Preface

The discovery in 1958 of magnetically trapped energetic particles surrounding planet Earth [1, 2] marked the beginnings of a new scientific discipline—Space Physics. Early US and Soviet satellites provided a wealth of initial data on the radiation belts; internationally coordinated satellite missions in the 1970s led to the study of relevant source, acceleration and diffusion processes. In the 1980s and 1990s, with the Pioneer, Galileo and Cassini missions, space physics turned its interest to the outer planets; terrestrial radiation belt studies fell somewhat into a hiatus. The situation, however, is changing, with several satellite missions like the NASA Van Allen Probes, the Canadian Outer Radiation Belt Injection, Transport, Acceleration, and Loss Satellite (ORBITALS) and the Japanese Energization and Radiation in Geospace (ERG) mission, all designed to enhance our understanding of radiation belt dynamics. After only a few months in orbit, the Van Allen Probes duo already has provided unsuspected and tantalizing results on the dynamics of geomagnetically trapped particles [3, 4]. This whole development—we may call it rejuvenation of radiation belt physics—will also require revitalizing the theoretical studies of all physical processes involved.

This book represents a thorough revision, expansion and update of Juan G. Roederer's 1970 "classic" *Dynamics of Geomagnetically Trapped Radiation* [5] (Russian translation: *Dinamika Radiatsiy, Zakhvatchennoi Geomagnitnim Polyem*, Publishing House Mir 1971, Moscow). It is complemented and amplified with material from more recent lecture notes from both authors and their research experience in the study of terrestrial and Jovian magnetospheres.

Like its old predecessor, *this is not a cookbook* with recipes that can be followed blindly to obtain some concrete results. Emphasis is not on the "what" but on the "why" of things out there in near-earth space. Our goal is not to tell but to explain, leading the student and beginning scientist on a path of *understanding* the underlying physics—rather than just offering a collection of laws and rules and their mathematical expressions. Our hope is to provide the reader with the mental tools to find out, not only what physical variables are related to each other, but *why* they are related at all, and to seek new causal relationships as new experimental results become available. In short, old equations can be looked up in books—new ones must be created by the brain. Essential theoretical complements for the present volume are Northrop's classic booklet [6], mathematically rigorous but highly compressed (it has everything!), and the detailed and highly didactical treatise by Rossi and Olbert [7] (it's all relativistic!).

The senior author (JGR) thanks his wife Beatriz for her infinite patience when, after almost 20 years of "active retirement" and his defection into a biology-related research field, he decided to come back to the good old topic on which he once built his career as a young scientist. The junior author (HZ) wishes to express her gratitude to Robert McCoy, Director of the Geophysical Institute, for his encouragement and the financial support from the Geophysical Institute for her time to participate in this enterprise. And both authors are immensely grateful to Solène Lejosne, who during the course of her thesis work at the University of Toulouse (France) [8] had contacted JGR with a number of very pointed questions and comments on his old book. On that basis we took the fortunate decision to ask Ms. Lejosne to help us as an unbiased critical reviewer from a grad-student's perspective, which she did with painstaking precision and dedication. Her collaboration confirmed the long-suspected fact that young scientists are far more valuable judges of the pedagogical value of a written piece than older experts!

The prerequisites for understanding the physics discussed in this book are some basic understanding of the magnetosphere and its principal regions and perturbations, and a reasonably good knowledge of electromagnetism, up to and including Maxwell's equations. To remind the reader of some necessary tools from that latter discipline, we have written Appendix A.1 which, rather than a mere presentation of algebraic, differential and geometric vector relationships between magnetic and electric field quantities needed in the main text, presents a discussion of the *physical meaning and field-topological consequences* of such relationships. We briefly elaborate in a perhaps less traditional way on Maxwell's equations and some important conceptual aspects that are germane to a better understanding of cause-and-effect relationships in the physics of radiation belts and magnetospheric plasmas. We strongly recommend that Appendix A.1 be read first.

