Preface

Lie la lie, lie la la-lie lie la-lie.
There must be fifty ways to leave your lover.
Oh, still crazy after all these years.

PAUL SIMON[1]

Think about going to a lonely island for some substantial time and that you are supposed to decide what books to take with you. This book is then a serious alternative: it does not only guarantee a good night's sleep (reading in the late evening) but also offers you a survival kit in your urgent regression problems (definitely met at the day time on any lonely island, see for example Photograph 1, p. ii).

Our experience is that even though a huge amount of the formulas related to linear models is available in the statistical literature, it is not always so easy to catch them when needed. The purpose of this book is to collect together a good bunch of helpful rules—within a limited number of pages, however. They all exist in literature but are pretty much scattered. The first version (technical report) of the *Formulas* appeared in 1996 (54 pages) and the fourth one in 2008. Since those days, the authors have never left home without the *Formulas*.

This book is not a regular textbook—this is supporting material for courses given in linear regression (and also in multivariate statistical analysis); such courses are extremely common in universities providing teaching in quantitative statistical analysis. We assume that the reader is somewhat familiar with linear algebra, matrix calculus, linear statistical models, and multivariate statistical analysis, although a thorough knowledge is not needed, one year of undergraduate study of linear algebra and statistics is expected. A short course in regression would also be necessary before traveling with our book. Here are some examples of smooth introductions to regression: Chatterjee & Hadi (2012) (first ed. 1977), Draper & Smith (1998) (first ed. 1966), Seber & Lee (2003) (first ed. 1977), and Weisberg (2005) (first ed. 1980).

The term *regression* itself has an exceptionally interesting history: see the excellent chapter entitled *Regression towards Mean* in Stigler (1999), where (on p. 177) he says that the story of Francis Galton's (1822–1911) discovery of regression is "an exciting one, involving science, experiment, mathematics, simulation, and one of the great thought experiments of all time".

[1] From (1) *The Boxer*, a folk rock ballad written by Paul Simon in 1968 and first recorded by Simon & Garfunkel, (2) *50 Ways to Leave Your Lover*, a 1975 song by Paul Simon, from his album "Still Crazy After All These Years", (3) *Still Crazy After All These Years*, a 1975 song by Paul Simon and title track from his album "Still Crazy After All These Years".

This book is neither a real handbook: by a handbook we understand a thorough representation of a particular area. There are some recent handbook-type books dealing with matrix algebra helpful for statistics. The book by Seber (2008) should be mentioned in particular. Some further books are, for example, by Abadir & Magnus (2005) and Bernstein (2009). Quick visits to matrices in linear models and multivariate analysis appear in Puntanen, Seber & Styan (2013) and in Puntanen & Styan (2013).

We do not provide any proofs nor references. The book by Puntanen, Styan & Isotalo (2011) offers many proofs for the formulas. The website http://www.sis.uta.fi/tilasto/matrixtricks supports both these books by additional material.

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