Chapter 1
Introduction

Recent advances in technology have led to dynamical systems with increasing complexity, which in turn demand for more and more efficient and reliable control systems. It has been widely recognized that the requirements of specific behaviors and stringent performances call for the inclusion of possible failure prevention in a modern control design. Therefore, either due to security reasons or efficiency necessity, a system failure is a critical issue to be considered in the design of a controller in modern technology. In view of this, dynamical systems that are subject to abrupt changes have been a theme of increasing investigation in recent years and a variety of different approaches to analyze this class of systems has emerged over the last decades. A particularly interesting class of models within this framework is the so-called *Markov jump linear systems* (MJLS), which is the subject matter of this book. The goal of this first chapter is to highlight, in a rather informal way, some of the main characteristics of MJLS, through some illustrative examples of possible applications of this class of systems.

1.1 Markov Jump Linear Systems

One of the main challenges when modeling a dynamical system is to find the best trade-off between the mathematical complexity of the resulting equations and the capability of obtaining a tractable problem. Thus, it is of overriding importance to reach a proper balance in the settling of this issue while keeping in mind the performance requirements. For instance, in many situations the use of robust control techniques and some classical sensitivity analysis for time-invariant linear models can handle in a simple and straightforward way the control problem of dynamical systems subject to changes in their dynamics. However, if these changes significantly alter the dynamical behavior of the system, these approaches may no longer be adequate to meet the performance requirements. Within this scenario, the introduction of some degree of specialization on the modeling of the dynamical system in order to accommodate these changes is inevitable and perhaps even desirable.

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The modeling of dynamic systems subject to abrupt changes in their dynamics has been receiving lately a great deal of attention. These changes can be due, for instance, to abrupt environmental disturbances, to actuator or sensor failure or repairs, to the switching between economic scenarios, to abrupt changes in the operation point for a non-linear plant, etc. Therefore, it is important to introduce mathematical models that take into account these kind of events and to develop control systems capable of maintaining an acceptable behavior and meeting some performance requirements even in the presence of abrupt changes in the system dynamics.

In the case in which the dynamics of the system is subject to abrupt changes, one can consider, for instance, that these changes are due to switching (*jump*) among well-defined models. To illustrate this situation, consider a continuous-time dynamical system that is, at a certain moment, well described by a model $\mathcal{G}_1$. Suppose that, after a certain amount of time, this system is subject to abrupt changes that cause it to be described by a different model, say $\mathcal{G}_2$. More generally, one can imagine that the system is subject to a series of possible qualitative changes that make it to switch, over time, among a countable set of models, for example, $\{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_N\}$. One can associate each of these models to an *operation mode* of the system, or just *mode*, and say that the system *jumps* from one mode to the other or that there are *transitions* between them. A central issue on this approach is how to append the jumps into the model. A first step is to consider, for instance, that the jumps occur in a random way, i.e., the mechanism that rules the switching between the aforementioned models is random. In addition, one can assume that this process (hereinafter $\theta(t)$) just indicates which model, among the $\{\mathcal{G}_i, i = 1, 2, \dots, N\}$, is running the system, i.e., $\theta(t) = i$ means that $\mathcal{G}_i$ is running the system. Furthermore, it would be desirable to have some a priori information on the way in which the system jumps from one operation mode to another (the transition mechanism). A random process which bears these features and has been used with a great deal of success for modeling these situations is the Markov chain.

Within this framework, a particularly interesting class of models is the so-called Markov jump linear systems (from now on MJLS). Since its introduction, this class of models has an intimate connection with systems which are vulnerable to abrupt changes in their structure, and the associated literature surrounding this subject is now fairly extensive. Due, in part, to a large coherent body of theoretical results on these systems, MJLS has been used recently in a number of applications on a variety of fields, including robotics, air vehicles, economics, and some issues in wireless communication, among others. For instance, it was mentioned in [266] that the results achieved by MJLS, when applied to the synthesis problem of wing deployment of an uncrewed air vehicle, were quite encouraging. As mentioned before, the basic idea is to consider a family of continuous-time linear systems, which will represent the possible modes of operation of the real system. The modal transition is given by a Markov chain represented, as before, by $\theta(t)$, which is also known in the literature as the operation mode (or the Markov state). This class of systems will be the focus of investigation of the present book. We will restrict ourselves in this book to the case in which all operation modes are continuous-time linear models and the jumps from one mode of operation to another follow a continuous-time

Markov chain taking values in a *finite set* $\{1, \ldots, N\}$. In this scenario, it is possible to develop a unified and coherent body of concepts and results for stability, filtering, and control as well as to present controller and filter design procedures.