Chapter 1 develops the adiabatic theory of particle motion from first principles. It is presented as a prime example of *physics as the art of modeling*, in which a complex real system is replaced by a highly simplified *virtual*, i.e., imagined one. In our case, the original system is a charged particle in complex multi-periodic motion in a magnetic field, replaced by a so-called *guiding center particle* (a model!) of equal mass and charge, moving in a smooth, uncomplicated way (having averaged out the smaller scale turns and loops of the real particle). The instantaneous position of this virtual particle is the *guiding center*, a point in space whose coordinates depend on the local magnetic field and the properties of the real particle. In addition, we show why the guiding center particle is endowed with a *magnetic moment* which impersonates average electromagnetic properties of the rapidly cycling motion of the original particle. The notion of *drift velocity* is introduced, and other fundamental physical magnitudes germane to adiabatic motion are defined; drift velocities are classified into zero order (independent of the particle's dynamic properties), first order and higher order. The *first adiabatic* *invariant* is defined and a simple demonstration of its near-constancy under certain restrictions (the adiabatic conditions) is given. We then discuss in detail what happens to a charged particle in its cyclotron motion when the magnetic field is slowly time-dependent, demonstrating the fundamental role of the often neglected magnetic vector potential. Examples are given for zero and first order drifts in simple field configurations. However simple, these examples (e.g., ion pick-up, adiabatic breakdown, closed vs. open drift orbits and their separatrix, co-rotating vs. convecting regions in an externally imposed electric field) illustrate some important basic properties of particles trapped in the equatorial region of the magnetosphere. In the course of this chapter, we emphasize that *magnetic field lines* are purely geometric entities, useful for one's mental representation of magnetic field configurations but, like the notion of a guiding center particle, devoid of any physical reality; nonetheless we give a phenomenological definition of field line velocity—again putting to good use the vector potential and its local time variation.

Chapter 2 formalizes the definition of drift velocity in general terms for arbitrary field configurations; some specific second order drifts are discussed. We show *why* guiding center particles follow curved magnetic field lines (a non-trivial fact) and discuss the conditions for that to happen. Since drifts are, by definition, perpendicular to the local magnetic field vector, we then examine the *parallel motion* of the guiding center, introducing the all-important concepts of *particle trapping*, *mirror points* and periodic *bounce motion*. In discussing the concept of bounce period we show that, like a pendulum at rest, even non-bouncing equatorial (90°-pitch angle) particles have an intrinsic bounce period (with which external perturbations can resonate). In the second part of this chapter, we take a close look at the guiding center particle's parallel acceleration and total kinetic energy change along a magnetic field line in the presence of a field-aligned electric field, deriving the *energy equation* and the so-called *betatron and Fermi accelerations*. The chapter ends with a thorough discussion of the effects of given potential (conservative) *parallel electrostatic fields* on a particle's bounce motion, identifying distinct regions of behavior in a parallel/perpendicular velocity map—a subject of importance in auroral physics.

Chapter 3, based mainly on the 1970 edition [5], defines the fundamental concept of trapped particle *drift shells*. Two additional field-geometric concepts are introduced and added to the concept of guiding center: the *guiding field line* and the *guiding drift shell*. All this leads to the definition of the *second adiabatic invariant* and several related simplified expressions thereof valid for some special cases. The conservation of the second invariant is demonstrated (in Appendix A.2). Using this theorem, it is possible to trace drift shells in general fields, and despite the "cookbook" disclaimer at the beginning of this Preface, some practical recipes are given for numerical methods to accomplish this. The concepts of *shell splitting* and *pseudo-trapping* in azimuthally asymmetric fields are discussed, and consequences for particle trapping and diffusion are mentioned. Several examples are analyzed in detail and analytical expressions for near-equatorial particles are given. A special look is taken at the dipole field as a first approximation of the geomagnetic field. Old but still much used quantities are introduced and discussed, such as the *L-value* and the system of *invariant coordinates*. We examine the situation of near-equatorial particles (pitch angles near 90°), for whose drift orbits analytical relationships can be written down for first-order magnetospheric field approximations. The last part of this chapter deals with slowly time-varying fields and the resulting effects on drift shells. We introduce the *third adiabatic invariant*, a purely field-geometric quantity (the magnetic flux enclosed by a drift shell), and demonstrate (in Appendix A.3) its constancy under adiabatic conditions. The process during the adiabatic change is examined "under a microscope", showing that adiabatic constancy is not absolute but valid only when averaged over a drift period: under time-dependent conditions, identical particles on the same drift shell share a common drift shell only at times that are integer multiples of their drift period. Without proof, we mention that this really is also true for the other two adiabatic invariants and their related periodicities. A generalized L-parameter, or "*L-star*" is introduced, and a general method for its calculation is presented.

