Preface

The history of mathematical and numerical finance starts in 1900, with the seminal thesis of Louis Bachelier, *Théorie de la Spéculation*, which introduced Brownian motion in order to model stock price movements and evaluate options. Not only did this remarkable work modeled the randomness of stock prices in a mathematical framework germane to the popular Nobel Prize in Economics winning solution proposed by Fischer Black, Myron Scholes and Robert Merton in 1973, but it also laid the foundation for some key concepts of stochastic analysis.

The celebrated Black-Scholes-Merton pricing paradigm which took the financial industry by storm, is not limited to the Samuelson's geometric Brownian motion model. However, it is based on a series of unrealistic assumptions, including Gaussian return fluctuations, constant volatility, risk-free interest rates, full liquidity, absence of frictions, no price impact from large or frequent trades, ..., and the list could go on. Furthermore, the original pricing arguments do not directly apply to derivatives with non-European exercises such as American options, without another level of sophistication and approximation.

The last two decades have seen a rapid development of increasingly realistic and sophisticated stochastic models and methods for pricing, hedging and risk management in rapidly growing markets, with more unfathomable financial products. Modern finance is becoming increasingly technical, requiring the use of complicated mathematical models, and involving numerical techniques based on theoretical results from subfields of mathematics ranging from stochastic analysis, dynamical system theory, nonlinear integro-differential equations, game theory, optimal control and dynamic programming, to statistical learning and information theory. Situated at the confluence of applied mathematics, computer sciences and economics, quantitative finance distinguishes itself through its wide range of themes, and its interaction with a broad spectrum of scientific domains.

Any attempt at capturing all the fundamental developments which occurred in quantitative finance would simply be an impossible undertaking. With this volume, we aim at something less ambitious, more focused, and hopefully more useful, offering a collection of representative articles in the area of computational finance. Our objective is to bring financial professionals, economists and mathematicians closer together by raising the level of awareness of the new challenges emerging in quantitative finance, and offering a lucid exposé of recent numerical solutions developed by researchers working in stochastic analysis and financial mathematics. The book should be of interest to practitioners, academics, graduate students in financial mathematics, but also to probabilists, statisticians and more generally, applied mathematicians.

Research in financial mathematics is the driver for a great variety of numerical applications: parameter estimation, calibration of valuation models, derivative pricing, sensitivity analysis, hedging in incomplete markets, credit risk, risk and uncertainty quantification, portfolio optimization.... Resisting the temptation to cover as broad a range of applications as possible, we chose to focus on one particularly interesting issue, namely pricing and hedging of instruments with exercises of the American type. Our choice is motivated by the fact that optimal stopping problems offer a unique test-bed for research which goes beyond the edges of mathematical finance, at the crossroad of stochastic control and operations research. As a result, the numerical methods developed for these specific problems can in general be extended to a wide range of other stochastic control problems.

The present volume includes the works presented by the participants of the *Workshop on Numerical Methods in Finance*, organized at the INRIA Bordeaux-Sud Ouest Center and at the Mathematical Institute in Bordeaux, in June 2010. The editors are grateful to EDF R&D and the FiME Lab (*Finance for Energy Market Research Centre*) without which this book would not have been possible. We are also grateful to all the authors who accepted to contribute their works, and to the anonymous referees who reviewed the original submissions. Their insightful comments and their constructive reports helped improve the quality of the final product.

The book is organized into three logically delineated parts, and the remaining of this foreword is devoted to the description of the contents of the contributions included in these three parts.

The investigation of interacting particle methods has become a very active area of research in scientific computing. These methods are rooted in the pioneering work of Feynman and Kac in high-energy physics over half a century ago. The systematic formalization of these ideas into a rigorous mathematical theory is more recent, essentially dating to the 1990s. They are now used in a wide variety of domains, such as rare event estimation and simulation, filtering and stochastic optimization. However, the development and the applications of these algorithms to mathematical finance problems is still in its infancy. The first part of this book is devoted to this subject.

