Chapter 2
Kinematics of Micropolar Continuum

In this chapter we briefly recall general kinematical relations for a micropolar continuum. For a comprehensive approach, we refer the reader to [1, 2]. The symbolic (direct) tensor notation follows the one by [3, 4], see also Appendix A.

The description of motion of a particle of a micropolar continuum (medium) is based on the assumption that every particle of the micropolar body has six degrees of freedom, see [1, 2]. This is similar to the description of a rigid body in classical mechanics. Three of the degrees of freedom are translational as in classic elasticity, and other three degrees are orientational or rotational.

In the actual configuration $\chi$ at instant $t$, the position of a particle of micropolar continuum is given by the position vector $r$. The particle orientation is defined by an orthonormal trihedron $d_k$ ($k = 1, 2, 3$) whose vectors are called directors. The two vector fields $r$ and $d_k$ define the translational and rotational motions of a particle.

To describe the medium relative deformation, we use some fixed position of the body that may be taken at $t = 0$ or another fixed instant; we call this position the reference configuration $\kappa$. Here the state of particle is defined by the position vector $R$, whereas its orientation by directors $D_k$ (cf. Fig. 2.1). Let us note that as the reference configuration can be chosen not only the real state but also any one.

The motion of a micropolar continuum can be described by the following vectorial fields

$$r = r(R, t), \quad d_k = d_k(R, t). \quad (2.1)$$

In the process of deformation the trihedron $d_k$ stays orthonormal, $d_k \cdot d_m = \delta_{km}$. The change of the directors can be described by an orthogonal tensor that is

$$H = d_k \otimes D_k.$$

$H$ is called the microrotation tensor. So $r$ describes the position of the particle of the continuum at time $t$, whereas $H$ defines its orientation. The orientation of
D_k and d_k can be selected the same, so H is proper orthogonal. Hence, the micropolar continuum deformation can be described by the following relations

\[ \mathbf{r} = \mathbf{r}(\mathbf{R}, t), \quad \mathbf{H} = \mathbf{H}(\mathbf{R}, t). \tag{2.2} \]

The linear velocity is given by the relation

\[ \mathbf{v} = \dot{\mathbf{r}}. \tag{2.3} \]

For brevity, we use the notation \( \frac{d}{dt}(...) \equiv \frac{d}{dt}(...) \), where \( \frac{d}{dt} \) denotes the material derivative with respect to \( t \). As in classical mechanics, see (B.9), the angular velocity vector, called microgyration vector, is given by

\[ \mathbf{\omega} = -\frac{1}{2} \left( \mathbf{H}^T \cdot \dot{\mathbf{H}} \right)_\times, \tag{2.4} \]

where the dot denotes the dot (inner) product and \( (...)^T \)-transposed. The symbol \( (...)_\times \) stands for the vector invariant of a second-order tensor (cf. (A.4)). In particular,
for a dyad $\mathbf{a} \otimes \mathbf{b}$ we have $(\mathbf{a} \otimes \mathbf{b}) \times = \mathbf{a} \times \mathbf{b}$, where $\times$ is the vector (cross) product. Relation (2.4) means that $\omega$ is the axial vector associated with the skew-symmetric tensor $\mathbf{H}^T \cdot \dot{\mathbf{H}}$.

References

Foundations of Micropolar Mechanics
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