Preface

Motivation A combined search at `Mathscinet` and `Zentralblatt` shows more than 800 articles with the expression "condition number" in their title. It is reasonable to assume that the number of articles dealing with conditioning, in one way or another, is a substantial multiple of this quantity. This is not surprising. The occurrence of condition numbers in the accuracy analysis of numerical algorithms is pervasive, and its origins are tied to those of the digital computer. Indeed, the expression "condition number" itself was first introduced in 1948, in a paper by Alan M. Turing in which he studied the propagation of errors for linear equation solving with the then nascent computing machinery [221]. The same subject occupied John von Neumann and Herman H. Goldstine, who independently found results similar to those of Turing [226]. Ever since then, condition numbers have played a leading role in the study of both accuracy and complexity of numerical algorithms.

To the best of our knowledge, and in stark contrast to this prominence, there is no book on the subject of conditioning. Admittedly, most books on numerical analysis have a section or chapter devoted to it. But their emphasis is on algorithms, and the links between these algorithms and the condition of their data are not pursued beyond some basic level (for instance, they contain almost no instances of probabilistic analysis of algorithms via such analysis for the relevant condition numbers).

Our goal in writing this book has been to fill this gap. We have attempted to provide a unified view of conditioning by making condition numbers the primary object of study and by emphasizing the many aspects of condition numbers in their relation to numerical algorithms.

Structure The book is divided into three parts, which approximately correspond to themes of conditioning in linear algebra, linear programming, and polynomial equation solving, respectively. The increase in technical requirements for these subjects is reflected in the different paces for their expositions. Part I proceeds leisurely and can be used for a semester course at the undergraduate level. The tempo increases in Part II and reaches its peak in Part III with the exposition of the recent advances in and partial solutions to the 17th of the problems proposed by Steve Smale for the mathematicians of the 21st century, a set of results in which conditioning plays a paramount role [27, 28, 46].

As in a symphonic poem, these changes in cadence underlie a narration in which, as mentioned above, condition numbers are the main character. We introduce them, along with the cast of secondary characters making up the *dramatis personae* of this narration, in the *Overture* preceding Part I.

We mentioned above that Part I can be used for a semester course at the undergraduate level. Part II (with some minimal background from Part I) can be used as an undergraduate course as well (though a notch more advanced). Briefly stated, it is a "condition-based" exposition of linear programming that, unlike more elementary accounts based on the simplex algorithm, sets the grounds for similar expositions of convex programming. Part III is also a course on its own, now on computation with polynomial systems, but it is rather at the graduate level.

Overlapping with the primary division of the book into its three parts there is another taxonomy. Most of the results in this book deal with condition numbers of specific problems. Yet there are also a few discussions and general results applying either to condition numbers in general or to large classes of them. These discussions are in most of the *Overture*, the two *Intermezzi* between parts, Sects. 6.1, 6.8, 9.5, and 14.3, and Chaps. 20 and 21. Even though few, these pages draft a general theory of condition, and most of the remainder of the book can be seen as worked examples and applications of this theory.

The last structural attribute we want to mention derives from the technical characteristics of our subject, which prominently features probability estimates and, in Part III, demands some nonelementary geometry. A possible course of action in our writing could have been to act like Plato and deny access to our edifice to all those not familiar with geometry (and, in our case, probabilistic analysis). We proceeded differently. Most of the involved work in probability takes the form of estimates—of either distributions' tails or expectations—for random variables in a very specific context. We therefore included within the book a *Crash Course on Probability* providing a description of this context and the tools we use to compute these estimates. It goes without saying that probability theory is vast, and alternative choices in its toolkit could have been used as well. A penchant for brevity, however, prevented us to include these alternatives. The course is supplied in installments, six in total, and contains the proofs of most of its results. Geometry requirements are of a more heterogeneous nature, and consequently, we have dealt with them differently. Some subjects, such as Euclidean and spherical convexity, and the basic properties of projective spaces, are described in detail within the text. But we could not do so with the basic notions of algebraic, differential, and integral geometry. We therefore collected these notions in an appendix, providing only a few proofs.

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