In its most simple form, continuous-time MJLS are described as

$$\dot{x}(t) = A_{\theta(t)} x(t). \tag{1.1}$$

For a more general situation, known as the case with partial observations, continuous-time MJLS are represented by

$$\mathcal{G} = \begin{cases} dx(t) = A_{\theta(t)}(t)x(t)\,dt + B_{\theta(t)}(t)u(t)\,dt + J_{\theta(t)}(t)\,dw(t), \\ dy(t) = H_{\theta(t)}(t)x(t)\,dt + G_{\theta(t)}(t)\,dw(t), \\ z(t) = C_{\theta(t)}(t)x(t) + D_{\theta(t)}(t)u(t), \\ x(0) = x_0, \qquad \theta(0) = \theta_0, \end{cases} \tag{1.2}$$

with $x(t)$ standing for the state variable of the system, $u(t)$ the control variable, $y(t)$ the measured variable available to the controller, $z(t)$ the output of the system, and $w(t)$ is a Wiener process.

Although MJLS seem, at first sight, just an extension of linear systems, they differ from the latter in many instances. This is due, in particular, to some peculiar properties of these systems that can be included in the class of complex systems (roughly speaking, a system composed of interconnected parts that as a whole exhibit one or more properties not obvious from the properties of the individual parts). In order to give a small glimpse of this, let us consider a typical situation in which one of these peculiar properties is unveiled. Consider a homogeneous Markov chain θ with state space $\mathcal{S} = \{1, 2\}$ and transition rate matrix

$$\Pi = \begin{bmatrix} -\beta & \beta \\ \beta & -\beta \end{bmatrix}, \quad \beta > 0.$$

Some sample paths of θ, which in this case corresponds to a telegraph process with exponentially distributed waiting times, are shown in Fig. 1.1. Clearly, an increase in the transition rate gives rise to more frequent jumps (fast switching), with the converse applying in the slow switching case.

In this setting, consider system (1.1) evolving in $\mathbb{R}^2$, with initial condition $x(0) = [1\ -1]'$ and state matrices

$$A_1 = \begin{bmatrix} \frac{1}{2} & -1 \\ 0 & -2 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -2 & -1 \\ 0 & \frac{1}{2} \end{bmatrix}. \tag{1.3}$$

The state trajectory of the system, corresponding to the realizations of θ depicted in Fig. 1.1, is shown in Fig. 1.2. From these figures it is evident that the overall system behavior may vary considerably, depending on how fast the switching occurs. Even more remarkable is the fact that the state converges to the origin in the case $\beta = 3/2$,

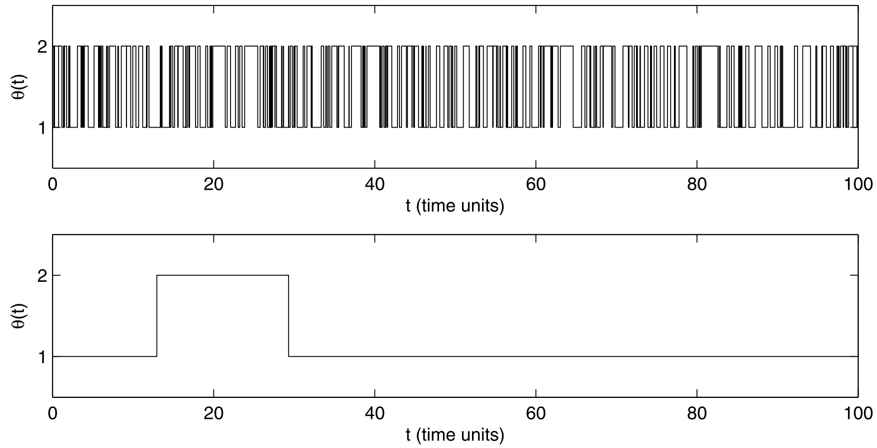


Fig. 1.1 Sample paths of θ for $\beta = 3/2$ (*top*) and $\beta = 3/100$ (*bottom*)

