Preface

This book provides an alternative approach to study the pre-kernel solution of transferable utility games based on a generalized conjugation theory from convex analysis. Although the pre-kernel solution possesses an appealing axiomatic foundation that lets one consider this solution concept as a standard of fairness without referring to the notion of "interpersonal comparison of utility" – it has, in addition, a foundation that is based on a noncooperative bargaining model – many scholars, nevertheless, regard the pre-kernel and its related solutions as obscure and too technically complex to be treated as a real alternative to the Shapley value. Comprehensible and efficient computability is widely regarded as a desirable feature to qualify a solution concept apart from its axiomatic foundation as a standard of fairness. Subjects will distrust a division rule that can only be computed and "fully understood" by a handful of experts. It remains the flavor of being fleeced by the proposer or mediator. In case that decision makers have the feeling of being treated unfairly, they will obstruct an agreement. Especially if they are not able to compute their own share by themselves. Here, the Shapley value – if it is accepted as a fair division rule – offers the clear advantage that everybody can calculate his own share simply by applying its formula.

A central task in real-life managerial problems concerns the sustainability of the resource. Especially in areas where multilateral agreements are nonbinding, compliance becomes a crucial issue in order to avoid the destruction of a natural resource. Compliance can be achieved when agents exercise self-constraint and refrain from using their powers to exploit one another. Then a solution can be obtained that is acceptable for all participants: a fair compromise. Such a compromise will be considered a fair outcome when it produces a common virtual world where compliance is reality and obstruction is held to account. Rather than considering fairness as some opaque concept, in order to advance our understanding of compliance on nonbinding agreements, we study fairness on the set of principles (axioms) which describe the pre-kernel. However, the pre-kernel is not fully understood to give a simple and efficient computation method to qualify as an attractive fair division rule. Other rules of distributive justice are usually considered more attractive for implementing fairness in economical or in

experimental situations, which might, however, have originated in its simplicity of computation and not on its normatively compelling foundation.

In this monograph, we review and then improve an approach that was invented by Meseguer-Artola (1997) to compute the pre-kernel of a cooperative game by the indirect function. The indirect function is known as the Fenchel-Moreau conjugation of the characteristic function introduced by Martínez-Legaz (1996). By following and extending the indirect function approach proposed by Meseguer-Artola (1997), we are able to characterize the pre-kernel of the grand coalition simply by the solution sets of a family of quadratic objective functions. Now, each function can be solved by means well known from analysis and linear algebra. For economic situations which offer strong incentive for mutual cooperation, the formerly difficult issue of a fair division of the proceeds in accordance with the pre-kernel can now be easily accomplished and verified by partners involved in a common venture. Henceforth, the rules of distributive justice (axioms) which characterize the pre-kernel as a fair compromise shall be deemed as an alternative to solve and stabilize the self-administration problem of a natural resource. For many subjects fairness is a nebulous notion. Subjects have different perceptions of what they consider to be fair or unfair. Enlarging the set of rules for distributive justice that can be devoted and agreed upon for self-managing a natural resource might help to avoid its destruction.

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