Chapter 4 comes down from the lofty heights of pure theory and introduces the concept of flux of an ensemble of adiabatically behaving particles. It defines the corresponding physical concepts of differential *directional flux* and that of the ubiquitous *distribution function* as average quantities linking actual particle ensembles with macroscopic, measurable quantities like the mass, charge, number and energy densities; bulk velocity; pressure and temperature. Particular attention is given to transformation of phase-space related coordinates and corresponding *transformations of the distribution function*. A whole section is dedicated to the *pressure tensors* and related definitions, and the physical meaning of their components, especially the perpendicular and parallel pressures in the case of a magnetically trapped ensemble of charged particles. In the definition of the perpendicular and parallel temperatures, we try to deactivate the usual tendency to associate these concepts with a Maxwellian distribution of particle velocities, showing that even a mono-energetic and mono-pitch angle ensemble of particles can have both. The example of a static distribution of trapped particles is examined in detail. Another section is dedicated to flux mapping in *invariant* Φ, J, M *space* and corresponding expressions, including their mutual transformations and relations to particle distribution functions in phase space and other invariant coordinate spaces. The chapter concludes with a brief discussion of trapped *particle diffusion*, with emphasis on the processes involved and their effects on the distribution function. The *Fokker-Planck equation* is derived for the radial diffusion case, and some general qualitative rules for the determination of *diffusion coefficients* are given.

The aim of Chap. 5 is to analyze and help understand the magnetospheric plasma as a self-organizing entity with self-generated electromagnetic fields (whereas in all previous chapters, the magnetic and electric fields were *given*, of sources external to the particle population). It starts with an introduction to *collisionless plasma physics* exclusively based on the understanding of adiabatic motion of individual particles. The concepts of *particle (kinetic) and guiding center fluids* are introduced as yet another example of "physics as the art of modeling". The corresponding distribution functions and their relations to macroscopic quantities are examined for the hypothetical case of identical particle ensembles, linking magnetization density with perpendicular pressure in a guiding center fluid. On the basis of very simple examples (throughout the text we call these "kindergarten examples"), the physical meanings of *equivalent and convective current densities* and their return circuits in a guiding center fluid are analyzed in detail, with emphasis on their origin in geometric aspects of cyclotron motion. Different classes of current densities are defined in general terms and their role in the generation of *magnetic stresses* in a particle ensemble is thoroughly examined. We finally turn to quasi-neutral mixtures of positive and negative particles, introducing the so-called *center of mass fluid*, discussing its properties and related equations. The concept of quasi-neutrality is examined and the plasma parameter known as *Debye length* is introduced; the reason why the magnetic field does not appear in the Debye length is discussed explicitly. All this leads to the plasma *momentum and magnetohydrodynamic equations*. The chapter concludes with the introduction of collisions and the formulation of the so-called *generalized Ohm equation*; the physical meaning of its terms is discussed as well its link to Maxwell's equations and the "chicken-and-egg" problem of whether currents drive fields or fields drive currents in a plasma. Several simplifications for some special situations are discussed, introducing concepts like *Hall conductivity*, *magnetic field diffusion*, *frozen-in magnetic field lines* and *Alfvén waves*.

To conclude this Preface and to facilitate the job of eventual book reviewers, we list some likely critical comments and give our pertinent replies/justifications:

1. *There are no problems, questions or exercises*. Correct. We believe that any instructor lecturing on this subject will be perfectly able to create meaningful problems which are not just mathematical "plug-in" exercises, and are tailored appropriately to the level of his/her class. Besides, throughout the text there are several standard statements such as "it is easy to show that ...". So just do that as an exercise!
2. *Where are the data?* Nowhere. For two reasons: the book is on theory and the really exciting new data are only now coming in.
3. *Why are magnetospheric models not discussed?* To show the fundamental physics of trapped particles, it is enough to deal with uncomplicated, unsophisticated models that still feature the most important characteristics of the real field, and which can even lead to analytical expressions.
4. *Where are the Euler coordinates?* In some footnotes. They are very useful tools for the math involved, but devoid of physical meaning (no direct, intuitive relation to the sources of the field).
5. *There are very few references, and my own work is not quoted*. True. But for instance in basic physics textbooks there usually are no references whenever Newton's or Maxwell's equations are used. Ours is a textbook, not a collection of review articles. Besides, there *are* some references—of papers which include abundant literature sources.
6. *The title of the book should be "Kinematics of Magnetically ... "*. Touché! But consider this: in any basic dynamics textbook, we find statements such as "let's apply a force *here*" or "consider a system of mass points *under the following*

constraints ...", with no words about the interaction processes responsible for those forces or constraints. And none are necessary if we just want to understand what happens and why.

Happy reading and, more importantly, happy understanding!

Fairbanks, Alaska, USA — Juan G. Roederer
August 2013 — Hui Zhang

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