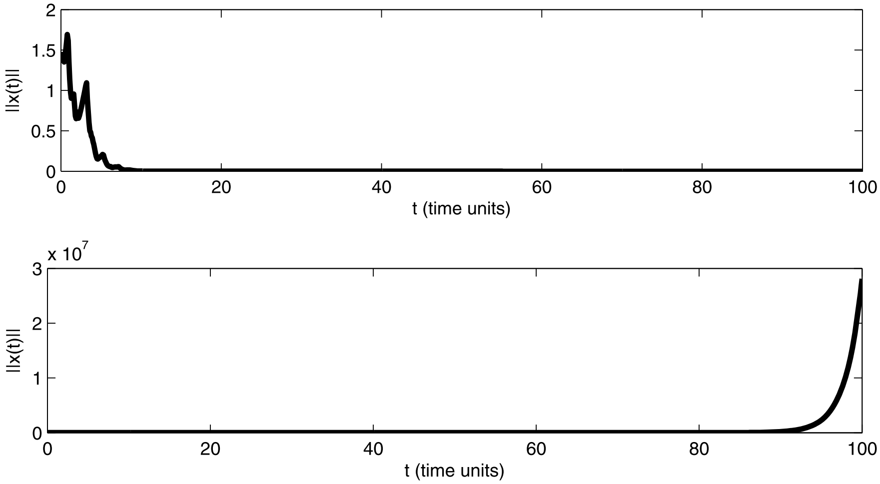


Fig. 1.2 Sample paths of $\|x(t)\|$ corresponding to $\beta = 3/2$ (*top*) and $\beta = 3/100$ (*bottom*) in (1.3)

in spite of neither A_1 nor A_2 being stable. Suppose now that the system matrices in (1.1) are replaced by

$$A_1 = \begin{bmatrix} -1 & 10 \\ 0 & -1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -1 & 0 \\ 10 & -1 \end{bmatrix}, \tag{1.4}$$

which are both stable, i.e., have all the eigenvalues with negative real parts. In this case the sample paths of $\|x(t)\|$ corresponding to the same trajectories of θ are depicted in Fig. 1.3. As these two situations suggest, Markovian switching between stable (unstable) systems may produce unstable (stable) dynamics. In fact, as it will

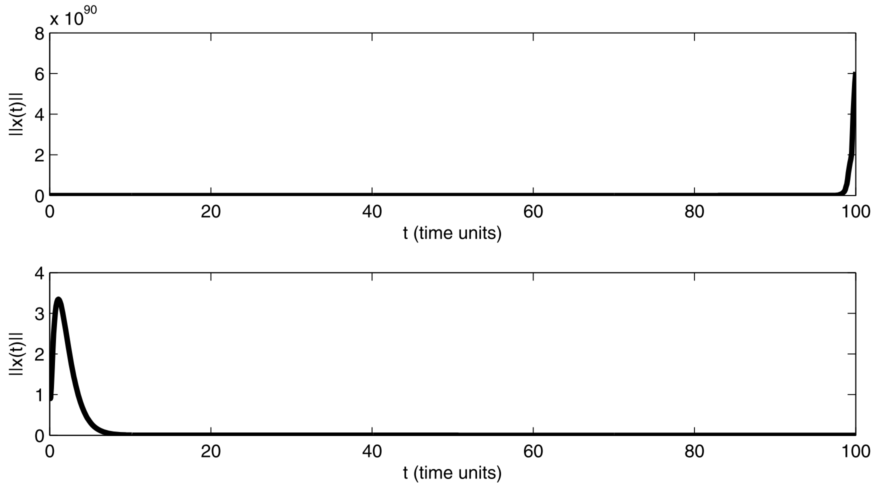


Fig. 1.3 Sample paths of $\|x(t)\|$ corresponding to $\beta = 3/2$ (*top*) and $\beta = 3/100$ (*bottom*) in (1.4)

be explicitly shown in Chap. 3, in the first case stability is equivalent to $\beta > 4/3$, whereas in the second situation the overall system is stable if and only if $\beta < 1/24$.

Some features which distinguish MJLS from the classical linear systems are:

- The stochastic process $\{x(t)\}$ alone is no longer a Markov process.
- As seen in the previous examples, the stability (instability) for each mode of operation does not guarantee the stability (instability) of the system as a whole.
- The optimal filter for the case in which $(x(t), \theta(t))$ are unknown is nonlinear and infinite-dimensional (in the filtering sense).
- Contrary to the linear time-invariant case, there is a fundamental limitation on the degree of robustness against linear perturbation that the H_∞ control of MJLS may offer (see [284]).

If the state space of the Markov chain is *infinitely countable*, there are even further distinctions as illustrated below:

- Mean-square stability is no longer equivalent to L^2-stability (see [148]).
- One cannot guarantee anymore that the maximal solution of the Riccati equation associated to the quadratic optimal control problem is a strong solution (see [17]).

In order to develop a theory for MJLS using an operator theoretical approach, the following steps are adopted:

(S.1) "Markovianize" the problem by considering as the state of the model the pair $(x(t), \theta(t))$.

(S.2) Establish a connection between $x(t)$ and measurable functions of the pair $(x(t), \theta(t))$.

(S.3) Devise adequate operators from (S.2) by considering the first and second moments.

