Preface

Statistical field theory deals with the behavior of classical or quantum systems consisting of an enormous number of degrees of freedom in and out of equilibrium. Quantum field theory provides a theoretical framework for constructing quantum mechanical models of systems with an infinite number of degrees of freedom. It is the natural language of particle physics and condensed matter physics. In the past decades the powerful methods in statistical physics and Euclidean quantum field theory have come closer and closer, with common tools based on the use of path integrals. The interpretation of Euclidean field theories as particular systems of statistical physics opened up new avenues to understand strongly coupled quantum systems or quantum field theories at zero or finite temperature. The powerful methods of statistical physics and stochastics can be applied to study for example the vacuum sector, effective action, thermodynamic potentials, correlation functions, finite size effects, nature of phase transitions or critical behavior of quantum systems.

The first chapters of this book contain a self contained introduction to path integrals in Euclidean quantum mechanics and statistical mechanics. The resulting high-dimensional integrals can be estimated with the help of Monte Carlo simulations based on Markov processes. The method is first introduced and then applied to ordinary integrals and to quantum mechanical systems. Thereby the most commonly used algorithms are explained in detail. Equipped with these stochastic methods we may use high performance computers as an “experimental” tool for a new brand of theoretical physics.

The book contains several chapters devoted to an introduction into simple lattice field theories and a variety of spin systems with discrete and continuous spins. An ideal guide to the fascinating area of phase transitions is provided by the ubiquitous Ising model. Despite its simplicity the model is often used to illustrate the key features of statistical systems and the methods available to understand these features. The Ising model has always played an important role in statistical physics, both at pedagogical and methodological levels. Almost all chapters in the middle part of the book begin with introducing methods, approximations, expansions or rigorous results by first considering the Ising model. In a next step we generalize from the Ising model to other lattice systems, for example Potts models, $O(N)$ models, scalar
field theories, gauge theories, and fermionic theories. For spin models and field the-
ories on a lattice it is often possible to derive rigorous results or bounds. Important
examples are the bounds provided by the mean field approximation, inequalities be-
tween correlation functions of ferromagnetic systems, and the proofs that there exist
spontaneously broken phases at low temperature or the duality transformations for
Abelian models which relate the weak coupling and strong coupling regions or the
low temperature and high temperature phases. All these interesting results are de-
erved and discussed with great care.

As an alternative to the lattice formulation of quantum field theories one may use
a variant of the flexible renormalization group methods. For example, implementing
(spacetime) symmetries is not so much an issue for a functional renormalization
group method as it sometimes is for a lattice regularization and hence the method is
somewhat complementary to the ab initio lattice approach. In cases where a lattice
regularization based on a positive Boltzmann factor fails, for example for gauge
theories at finite density, the functional method may work. Thus it is often a good
strategy to consider both methods when it comes to properties of strongly coupled
systems under extreme conditions. Knowledge of the renormalization group method
and in particular the flow of scale dependent functionals from the microscopic to the
macroscopic world is a key part of modern physics and thus we have devoted two
chapters to this method.

According to present day knowledge all fundamental interactions in nature are
described by gauge theories. Gauge theories can be formulated on a finite spacetime
lattice without spoiling the important local gauge invariance. Thereby the functional
integral turns into a finite-dimensional integral which can be handled by stochastic
means. Problems arise when one considers gauge fields in interaction with fermions
at finite temperature and non-zero baryon density. A lot of efforts have gone into
solving or at least circumventing these problems to simulate quantum chromody-
namics, the microscopic gauge theory underlying the strong interaction between
quarks and gluons. The last chapters of the book deal with gauge theories without
and with matter.

This book is based on an elaboration of lecture notes of the course Quantum Field
Theory II given by the author at the Friedrich-Schiller-University Jena. It is designed
for advanced undergraduate and beginning graduate students in physics and applied
mathematics. For this reason, its style is greatly pedagogical; it assumes only some
basics of mathematics, statistical physics, and quantum field theory. But the book
contains some more sophisticated concepts which may be useful to researchers in
the field as well. Although many textbooks on statistic physics and quantum field
theory are already available, they largely differ in contents from the present book.
Beginning with the path integral in quantum mechanics and with numerical meth-
ods to calculate ordinary integrals we bridge the gap to lattice gauge theories with
dynamical fermions. Each chapter ends with some problems which should be useful
for a better understanding of the material presented in the main text. At the end of
many chapters you also find listings of computer programs, either written in C or in
the freely available Matlab-clone Octave. Not only because of the restricted size of
the book I did not want to include lengthy simulation programs for gauge theories.
Acknowledgments  Over the years I have had the pleasure of collaborating and discussing many of the themes of this book with several of my teachers, colleagues and friends. First of all, I would like especially to thank the late Lochlain O’Raifeartaigh for the long and profitable collaboration on effective potentials, anomalies, and two-dimensional field theories, and for sharing his deep understanding of many aspects of symmetries and field theories. I would like to use this opportunity to warmly thank the academic teachers who have influenced me most—Jürg Fröhlich, Res Jost, John Lewis, Konrad Osterwalder, Eduard Stiefel, and especially Norbert Straumann. I assume that their influence on my way of thinking about quantum field theory and statistical physics might be visible in some parts of this book.

I have been fortunate in having the benefit of collaborations and discussions with many colleagues and friends and in particular with Manuel Asorey, Pierre van Baal, Janos Balog, Steven Blau, Jens Braun, Fred Cooper, Stefan Durr, Chris Ford, Lazlo Feher, Thomas Filk, Peter Forgacs, Christof Gattringer, Holger Gies, Tom Heinzl, Karl Jansen, Claus Kiefer, Kurt Langfeld, Axel Maas, Emil Mottola, Renato Musto, Jan Pawlowski, Ivo Sachs, Lorenz von Smekal, Thomas Strobl, Torsten Tok, Izumi Tsutsui, Sebastian Uhlmann, Matt Visser, Christian Wiesendanger, and Hiroshi Yoneyma. On several topics covered in the second and more advanced part of the book I collaborated intensively with my present and former Ph.D. students Georg Bergner, Falk Bruckmann, Leander Dittman, Marianne Heilmann, Tobias Kästner, Andreas Kirchberg, Daniel Körner, Dominque Länge, Franziska Synatschke-Czerwonka, Bjoern Wellegehausen and Christian Wozar. Last but not least I am indebted to Holger Gies and Kurt Langfeld for a critical reading of parts of the manuscript and Marianne Heilmann for translating the German lecture notes into English.

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Statistical Approach to Quantum Field Theory
An Introduction
Wipf, A.
2013, XVIII, 390 p. 133 illus., Softcover
ISBN: 978-3-642-33104-6