Equation (1.5) from Chap. 1 may also be used to define an efficiency or power coefficient $0 \leq c_P \leq 1$:

$$c_P = \frac{P}{\frac{1}{2} \rho A_r \cdot v^3}. \quad (2.1)$$

Wind turbine aerodynamic analysis frequently involves the derivation of useful equations and numbers for this quantity. Most (in fact, almost all) wind turbines use rotors which produce torque or moment of force

$$M = \frac{P}{\omega}. \quad (2.2)$$

with $\omega = \text{RPM} \cdot \pi/30$ the angular velocity. Comparing tip speed $v_{\text{Tip}} = \omega \cdot R_{\text{Tip}}$ and wind speed, we have

$$\lambda = \frac{v_{\text{Tip}}}{v_{\text{wind}}} = \frac{\omega \cdot R_{\text{Tip}}}{v_{\text{wind}}}. \quad (2.3)$$

the tip speed ration (TSR). Figure 2.1, sometimes called the map of wind turbines, gives an overview of efficiencies as a function of non-dimensional RPM. The numbers are estimated efficiencies only. A few remarks are germane to the discussion. From theory (Betz, Glauert), there are clear efficiency limits, but no theoretical maximum TSR. In contrast, the semiempirical curves for each type of wind turbine have a clearly defined maximum efficiency value.
2.1 Historical and State-of-the-Art Horizontal Axis Wind Turbines

The wind energy community is very proud of its long history. Some aspects of this history are presented in [2, 3]. Apparently, the oldest one [3] is the so-called *Persian* windmill (Fig. 2.2). It was first described around 900 AD and is viewed from our system of classification (see Sect. 2.6) as a drag-driven windmill with a vertical axis of rotation.
Somewhat later, the Dutch windmill appeared as the famous Windmill Psalter of 1279 [3], see Fig. 2.3. This represented a milestone in technological development: The axis of rotation changed from vertical to horizontal. But also from the point of view of aerodynamics, the Dutch concept began the slow movement toward another technological development: lift replacing drag. These two types of forces simply refer to forces perpendicular and in-line with the direction of flow. The reason this is not trivial stems from D’Alembert’s Paradox:

**Theorem 2.1**  *There are no forces on a solid body in an ideal flow regime.*

We will continue this discussion further in Chap. 3. These very early concepts survived for quite a few centuries. Only with the emergence of electrical generators and airplanes were these new technologies adapted, in the course of a few decades, to what is now called the standard horizontal axis wind turbine:

- horizontal axis of rotation
- three bladed
- driving forces mainly from lift
- upwind arrangement of rotor; tower downwind
- variable speed/constant TSR operation
- pitch control after rated power is reached.
2.2 Non-Standard HAWTs

With this glimpse of what a standard wind turbine should be, everything else is non-standard:

- no horizontal axis of rotation
- number of blades other than three (one, two, or more than three)
- drag forces play important role
- downwind arrangement of rotor; tower upwind
- constant speed operation
- so-called stall control after rated power is reached.

From these characteristics, we may derive a large number of different designs. Only a few of them became popular enough to acquire their own names (Fig. 2.4):

The American or Western-type turbine [3]. Chapter 1 uses a very high (10–50) number of blades which in most cases are flat plates with a small angle between plane of rotation and chord. These turbines were used mainly in the second half of the nineteenth century, see Fig. 2.5.

The Danish Way of extracting wind energy [4] used most of the now classical properties with a fixed-pitch blade arrangement and a constant RPM operation mode. The development of this design philosophy started in the 1940s and died off slowly in the 1990s.

2.3 Small Wind Turbines

Small wind turbines are defined by IEC [5] as a wind turbine with a rotor swept area no greater than 200 m². Therefore, the diameter is limited to 16 m. However, most of them have much smaller diameters starting at about 1 m. More can be found in [6]. Figure 2.6 gives an account of scaling. The main problem with safety approval of [5] is that it offers two very different methods:

- the usual aero elastic simulation modeling,
- a simplified load model.