These steps were followed in [152] (see also [77]), using the identities

$$x(t) = \sum_{i=1}^{N} x(t) 1_{\{\theta(t)=i\}} \quad \text{and} \quad x(t)x(t)^* = \sum_{i=1}^{N} x(t)x(t)^* 1_{\{\theta(t)=i\}},$$

where $1_{\{\theta(t)=i\}}$ represents the Dirac measure over the set $\{\theta(t)=i\}$ (see (2.2)), and * the transpose conjugate. The key point here is to work with $x(t)1_{\{\theta(t)=i\}}$ and $x(t)x(t)^*1_{\{\theta(t)=i\}}$, which are measurable functions of $(x(t), \theta(t))$, and obtain differential equations for the first and second moments in terms of some appropriate operators (see Sect. 3.3). This approach has uncovered many new differences between the MJLS and its linear classical counterpart, and has allowed the development of several new theoretical results and applications for MJLS. For instance, it is possible to devise a spectral criterion for mean-square stability and study stability radius in the same spirit as the one found in the linear system theory. What unifies the body of results in this book, and has certainly helped us to choose its content, is this particular approach, which we will call the *analytical point of view* (from now on APV).

It is worth mentioning two other distinctive approaches, which, together with the APV, have given rise to a host of important results on various topics of MJLS: the so-called Multiple Model (MM) approach and the Hidden Markov Model (HMM) approach. In the MM approach the idea is, roughly speaking, to devise strategies that decide in an efficient way which mode is running the system and work with the linear system associated to this mode (see, e.g., [22] for a comprehensive treatment on this subject). The HMM approach focuses on what is known in the control literature as a class of partially observed stochastic dynamical systems. The basic framework for the HMM consists of a Markov process $\theta(t)$ that is not directly observed but is hidden in a noisy observation process $y(t)$. Roughly speaking, the aim is to estimate the Markov process, given the related observations, and from this estimation to derive a control strategy for the hidden Markov process (usually the transition matrix of the chain depends on the control variable $u(t)$). See, e.g., [130] for a modern treatment on this topic.

The present book can be seen, roughly speaking, as a continuation of the book [81], which dealt with the discrete-time case, to the continuous-time case. From a practical viewpoint, perhaps the most important aspect which favors the study of continuous-time systems is that many models in science are based upon specific relations between the system variables and their *instantaneous* rates of variation (some examples being Newton's second law of classical mechanics, Maxwell's nonstationary equations of electromagnetics, the law of continuity of fluid transport phenomena, Verhulst's logistic equation, and Gompertz's model of tumor growth, among many others). In these cases, the direct application of discrete-time systems theory may be a potential cause for distortion to the model in the form, e.g., of spurious effects such as artificial energy dissipation, instability, oscillations, and a potentially cumbersome dependence on the discretization parameters (which may even be plagued by the curse of dimensionality, as pointed out in [114]). Hence, it

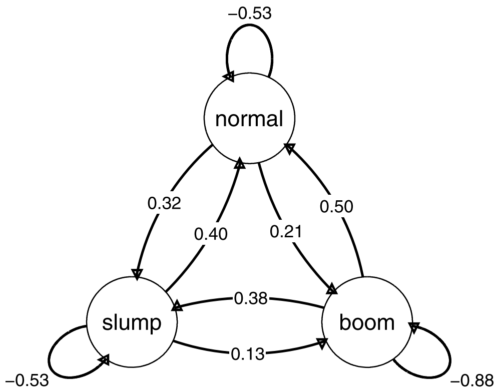


Fig. 1.4 Transitions between the different scenarios of a country's economy

is fair to say that, *prior to sampling*, a cautious study of the underlying continuous-time process should be carried out in order to ensure that such adversities would not hinder the desired specifications to be achieved in a given application.

1.2 Some Applications of MJLS

Since their inception in the early 1960s, MJLS have found many applications in a great variety of fields. These include unmanned air vehicles [266], solar power stations [275], satellite dynamics [226], economics [32, 33, 52, 108, 268], flight systems [174, 175], power systems [206, 211, 212, 290, 291], communication systems [1, 2, 208, 243], among many others. This section is devoted to a brief exposition of some selected topics regarding applications of MJLS, with special attention to those in continuous time.

The earliest application of MJLS to economics, introduced in the discrete-time scenario in [33], was based on Samuelson's multiplier–accelerator macroeconomic model (see [252, 286, 299]). The model studied in [33] consists of a very simplified relation for the dynamical evolution of a country's *national income* in terms of the *governmental expenditure*, which is weighted by two parameters (the *marginal propensity to save* and the *accelerator coefficient*). Based on the historical data obtained from the U.S. Department of Commerce from years 1929 until 1971, [33] assumed that the state of the economy could be roughly lumped in three possible operation modes ("normal", "boom", and "slump") and that the switching between them could be modeled as a homogeneous Markov chain, with the transition rates depicted in Fig. 1.4. The subsequent problem considered in [33] corresponds to an MJLS version of the optimal linear quadratic control setup. This application, which was also analyzed in [81] for the discrete-time case, will be further studied in Chap. 10.

