Chapter 2
Particle Physics and the Neutrino

2.1 Introduction to Particle Physics

The Standard Model of particle physics has been incredibly successful at describing fundamental particles and their interactions, for a comprehensive introduction and overview see e.g. [1–3]. The Standard Model is an $SU(3) \otimes SU(2) \otimes U(1)$ gauge theory that consists of fermions (quarks and leptons) and fundamental forces (electromagnetic, weak nuclear and strong nuclear) that are mediated by force carrying particles, bosons.

2.1.1 Quarks

There are six different types (or flavours) of quarks which are divided into three generations:

\[
\begin{pmatrix}
u \\
d \\
c \\
s \\
t \\
b
\end{pmatrix}, \begin{pmatrix}
u \\
d \\
c \\
s \\
t \\
b
\end{pmatrix}, \begin{pmatrix}
u \\
d \\
c \\
s \\
t \\
b
\end{pmatrix}
\]  

(2.1.1)

where $u, c, t$ have charge $+\frac{2}{3}$ and $d, s, b$ have charge $-\frac{1}{3}$. The generations are grouped by mass, with $u, d$ being the least and $t, b$ the most massive. Quarks are fermions and have $\frac{1}{2}$-integer spin. Quarks also possess one of three colour charges, a property of quantum chromodynamics that relates to the strong-nuclear force interactions.

2.1.2 Leptons

As with quarks, leptons are divided into three generations, which are defined by their flavour:

The upper particle in each flavour set is a lepton with charge $-1$, the lower particle, $\nu_i$, is a chargeless neutrino of the corresponding flavour. The charged leptons above are ordered by mass, with $e$ being the least and $\tau$ the most massive. For a description of neutrino masses, see Sect. 2.2. Leptons are fermions, with $\frac{1}{2}$-integer spin. Leptons are colourless and as such do not interact via the strong force.

### 2.1.3 Bosons

The four fundamental forces are mediated via force carriers which have integer spin, called bosons. The electromagnetic force is mediated via the massless photon, $\gamma$. The electromagnetic force will only couple to those particles that carry charge. The strong nuclear force is mediated via gluons, $g$, and couples to particles with colour charge. The strength of the strong force increases with separation, resulting in confinement of quarks in colourless groupings known as hadrons. The weak nuclear force, transmitted via three bosons $W^\pm$ and $Z$, couples to particles with weak isospin. All quarks and leptons carry weak isospin. Although the fundamental strength of the weak force is of the same order as the electromagnetic force, the massive nature of the $W^\pm$ and $Z$, 80.4 and 91.2 GeV respectively, makes the force appear weak and short ranged. Gravity is the weakest force and, although it is not included in the Standard Model, is thought to be mediated by a spin 2 boson known as a graviton.

Quarks, therefore, couple to all fundamental forces. Charged leptons do not feel the strong force. Neutrinos in the Standard Model, meanwhile, feel only the weak force.

### 2.2 The Neutrino

The existence of the neutrino was first predicted by Wolfgang Pauli in 1930 to explain the apparent discrepancy between initial and final energy and momenta in beta decays. The particle, initially dubbed the neutron by Pauli, was named neutrino (little neutral one) by Enrico Fermi in 1934, after the discovery and naming of the heavier particle we know as the neutron.

Neutrinos are known to exist in three flavour states ($\nu_e$, $\nu_\mu$ and $\nu_\tau$) as partners to the three, heavier, charged leptons ($e$, $\mu$ and $\tau$). Much of the neutrino’s nature is yet to be established; investigation into the neutrino is seen as one of the most promising avenues for beyond Standard Model physics.

As neutrinos interact only weakly, they are only able to exchange one of the three weak force bosons, $W^\pm$ and $Z^0$, with these interactions known as

\[
\left(\begin{array}{c}
e \\
\nu_e \\
\end{array}\right), \left(\begin{array}{c}
\mu \\
\nu_\mu \\
\end{array}\right), \left(\begin{array}{c}
\tau \\
\nu_\tau \\
\end{array}\right)
\]

(2.1.2)
charged-current (CC) and neutral current (NC) respectively. It is these interactions that enable experiments to observe neutrinos, with observations of secondary leptons or through the search for hadronic recoil. Feynman diagrams of neutrino interactions are given in Figs. 2.1 and 2.2.