The first one implies the same amount of work as for a state-of-the-art turbine and is not economical in most cases. The second procedure is much easier (see [6]) but at the
2.3 Small Wind Turbines

Fig. 2.5 American or western windmill

Fig. 2.6 Scaling of blade masses of smaller wind turbines

expense of exceedingly high safety factors. As an example, the required blade-mass for the simple load model is shown in Fig. 2.6. The mass required by the simple load model has to be more than 300 kg, compared to only 120 kg for a blade designed without these high safety factors.
2.4 Vertical Axis Wind Turbines

As was explained earlier, vertical axis windmills and the subsequent vertical axis wind turbines seem to be older than those with an horizontal axis of rotation. Mainly due to the inventions of Darrieus [7] and Savonius [8]\(^1\) interest in these vertical turbines was renewed in the early twentieth century. Then, after the first so-called oil crisis in the beginning of the 1970s, many US [9–12] and German [13–19] vertical turbine development efforts were undertaken which lasted until the 1990s, while HAWTs also progressed. A summary may be found in [20]. Now, after some 20 years of dormancy, interest in VAWTs seems to be returning slowly, see for example [21] (Fig. 2.7).

One of the big advantages is the independence of directional change in wind. See Fig. 2.8 for a typical distribution of wind direction at a typical site. Also, heavy components may be installed close to the ground, as illustrated in Fig. 2.9. The largest VAWT manufactured so far was the so-called ÉOle-C made in Canada. Its height was about 100 m, the rotating mass was 880 metric tons, and the rated power was supposed to be 4 MW. Unfortunately, due to severe vibration problems, the rotational speed was limited to such low values that only 2 MW was reached [20].

It has to be noted that the flow mechanics behind these designs (see Problems 6.1 and 6.2 in Chap. 6) is much more interesting—but also more complicated—due to fact that the sphere of the influence of the rotor is modeled as a real volume and not a 2D disk as is assumed for the HAWT actuator disk model. Therefore, at least for each half revolution, the blades are operating in a wake, meaning that load fluctuations have a much greater influence on the blades. The resulting so-called aerodynamic

\(^1\) A more detailed discussion can be found in Sect. 2.6.
fatigue loads for a VAWT rotor blade are much higher and are one of the reasons that VAWTs are much more prone to earlier failure of components—mostly at joints—than are HAWTs.

One way out of this difficulty is the so-called Gyro-Mill or articulated VAWT [22], where the pitch angle (the angle of the chord line in relation to the circumference) is changed periodically so that—in an ideal case—the driving force remains almost constant (Fig. 2.10).

2.5 Diffuser Augmented Wind Turbines

As we have seen, the power contained in the wind is proportional to the swept area. An obvious extension of this concept is to look for wind-concentrating devices resembling a cone or funnel, see Fig. 2.11. Such devices are very common in wind turbines and are called draft tubes or suction tubes. From first principles of fluid mechanics, the exit area of such a device has to be larger than the inflow area. At a glance, this is clearly counterintuitive. Then, by closer inspection of the basic laws of conservation of mass and energy—called Bernoulli’s law—it follows that an increase in mass flow proportional to the area ratio $A_{\text{exit}}/A_{\text{inflow}}$ is possible if the flow follows the contour of the cone.

Unfortunately, nature is not that generous. The increase in diameter has to be very moderate. To be more precise, opening angles less than 10° have to be used to avoid what is called flow separation. It immediately follows that, to have reasonable area ratios, we have to use very long diffusers with very large weights. The engineering task then is to find a reasonable compromise—if possible at all. Serious work started in 1956 by Liley and Rainbird [23] and efforts up to 2007 are summarized by van Bussel [24]. Some applications to small wind turbines may be found at [25].
2.6 Drag-Driven Turbines

Drag was defined earlier as a force on a structure subjected to a stream of air in line with the flow, see Fig. 2.4. We therefore define a simple number, the drag coefficient (Fig. 2.12):

\[ c_D := \frac{D}{\frac{1}{2} \cdot A_r \cdot v^2}. \]  

(2.4)
To imagine this, we consider at first a simply translating sail of velocity $u$ and Area $A = c \cdot \ell$ and $v$ the wind velocity as usual. The power $P = D \cdot u$, is using Eq. (2.4):

$$P = \frac{\rho}{2} (v - u)^2 \cdot c_D \cdot c \cdot \ell \cdot u. \tag{2.5}$$

Now using $P_{\text{wind}} = \frac{\rho}{2} v^3$, we have:

$$c_P = c_D (1 - u/v)^2 (u/v). \tag{2.6}$$
Fig. 2.12 Collected performance data for diffusers, from van Bussel (2007)