Another practical application of MJLS, which, in the discrete-time case, has been extensively discussed in [81], is *the control of a solar power receiver*. In continuous time, this situation was initially considered in [275] with $\theta(t)$ representing

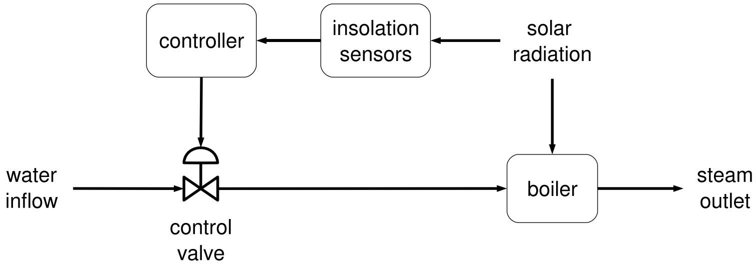


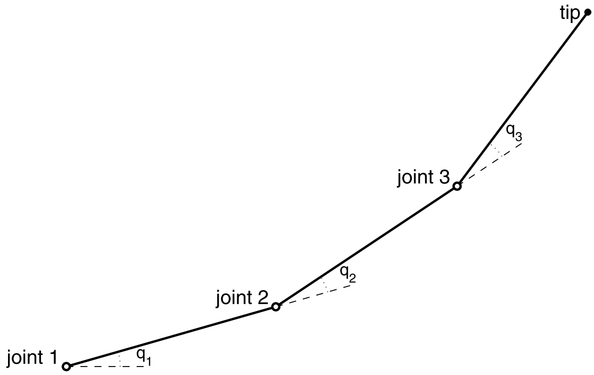
Fig. 1.5 Simplified boiler configuration

abrupt environmental changes between "sunny" and "cloudy" conditions, which are measured by sensors located on the plant. The role of control is to determine the feedwater flow rate which will enter the boiler, in such a way as to regulate the outflow temperature at the desired level. The boiler flow rate is strongly dependent upon the receiving insolation, and, as a result of this abrupt variability, several linearized models are required to characterize the evolution of the boiler when clouds interfere with the sun's rays. The control law described in [275] makes use of the state feedback and a measurement of $\theta(t)$ through the use of flux sensors on the receiver panels. In Fig. 1.5 a very simplified representation of the system is shown. Although several important blocks such as the boiler internal models for steam and metal dynamics, thermal couplings, and feedforward compensation loops are not displayed, their abstraction evidences the dependence of the control system on the abrupt variability of the received insolation.

Robotic manipulator arms are employed in a great deal of modern applications, which span areas as diverse as deep sea engineering, manufacturing processes, space technology, or teleoperated medicine, for instance. The adequate operation of such devices, however, is severely compromised by the occurrence of failures, which may be intolerable in safety-critical applications, for example. Furthermore, repairing the faulty arm may frequently turn out to be a difficult task during the course of operation (which may occur if, e.g., the robot is functioning in a hazardous environment). On this regard, a great deal of research has been carried out on the control of robotic arms with less actuators than degrees of freedom, commonly referred to as underactuated manipulators in the specialized literature [11, 12, 92]. In practical terms, a manipulator arm is said to be *underactuated* whenever the motor on at least one of its joints is in a passive state. The basic principle in the operation of these devices is then to explore the dynamic coupling that the active joints impose on the passive ones, in such a way as to drive the arm to the desired position in spite of its faulty condition.

The introduction of MJLS theory to tackle the control of underactuated robotic arms was made by Siqueira and Terra in [261], in the discrete-time setting. This approach was subsequently described in [81] and, more recently, brought to the continuous-time setting in [262, 263]. In Chap. 10 the robust control of the planar 3-link underactuated arm depicted in Fig. 1.6 is further studied by means of the results devised in this book.

Fig. 1.6 Geometric modeling of the planar 3-link arm



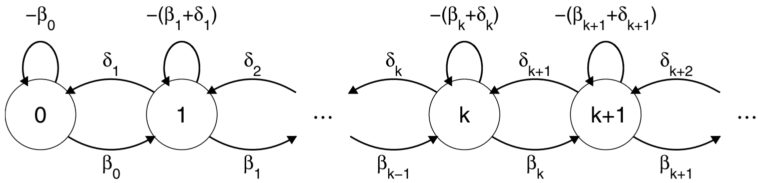


Fig. 1.7 Birth–death process considered in [175]

In yet another field, MJLS were employed in [175] for the stability analysis of *controlled flight systems affected by electromagnetic disturbances*. This problem stems from the susceptibility of electronic devices against external disturbances such as lightning, thermal noise, and radio signals, for instance, which is recognized as a potential cause for *computer upsets* in digital controllers. In order to cope with the resulting adversities, which may range from random bit errors to permanent computer failures, the strategy in [175] is to model the accumulative effect of external disturbances on the system by means of a continuous-time birth–death Markov process of the form indicated in Fig. 1.7. The rationale for this choice has been, as pointed out in [175], that by doing so the arrival of new disturbances is dictated by a Poisson process with exponentially distributed sojourn times.