### 2.2.1 Neutrino Oscillations

The three neutrino flavour states are themselves superpositions of three mass states, $m_1$, $m_2$ and $m_3$, with the relation between the flavour ($\alpha$) and mass ($i$) states given by:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

Here, $U$ is the PMNS mixing matrix (named for Pontecorvo, Maki, Nakagawa and Sakata), which defines neutrino mixing and oscillations:
\[ U = \begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
  -s_{12} c_{23} - c_{12} s_{23} & c_{12} c_{13} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\
  s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13}
\end{pmatrix} \begin{pmatrix} 1 & 0 \ 0 & 0 & e^{i\alpha} \ 0 & e^{i\beta} \end{pmatrix} \tag{2.2.2}\]

where \( s_{ij} = \sin(\theta_{ij}) \), \( c_{ij} = \cos(\theta_{ij}) \) and \( e^{i\delta} \) is a CP violating term with phase \( \delta \). The terms \( \alpha \) and \( \beta \) are Majorana CP violating phases. The matrix can be factorised as follows:

\[ U = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & c_{23} & s_{23} e^{-i\delta} \\
  0 & -s_{23} & c_{23} e^{-i\delta}
\end{pmatrix} \begin{pmatrix} c_{13} & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1
\end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\
  0 & e^{i\alpha} & 0 \\
  0 & 0 & e^{i\beta}
\end{pmatrix} \tag{2.2.3}\]

Equations 2.2.2 and 2.2.3 demonstrate that the three states of neutrinos will mix with one another. After creation in a flavour state, the neutrino will then propagate as a superposition of the mass eigenstates. Any subsequent interaction will then take place as a flavour state, with the interacting flavour dependent on the mixing parameters. This gives rise to the concept of neutrino flavour oscillation.

It is useful to consider a two flavour system to see how this mass propagation effects the flavour state measured by an observer. In this simplified case, a mixing matrix would be given by:

\[ \begin{pmatrix}
  \nu_\alpha \\
  \nu_\beta
\end{pmatrix} = \begin{pmatrix}
  \cos(\theta) & \sin(\theta) \\
  -\sin(\theta) & \cos(\theta)
\end{pmatrix} \begin{pmatrix} \nu_1 \\
  \nu_2
\end{pmatrix} \tag{2.2.4}\]

A neutrino created in weak eigenstate \( \nu_\alpha \) is therefore a combination of the two mass states \( \nu_1 \) and \( \nu_2 \), as defined by the mixing angle \( \theta \):

\[ |\nu_\alpha\rangle = \cos(\theta)|\nu_1\rangle + \sin(\theta)|\nu_2\rangle \tag{2.2.5}\]

After propagating distance \( L \), this neutrino will be in the following state:

\[ |\nu_x=L\rangle = \cos(\theta)e^{iE_1t}|\nu_1\rangle + \sin(\theta)e^{iE_2t}|\nu_2\rangle \tag{2.2.6}\]

where \( E_1, E_2 \) are the energies of the two mass eigenstates and \( e^{iE_1t}, e^{iE_2t} \) are their propagation as a function of time. The probability of an oscillation into the other flavour eigenstate when measured is thus:

\[ P_{\nu_\alpha \to \nu_\beta} = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| (-\sin(\theta)|\nu_1\rangle + \cos(\theta)|\nu_2\rangle) \left( \cos(\theta)e^{iE_1t}|\nu_1\rangle + \sin(\theta)e^{iE_2t}|\nu_2\rangle \right) \right|^2 \\
= \left| (-\sin(\theta)\cos(\theta)e^{iE_1t} + \cos(\theta)\sin(\theta)e^{iE_2t}) \right|^2 \\
= \cos(\theta)\sin(\theta) \left( e^{iE_1t} - e^{iE_2t} \right)^2 \\
= \sin^2(2\theta) \sin^2 \left( \frac{E_2 - E_1}{2} t \right) \tag{2.2.7}\]
Assuming \( E_i \gg m_i \), \( E_i = \sqrt{m_i^2 + p^2} \), the probability of an oscillation becomes:

\[
P_{\nu_\alpha \to \nu_\beta} = \sin^2(2\theta) \sin^2 \left( \frac{1.27 \Delta (m_{12}^2) L}{E} \right)
\]  

(2.2.8)

where \( \Delta (m_{12}^2) \) is the difference in mass between the two mass eigenstates in eV, \( L \) is the distance travelled in km and \( E \) is the neutrino energy in GeV.

### 2.2.2 Measurement of Mixing Parameters

It was the observation of a flux deficit of Solar \( \nu_e \), first observed by the Homestake Mine experiments in the 1960s and 1970s [4], that led to the realisation of neutrino flavour oscillation. Further experiments such as the Sudbury Neutrino Observatory (SNO) [5] were designed to measure the solar neutrino flux. This led to the determination of the so called ‘Solar’ neutrino oscillation parameters (\( \nu_e \to \nu_{\mu,\tau} \) for \( \theta_{12} \) and \( \Delta (m_{12}^2) \)).