In the first contribution, we give a general overview in the form of an introductory survey of the mathematics of particle methods. As motivation, we show that several important numerical problems encountered in finance can be reduced to the computation of Feynman-Kac expectations. We recall the main principles and results of the theory behind interacting particle numerical methods, and show how they can be applied to European and American option pricing, and sensitivity analysis. In quantitative finance, stochastic volatility models offer a natural instance of a partially observed system, and filtering techniques have often been used to analyze these models. In [16], V. Genon-Catalot, T. Jeantheau and C. Laredo use particle methods for their estimation. A rather unexpected application of particle methods was proposed by R. Carmona, J. P. Fouque and D. Vestal in [4], and R. Carmona and S. Crepey in [3], for the computation of the probabilities of simultaneous defaults in large credit portfolios. In both cases, the method is based on the path-breaking paper of P. Del Moral and J. Garnier [10] on the use of Feynman-Kac expectations and particle methods to the computation of rare events. As an example of still another financial application, the second part of the contribution focuses on the use of particle methods for the solution of stochastic control problems with partial observations. Even if most of the financial mathematics literature assumes perfect observations, in concrete situations, investors and traders only have a partial knowledge of the parameters involved in the models used for pricing and hedging. From a pure mathematical point of view, partial observation problems are equivalent to their perfect observation versions, up to an *infinite* state space enlargement. In this new framework, the reference Markov evolution process is now defined in terms of the current observation and the hidden original process represented by a filtering equation. This strategy is surveyed in the recent article by V. S. Borkar [2] and the pioneering article by J. J. Florentin [18] in the early 1960s. In this contribution, we show that, the strength of particle techniques is that they make it possible to turn a stochastic control problem with partial observations into a fully observed problem associated with an easy to sample particle Markov chain approximation model.

The use of particle methods for partially observed control problems is further developed in the next two contributions to this first part.

The contribution of B. R. Rambharat introduces a new particle filter methodology to price American-style options on underlying investments governed by partially observed stochastic volatility models. In contrast to the majority of the research on American option valuation which assumes that all sources of randomness are fully observable, the author designs a pricing algorithm for stochastic asset evolution models with partial information on the volatility process. Posterior inference on these unobservables is accomplished by using a sequential Monte-Carlo methodology. The corresponding convergence analysis can be found in [6–8, 14] and in [21].

The contribution by M. Ludkovski presents an hybrid methodology for solving stochastic control problems with partial observations, relying on modern particle filter techniques combined with Longstaff-Schwartz style regression procedures. This contribution is very much in the spirit of the thorough discussion of particle estimation methods which can be found in [11, 13, 15]. And the series of articles [9, 11, 12, 20, 22].

Many financial applications can be modeled as stochastic optimization problems. Stochastic dynamic programming is a time-honored approach to the search for a solution to these problems. The main practical impediment in the implementation of the dynamic programming principle is the recursive computation of the conditional expectations appearing in the backward induction. A great variety of numerical methods, including plain Monte-Carlo simulation, importance sampling, least squares regression, integral transform, partial differential equations techniques, ... have been brought to bear in hope to find reasonable solutions.

The second part of the present volume, is entirely devoted to this problem. It presents a wide range of methods introduced recently in order to compute conditional expectations for pricing American/Bermudan options and numerical solutions of Backward Stochastic Differential Equations (BSDEs). We distinguish two kinds of approach depending on the type of assumption made on the underlying price model. Five articles consider the general case of Markovian price models with Monte-Carlo or quantization methods, and two articles are more specifically focused on exponential Lévy price models with integral transform and Partial Integro Differential Equation (PIDE) methods.

The first contribution of the second part is by P. Del Moral, B. Rémillard and S. Rubenthaler. It presents a new approximation method combining Monte-Carlo simulations and linear interpolation techniques which preserve the monotonicity and the convexity of the American option value function. The article also provides an overview of three classes of algorithms for the valuation of American options: deterministic and stochastic tree based methods, traditional Partial Differential Equation (PDE) techniques, and Longstaff-Schwartz style functional regression style methods. Illustrations and comparisons of the performance of these numerical methods in the case of American put options when the underlying interest is a geometric Brownian motion or a N-GARCH process.

The contribution of B. Rémillard, A. Hocquard, H. Langlois and N. Papageorgiou proposes an approximation of the price of an American-style option based on hedging with the underlying assets at discrete times. The authors provide an optimal hedging solution which minimizes the variance of the hedging error. A key feature is that the choice of the traditional risk-neutral measure is bypassed, the variance of the hedging error being minimized under the objective measure. The authors present the results of a Monte-Carlo experiment in which the hedging performance of the solution is evaluated. For asset returns which are either Gaussian, Variance Gamma, or general Lévy processes, they show that the proposed solution results in lower root mean square hedging error than with traditional delta hedging procedures.

The contribution of G. Pagès and B. Wilbertz provides a very nice review of pure quantization methods for pricing multiple exercise options. These numerical techniques rely on approximating a given random variable by a random variable taking values in some judiciously chosen finite grid. Quantization trees are the result of the quantization of the sequence of random states of a Markov chain. They lead to deterministic numerical methods allowing a straightforward implementation of the Dynamic Programming Principle (DPP) for optimal stopping and stochastic control problems. The authors also present a unified discussion of quantization methods based on Voronoï and Delaunay tessellations, and illustrate the performance of both methods with several numerical examples.