An important issue in this situation is that, while the aircraft dynamics treated in [175, Sect. IV.B] evolves in continuous time, the occurrence of electromagnetic disturbances takes place in the sampled-data digital controller implementation. This leaves open an interesting conjecture of whose aspects of continuous- and discrete-time MJLS theory should be relevant to the problem at hand.

In [211, 212] the *modeling and control of power systems* subject to Markov jumps has been addressed in the continuous-time scenario. In this case the switching mechanism is used to model random changes in the load, generating unit outages, and transmission line faults, for instance. As shown in [212], intermittent couplings between electrical machines operating in a network can render the overall system unstable in a stochastic sense, a result which somewhat resembles the analysis previously carried out for (1.1) in the cases (1.3) and (1.4). In [211] the main results were applied to the problem of dynamic security assessment, which amounts to determining, with desired probability, whether certain parameters of the electrical system are guaranteed to remain within a safe region of operation at a given period. A "security measure" was defined, which corresponds to a quantitative indicator of the vulnerability of the current system state and network topology to stochastic contingency events. Two types of switchings are considered therein: *primary* ones, driven by a continuous-time Markov process taking values in a finite set, and *secondary* events, which are modeled by state-dependent (controlled) jumps. Recent advances in the control of power systems have also been reported in [206, 290, 291] by means of decentralized control methods and the S-procedure. An alternative account of this problem via robust control methods is presented in details in Sect. 10.3.

The *modeling of communication systems* via MJLS is by now another promising trend in the applications front. In discrete-time this is boldly motivated by the connection between MJLS and the Gilbert–Elliott model for burst communication channels (see [129, 167, 172, 173, 186, 244, 254]), which in its simplest form corresponds to a two-state Markov chain. A convenient feature of these models (besides their relative simplicity) is that they are capable of describing the fact that eventual packet losses typically occur during *intervals* of time (e.g., while a wireless link is obstructed). In other words, isolated packet losses are not common events, and it usually takes a while before communication, once lost, is restored. As pointed out in [167, 173, 186, 254], large packet-loss rates imply poor performance or even instability, and therefore controllers implemented within a network may provide considerably better results if their design takes into account the probabilistic nature of the network.

In the continuous-time scenario, references [1, 2] treated the problem of *dynamic routing* in mobile networks. In loose terms, this amounts to determining a route within the network topology constraints, through which information packets will travel from one given node to another. Of course, many of these routing operations will typically occur at the same time and between different nodes and directions, so that the routing algorithm must be able to provide a satisfactory quality of service for all customers (for example, by delivering packets in the shortest time, with as little lag as possible, and with a very low packet loss rate), without violating physical constraints such as link capacity, memory size (queueing length), or power availability. The approach in [1, 2] considers that sudden variations, modeled by MJLS, occur in the network due to, e.g., mobility and topological variations. The consideration of *time delay* in the underlying model is of major importance, owing to the time that packets must wait on a queue before being processed, together with the fact that the network nodes are geographically separated. Furthermore, this latter constraint motivated the consideration of a *decentralized* control scheme. The ultimate problem considered in [1, 2] was the minimization of the worst-case queueing length, with the aid of H_∞ control methods.

1.3 Prerequisites and General Remarks

As prerequisite for this book, it is desirable some knowledge on the classical linear control theory (stability results, the linear quadratic Gaussian (LQG) control problem, H_∞ control problem, and Riccati equations), some familiarity with continuous-time Markov chains and probability theory, and some basic knowledge of operator theory.

In this book we follow an approach that combines probability and operator theory to develop the results. By doing this we believe that the book provides a unified and rigorous treatment of recent results for the control theory of continuous-time MJLS. Most of the material included in the book was published after 1990. The goal is to provide a complete and unified picture on the main topics of the control theory of continuous-time MJLS such as mean-square stability, quadratic control and H_2 control for the complete and partial observations (also called partial information) cases, associated coupled differential and algebraic Riccati equations, linear filtering, and H_∞ control. The book also intends to present some design algorithms mainly based on linear matrix inequalities (LMIs) optimization tool packages.

