Preface

This book can be considered a continuation of *The Regularity of Minimal Surfaces* by Ulrich Dierkes, Stefan Hildebrandt and Anthony Tromba, Volume 340 of the *Grundlehren der Mathematischen Wissenschaften*.

The central theme is the study of branch points for minimal surfaces with the goal of providing a new approach to the elementary question of whether minima of area or energy must be immersed.

One of the main difficulties with the current theory of branch points is the transparency and sophistication of the proofs of the main theorems. For example, Osserman’s original 1970 cut and paste proof, that absolute minima are free of interior branch points remains, for the most part, open only to experts. Furthermore, before the appearance of this volume, no complete proof has appeared in one place.

In the 1960’s the development of global nonlinear analysis and the idea of doing calculus or analysis on infinite dimensional manifolds had created a great deal of excitement, especially through the pioneering work of Jim Eells, Dick Palais and Steve Smale.

The goal of this book is to develop entirely new and elementary methods, in the spirit of global analysis, to address this beautiful question via energy (Dirichlet’s energy) as opposed to area. We will do something that rarely, if ever, has been done in the calculus of variations, namely calculate arbitrarily high orders of derivatives of energy. This method also applies to boundary branch points for minimal surfaces with smooth, but not analytic boundaries, a question that heretofore has not been addressed.

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My career owes a great debt to all of these wonderful mathematicians.

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