In their contribution, B. Bouchard and X. Warin propose two efficient pricing and hedging algorithms enhancing the Longstaff-Schwartz and the Malliavin Monte-Carlo methods for American exercise valuation. The first procedure improves the functional regression approximation using an original adaptive local basis approach. In the second one, the authors propose a clever idea to reduce the complexity of the Malliavin approximate backward dynamic program. Numerical experiments are provided, and comparisons with quantization grid methods are reported in the case of a d – dimensional Black-Scholes model.

The article by C. Bender and J. Steiner starts with a review of the least-squares Monte-Carlo approaches for solving BSDEs, very much in the spirit of the thorough discussion which can be found in the seminal article of E. Gobet, J.-P. Lemor and X. Warin [17]. In the second part of their contribution, the authors present an original function basis martingale approach and they show how to simplify the least-squares Monte-Carlo scheme using the martingale property provided by the random basis functions. Finally, the authors report different numerical results showing that their method has better stability properties than the original Longstaff – Schwartz procedure.

The article by L. J. Powers, J. Nešlehovà and D. A. Stephens explores the properties of diffusion approximations of infinite activity Lévy processes. These stochastic processes have an infinite number of small jumps in any finite time interval. For this reason, they are often used to model the micro structure of complex financial markets. The central idea behind diffusion approximation techniques is to replace the small jump component by a small Brownian motion. The authors investigate numerically the performance of these approximations for pricing American options in exponential Lévy models. The behavior of the approximation close to, and far from the free exercise boundary is investigated numerically using stochastic (Monte-Carlo) and deterministic (finite element) methods.

In their contribution, B. Zhang and C.W. Oosterlee present a Fourier cosine expansion scheme for pricing Bermudan options in Lévy asset price models. The authors suggest an improvement based on the Put-Call parity, and the performance of the improved algorithm is illustrated by simulations.

The third part of this volume is devoted to a generalization of the classical American option which is particularly relevant in the energy and commodity markets. Indeed, the physical nature of many of the instruments traded on these markets implies that a great number of contracts allow the holder to exercise an option (e.g. choice of the volume of the commodity actually delivered) multiple times, with flexibility in terms of both the quantity and the time exercised. The specification of such contracts can be very complex, with constraints on the volume exercised at any given time, and on the total volume exercised throughout the life of the option. This type of option, generically known as *swing option*, can also be used in the spirit of the theory of real options to capture optionality in the management of physical assets such as power plants or gas storage facilities, possibly integrating additional constraints to take into account the physical characteristics of the asset in question. This application demonstrates the close link mentioned earlier between computational finance and operations research. Some of the techniques presented in the second part of the book for standard American options, can be extended and applied in this more general context of this part.

The theoretical foundations needed for the study of these instruments require the analysis of complex stochastic control problems. It is a rather new field of research and we refer the reader to the groundbreaking articles of R. Carmona and N. Touzi [5] and the more recent treatment using quantization techniques by O. Bardou, S. Bouthemy and G. Pagès [1].

The first contribution by K. Wiebauer is of a survey nature. It provides an overview, with illustrative examples, of some common option types models with multiple exercise rights in the electricity and gas markets.

The contribution of M. Bernhart, H. Pham, P. Tankov and X. Warin introduces a new probabilistic approach for pricing swing options using the BSDE representation of impulse control problems with constrained jumps introduced recently by I. Kharroubi, H. Pham, J. Ma and J. Zhang in [19]. The authors introduce an original penalization procedure to deal with the constraints on the jumps. The effective pricing algorithm combines a time discretization approximation scheme with classical least square Monte-Carlo approximations. Finally, numerical simulations are provided, and the role of the different parameters of the model (jump intensity, penalization coefficient and time step) are discussed.

The article by F. Turboult and Y. Youlal introduces a new Monte-Carlo methodology for pricing multiple exercise options with a single source of uncertainty. It is based on the search for the optimal exercise boundary characterized as the point maximizing an expectation which can be estimated by Monte-Carlo methods. The proposed algorithm is proved to achieve a precision of the same order as Longstaff-Schwartz', for a lower computational complexity as confirmed by the results of numerical experiments.

Gas storage management models and related stochastic optimization problems are discussed in the article by X. Warin. The optimal asset management problem is approximated by a bang-bang stochastic control problem. The optimal hedging strategies developed in this article are computed using conditional tangent techniques. The mathematical foundations of these algorithms are presented in the complementary joint contribution of the author and B. Bouchard.

The article by J. F. Bonnans, Z. Cen and T. Christel analyzes a model of medium term commodity contract management when the state variable is multi-dimensional and the randomness enters the prices only at the times at which the commodities are exchanged. The authors provide a sensitivity analysis with respect to parameters driving the price. The stochastic price evolution and the Bellman value functions are approximated using a Voronoï quantization grid. The main contribution of this paper is the application of the Danskin's theorem for the computation of sensitivities of the stochastic dynamic decision problem.

USA *R. Carmona*
France *P. Del Moral, P. Hu, N. Oudjane*

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