One of the objectives of the book is to introduce, as far as possible in a friendly way, a bent of the MJLS theory that we have named here as the analytical point of view. We believe (and do hope) that experts in linear systems with Markov jump parameters will find in this book the minimal essential tools to follow this approach. Moreover, the stochastic control problems for MJLS considered in this book provide one of those few cases in the stochastic control field in which explicit solutions can be obtained, being a useful material for a course and for introducing students into an interesting and active research area. In addition, we do hope to motivate the reader, especially the graduate students, in such a way that this book could be a starting point for further developments and applications of continuous-time MJLS. From the application point of view we believe that the book provides a powerful theory with potential application in systems whose dynamics are subject to abrupt changes, as those found in safety-critical and high-integrity systems, industrial plants, economic systems, etc.

1.4 Overview of the Chapters

We next present a brief overview of the contents of the chapters of the book.

Chapter 2 is dedicated to present some background material needed throughout the book, as the notation, norms, and spaces that are appropriate for our approach. It also presents some important auxiliary results, especially related to the stability concepts to be considered in Chap. 3. A few facts on Markov chains and the bear essential of infinitesimal generator is also included. We also recall some basic facts regarding LMIs, which are useful for the design techniques and the H_∞ control problem.

Chapter 3 deals with mean-square stability for continuous-time MJLS. It is shown that mean-square stability is equivalent to the maximal real part of the eigenvalues of an augmented matrix being less than zero or to the existence of a solution of a Lyapunov equation. As aforementioned, stability of all modes of operation is neither necessary nor sufficient for global stability of the system. The criterion based on the eigenvalues of an augmented matrix reveals that a balance between the modes and the transition rate matrix is essential for mean-square stability.

It is worth pointing out that the setup of Chap. 3 is on the domain of complex matrices and vectors. By doing this we can use a useful result on the decomposition of a matrix into some positive semidefinite matrices, which is very important to prove the equivalence on stability results. But it is shown later on in this chapter that the results obtained are valid even in the setup of real matrices and vectors. In order to simplify the notation and proofs, in the remaining chapters of the book we consider just the real case.

Chapter 4 analyzes the quadratic optimal control problem for MJLS in the usual finite- and infinite-time horizon framework. We consider in this chapter that the controller has access to both the state variable $x(t)$ and the jump parameter $\theta(t)$. The case in which the controller has access only to an output $y(t)$ and $\theta(t)$ is considered in Chap. 6 and called the partial observation (or partial information) case. The solution for the quadratic optimal control problems of Chap. 4 relies, in part, on the study of a finite set of coupled differential and algebraic Riccati equations (CDRE and CARE, respectively). These equations are studied in the Appendix A.

Chapter 5 restudies the infinite-horizon quadratic optimal control for MJLS but now from another point of view, usually known in the literature of linear systems as the H_2 control. The advantage of the H_2 approach is that it allows one to consider parametric uncertainties and solve the problem using LMIs optimization tools.

Chapter 6 deals with the finite-horizon quadratic optimal control problem and the H_2 control problem of continuous-time MJLS for the partial information case. The main result shown is that the optimal control is obtained from two sets of coupled differential (for the finite-horizon case) and algebraic (for the H_2 case) Riccati equations, one set associated with the optimal control problem when the state variable is available, as analyzed in Chaps. 4 and 5, and the other set associated with the optimal filtering problem. This establishes the so-called separation principle for continuous-time MJLS.

Chapter 7 aims to derive the best linear mean square estimator of continuous-time MJLS assuming that only an output $y(t)$ is available. It is important to emphasize that in this chapter we assume that the jump parameter $\theta(t)$ is not known. The idea is to derive a filter which bears those desirable properties of the Kalman filter: *a recursive scheme suitable for computer implementation which allows some offline computation that alleviates the computational burden*. The linear filter has dimension Nn (where n denotes the dimension of the state vector, and N the number of states of the Markov chain). Both the finite-horizon and stationary cases are considered.

Chapter 8 is devoted to the H_∞ control of Markov jump linear systems, in the infinite-horizon setting. The statement of a bounded real lemma is the starting point toward a complete LMIs characterization of static state feedback, as well as of full-order dynamic output feedback stabilizing controllers that guarantee that a prescribed closed-loop H_∞ performance is attained. The main results include explicit formulas and the corresponding algorithms for designing the controllers of interest.

Chapters 9 and 10 are intended to conclude the book assembling some problems in the Markov jump context and the tools to solve them.