Fig. 2.13 Drag-driven turbines

Setting $a := u/v$ and solving for $a$ by making $\frac{dc_P}{da} = 0$, we see that the maximum power of such a drag-driven vehicle may not exceed

$$c_P^{\text{max}, D} = \frac{4}{27} \cdot c_D \text{ at } a = \frac{1}{3}$$

(2.7)

$c_P^{\text{max}, D} \approx 0.3$ if $u = 1/3 v$. For example, a sailing boat at wind force 7 (Beaufort $\approx 30$ knots $\approx 16.2$ m/s) may not travel faster than 10 Knots. If it has 30-$m^2$ sail area, the maximum power will be $P = 24$ kW.

The next step is to discuss the Persian wind mill or the closely related anemometer, see Fig. 2.13. The idea is to use a specially shaped body (semi-sphere) which has different $c_D$ when blown from one side or the other. A common pair of values for $c_D$ is $c_{D^+} = 1.33$ and $c_{D^-} = 0.33$. We then arrive at:

$$F_+ = c_{D^+} \cdot \frac{\rho}{2} A_r (u - \Omega r)$$

(2.8)

$$F_- = c_{D^-} \cdot \frac{\rho}{2} A_r (u + \Omega r) \text{ and finally}$$

(2.9)

$$c_P = (F_+ - F_-) \cdot v/A_r = \lambda \left( c_1 - c_2 \cdot \lambda + c_1 \cdot \lambda^2 \right)$$

(2.10)
2.6 Drag-Driven Turbines

\[ c_1 = c_{D+} - c_{D-} \quad \text{and} \quad c_2 = 2 \cdot (c_{D+} + c_{D-}). \]

Figure 2.14 shows that a very small efficiency at very low TSR \((c_{D+} = 1.33 \quad \text{and} \quad c_{D-} = 0.33)\).

\[ cp \]

\[ 0 \quad 0.02 \quad 0.04 \quad 0.06 \quad 0.08 \quad 0.1 \]

\[ 0 \quad 0.05 \quad 0.1 \quad 0.15 \quad 0.2 \quad 0.25 \quad 0.3 \quad 0.35 \quad 0.4 \]

\[ \text{TSR} \]

\[ \text{Efficiency of a Persian wind-mill} \]

**Fig. 2.14** Performance curve of an anemometer

**Fig. 2.15** Model of a counter-rotating wind turbine in a wind tunnel, diameter = 800 mm. Reproduced with permission of HEIG-VD, Iverdon-les-Bains, Switzerland

2.7 Counter-Rotating Wind Turbines

For some ship propellers and helicopters, there have been efforts to use to rotors rotating in opposite direction to improve the efficiency of the whole system. Unlike propellers and helicopters, swirl losses (see Sect. 5.2) appear to be small in wind
turbines. Nevertheless, many investigations of swirl losses [26, 27] have been undertaken. Figure 2.15 shows a model wind turbine which was evaluated during a wind tunnel experiment in 2009 in Geneva. Clearly, an improvement of about 10% was seen in performance but somewhat obscured by a rather large and uncertain blockage correction, which is of utmost importance to consider when comparing wind tunnel experiments with a freely expanding wake. From a theoretical point of view, this type of turbine is very interesting, because a swirl component does not have to be included. We will come back to this point in Chap. 6 (Fig. 2.16).

### 2.8 Concluding Remarks

We finish this chapter by noting that a myriad of other turbine types exist, some of which are based on very specific fluid mechanics principles or ideas. Even a pinwheel (toy) turbine was investigated experimentally [30]. The reader may consult the older literature [31, 32] for many highly entertaining examples.

### 2.9 Problems

**Problem 2.1** Derive Eq. 2.10 and find an expression for \( c \) as a function of \( c_{D+} \) and \( c_{D-} \).

**Problem 2.2** Estimate the increase of power for a DAWT with the following properties: \( D_{\text{Rotor}} = 1 \text{ m}, D_{\text{exit}} = 1.17 \text{ m} \) and efficiency of diffuser \( \eta_{\text{diff}} = 0.85 \).
Problem 2.3 Determine if a counter-rotating turbine consists of one turbine or two and give your reasons.

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