precursor atoms)
- The differential equations for the space and time dependent temperature fields in the fuel, the cladding and coolant of the reactor core (including the material properties of the different materials, e.g. thermal conductivity, heat capacity, etc.)
- The equations for the feedback effects affecting the effective multiplication factor, \( k_{\text{eff}} \), as, e.g. the fuel-Doppler-temperature coefficient and the moderator/coolant-temperature coefficient
- The equations for the time dependent temperatures and pressures at the inlet of the reactor core caused by perturbations on the secondary side of the steam exchangers.

Not in all cases all parts of these coupled systems of differential equations must be solved together. In case of relatively fast variations of the physical core characteristics (time range of seconds or less), e.g. the core inlet coolant temperature can be considered to remain constant, as the steam generators parameters change only slowly.

In many cases the instationary neutron kinetics can be approximated by a system of coupled ordinary differential equations with initial conditions. In this case the prompt and delayed neutrons are represented by one ordinary differential equation and six differential equations for the precursor atoms which emit the delayed neutrons by radioactive decay. This leads to seven ordinary differential equations. The solution of these systems of coupled ordinary differential equations shows that three ranges of \( k_{\text{eff}} \) are of importance:

For supercriticality \( k_{\text{eff}} > 1 \) two ranges of \( k_{\text{eff}} \) must be distinguished.

- The range between \( k_{\text{eff}} = 1 \) and \( k_{\text{eff}} < 1 + \beta \) in which the multiplying chain reaction is determined by the \textbf{delayed neutrons}. In this range of \( k_{\text{eff}} \) the relatively slowly originating delayed neutrons (from the radioactive decay of precursor atoms (see Sect. 2.1)) allow the control of the nuclear reactor in a time range of seconds to minutes. Control procedures by moving control rods or changes of the concentration of boric acid in the coolant water are performed in this range as displayed by Fig. 2.11.
- The range of \( k_{\text{eff}} > 1 + \beta \) in which the multiplying chain reaction is determined by the \textbf{prompt neutrons} originating promptly from the fission process
In this range the time difference of successive neutron generations is given by the lifetime of the prompt neutrons which is about $2.5 \times 10^{-5}$ s for LWRs with U-235 enriched uranium fuel. This means that the chain reaction multiplies very fast. The very rapidly increasing number of fissions and the reactor power as well as the fuel temperatures are, however, reduced by the countering negative fuel-Doppler-temperature coefficient. This reduces the power and after having attained a certain peak level the power drops again. LWRs are designed such that the $k_{eff}$ is limited such that the energy released by the power peak is small and limited.

- The range of $k_{eff} < 1$ if control/absorber elements are inserted into the reactor core or the concentration of boric acid is increased in the cooling water. In this case the power drops rapidly within seconds to the afterheat level.

Figure 2.11 explains the three ranges of $k_{eff}$ which are important for the description of the non-steady or instationary behavior of nuclear reactors. All control procedures, e.g. withdrawal of control/absorber elements or increase of the boric acid concentration in the coolant water are performed in the delayed prompt critical range of $k_{eff}$. The design of an LWR core must guarantee that the super prompt critical range is never attained during normal operation. In case that the super prompt critical range should be attained during accidental condition, then the fuel-Doppler-temperature coefficient will limit the energy released in a power peak. In addition rapid automatic reactor shut down by control/absorber rods will limit the damage to the reactor core.

Similarly, the negative moderator/coolant-temperature coefficient or void coefficient will drastically reduce the reactor power in case of depressurization of the primary coolant system by large scale pipe breaks or faulty opening of valves in the primary system.
References

The Risks of Nuclear Energy Technology
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