In the Appendix A some results on coupled differential and algebraic Riccati equations (CDRE and CARE, respectively) associated to the control problem are presented. Initially, we consider the problem of uniqueness, existence, positive definiteness, and continuity of the solution of the CDRE. After that we study the CARE, dealing essentially with conditions for the existence of solutions and asymptotic convergence, based on the concepts of mean-square stabilizability and detectability seen in Chap. 3. Regarding the existence of a solution, we are particularly interested in maximal and stabilizing solutions.

In the Appendix B we derive some auxiliary results related to an adjoint operator for MJLS used in the separation principle presented in Chap. 6.

1.5 Historical Remarks

The study of dynamical systems with *random parameters* can be traced back at least to [196, 200–202], and [142]. Due to the importance and complexity of the theme, a variety of techniques has emerged, and an extraordinary burst of publications associated with terminologies such as *multiple model*, *switching systems*, *hidden Markov models* and *Markov jump linear systems*, *inter alia*, has appeared in the specialized literature. Even for those approaches which embraced the idea of modeling the random parameter as a Markov chain, the methodologies were different. Each one of these topics have charted its own course, and the associated literature is by now huge. Therefore, it is out of the scope here to go into details on all of these topics (see, e.g., [22, 130, 232] and references therein for an account on multiple model, hidden Markov model, and switching systems, respectively). We focus instead on the MJLS case.

Regarding MJLS, the seminal papers [270] and [303] set the stage for future research in this area. In the first one, the jump linear quadratic (JLQ) control problem was considered for the finite-horizon setting, via a maximum principle approach (see also [269]). In the other one, dynamic programming was used, and the infinite-horizon case was also treated. In this case, although the objective had been carried out successfully, a technical inconvenience was related to the choice of the adopted stability criteria, which did not seem to be fully adequate. This, in turn, has entailed a great deal of challenges for future research. In the 1970s we can mention the papers [32, 248, 273, 274] and [33] dealing with the JLQ control problem, where the latter seems to be the first one that treated the discrete-time version of the optimal quadratic control problem for the finite-time horizon case (see also [20] for the MM approach).

Due, in part, to the lack of a suitable concept for stability, it took a while for the theory flourish. For instance, it elapsed more than ten years to appear some of the key papers that put mean-square stability (stochastic stability, or L_2-stability), mean square stabilizability, and mean square detectability for MJLS in a solid ground. Without any intention of being exhaustive here, we mention, for instance, [43, 48, 71, 77, 79, 118, 137, 147, 148, 150–152, 156, 164, 170, 170, 187, 189–191, 210, 221, 230, 231, 245, 278, 282, 284], for a sample of papers dealing with *stability and stabilizability* for MJLS. For the case in which the state space of the Markov chain is *countably infinite*, it was shown in [78] and [148] that mean-square and stochastic stability (L_2-stability) are no longer equivalent (see also [147]). Another critical issue was an adequate concept for the solution of the Riccati equation associated to MJLS (*coupled Riccati equations*). As far as the authors are aware of, it was in [156] where the concept of mean-square solution for the coupled Riccati equations was first proposed. For a glimpse on some issues dealing with coupled Riccati equations, see, for instance, [4–6, 17, 66, 67, 149, 156, 246]. For *control problems* (optimal, adaptive, H_∞, H_2, robust, receding horizon, singularly perturbed, partial observations, etc.), we mention, for instance, [18, 38, 44, 46, 47, 59, 60, 69, 70, 73–75, 78, 85, 87, 93, 94, 101, 104, 108, 111, 119, 120, 122, 125, 134, 143–146, 158, 159, 171, 176, 188, 190, 224, 229, 240, 258, 279, 307]. For the *filtering problem*, the reader is referred, for instance, to [9, 35–37, 65, 82–84, 126, 127, 132, 155, 160, 217, 321, 322]. The case with *delay* was treated, for instance, in [24, 44, 45, 55, 165, 209, 218, 257, 297]. The case with *uncertainty* (including uncertainty in the transition matrix of the Markov chain) was considered in [15, 26, 42, 256, 305, 306, 316–319]. *Separation principles* were derived, for instance, in [80, 89, 90, 153]. *Structural properties* such as controllability, detectability, and observability have been studied, for instance, in [61, 63, 187, 189, 222]. See [287, 288] for problems related to detection and identification for MJLS. In addition, there is by now a growing conviction that MJLS provide models of wide applicability (see, e.g., [14] and [266]). The evidence in favor of such a proposition has been amassing rapidly over the last decades. We mention [1, 2, 8, 14, 21, 28, 32, 39, 52, 57, 108, 124, 133, 141, 173–175, 184, 186, 197, 198, 208, 213, 219, 220, 225, 226, 228, 241, 243, 249, 253, 254, 266, 275] and [323], as works dealing with *applications* of this class of systems (see also the books [22, 40, 41, 81, 121, 223, 271, 309] and references therein).

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