Other experiments, such as Kamiokande [6] and Super-Kamiokande [7], use cosmic-ray-induced \( K^\pm \) and \( \pi^\pm \) decay chains (Eq. 2.2.9) to measure the ‘atmospheric’ oscillation parameters (\( \nu_\mu \to \nu_\tau \) for \( \theta_{23} \) and \( \Delta (m_{23}^2) \)):

\[
\pi^+ (\pi^-) \to \mu^+ \nu_\mu (\mu^- \bar{\nu}_\mu)
\]

(2.2.9)

\( K^\pm \) will decay to produce the same end state, but may also decay to produce final states containing \( \pi^0 \mu^\pm \bar{\nu}_\mu \) or \( \pi^0 e^\pm \bar{\nu}_e \). Any \( \mu^\pm \) produced will decay (although while the meson decay will occur in the atmosphere, the resultant \( \mu \) may penetrate into the Earth).

Equation 2.2.8 demonstrates that, for two flavour neutrino oscillations, there will be an optimal \( L/E \) value at which mixing will be maximal. In the last decade a number of experiments, e.g [8–10], have taken advantage of this, using neutrinos produced at accelerators to make precision measurements of both atmospheric and solar neutrino oscillation parameters, as well as place limits on the remaining oscillation parameters, \( \theta_{13} \) and \( \Delta (m_{13}^2) \) (see Table 2.1).

### 2.2.3 Neutrino Mass

Neutrinos are known to have finite mass, otherwise flavour oscillation would not be possible. While neutrino oscillation measurements provide the absolute difference in mass between the neutrino mass eigenstates, the ordering of these masses is not known. This is known as the neutrino mass hierarchy problem.
Table 2.1  Neutrino oscillation parameters, taken from [11] unless stated otherwise. \( \theta_{ij} \) and \( \Delta \left( m^2_{ij} \right) \) are the mixing angles and the squared mass splittings respectively

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin^2(2\theta_{12}) )</td>
<td>0.861^{+0.026}_{-0.022}</td>
</tr>
<tr>
<td>( \Delta \left( m^2_{12} \right) )</td>
<td>( (7.59 \pm 0.21) \times 10^{-5} \text{ eV}^2 )</td>
</tr>
<tr>
<td>( \sin^2(2\theta_{23}) )</td>
<td>&gt;0.92</td>
</tr>
<tr>
<td>( \Delta \left( m^2_{23} \right) )</td>
<td>( (2.32^{+0.12}_{-0.08}) \times 10^{-3} \text{ eV}^2 ) [12]</td>
</tr>
<tr>
<td>( \sin^2(2\theta_{13}) )</td>
<td>&lt;0.15</td>
</tr>
</tbody>
</table>

Table 2.2  Constraints on the neutrino mass

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mass limit</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{i,\text{heaviest}} )</td>
<td>( &gt;0.05 \text{ eV} )</td>
<td>Oscillation ((\Delta m^2_{23})[11])</td>
</tr>
<tr>
<td>( \Sigma m_i )</td>
<td>(&lt;2 \text{ eV} )</td>
<td>Cosmology [11]</td>
</tr>
<tr>
<td>( m_\beta )</td>
<td>(&lt;2.0 \text{ eV} )</td>
<td>( \beta )-decay [14]</td>
</tr>
</tbody>
</table>

Limits on the mass eigenstates have been obtained through neutrino oscillation experiments and cosmological\(^1\) measurements. Experimental measurements of the energy spectrum endpoint of \( \beta \) decay provides a further constraint on the electron neutrino mass, \( m_{\nu_e} \). These values are summarised in Table 2.2.

As the neutrino has mass, it is possible for the neutrino to be its own anti-particle, known as a Majorana particle. In this scenario \( \nu \) are purely left-handed particles and \( \bar{\nu} \) are purely right-handed (i.e. \( \nu \) viewed in a frame in which its helicity is flipped). It is possible to determine whether the neutrino is a Majorana particle through measurements of double \( \beta \)-decay, specifically searching for neutrinoless double \( \beta \)-decay. Through these experiments, combined with further cosmological measurements and single \( \beta \)-decay measurements, it is hoped that constraints on neutrino mass, neutrino mass hierarchy and the Majorana/Dirac nature of the neutrino, will be found.

References

1. F. Halzen, A.D. Martin, Quarks and Leptons (Wiley, New York, 1985)

\(^1\) Galaxy surveys and anisotropy analysis in the CMB both place limits of \( \Sigma m_i < 2 \text{ eV} \), combinations of multiple cosmological probes place a limit of \( \Sigma m_i < 0.17 \text{ eV} \) at the 95% CL [13].
10. The T2K Collaboration, K. Abe et al., hep-ex/1106.2822
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