Air Elimination in Milk

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Abstract This paper presents the work done to model a novel design for accurate volume measurement of milk. The new design proposes the addition of an air elimination vessel to the current milk pumping systems. This is an extra tank between a farmer's milk storage tank and the pumping storage system on a lorry designed to collect the milk from farms. The purpose of the air elimination vessel is to allow a pool of milk to accumulate so that air bubbles entrained with the milk can be removed and pumped out by a separate pump for air. The paper first discusses the flow of milk inside the air elimination vessel. This is modelled by a system of partial differential equations similar to the St Venant equations but modified for a cylindrical environment. The paper then discusses bubble formation and flow and predicts the level of milk required in the air elimination vessel for bubbles of air to rise and be pumped out. This is modelled by a single ordinary differential equation derived from momentum conservation.

1 Introduction

The aim of this project is to increase the measurement accuracy of the volume of milk being pumped from a farmer's tank to a storage tank on a lorry which travels from farm to farm collecting milk. Inaccuracies are introduced to the volume measurement by air, in the form of bubbles and foam, being sucked through the volume measurement device and registering as milk. The very simplified schematic given in Fig. 1 shows that when the level of milk in the farmer's tank is less than the width of the pipe draining it, air from the atmosphere will also be pumped. This air will create bubbles and foam which cause the inaccuracy in measurement.

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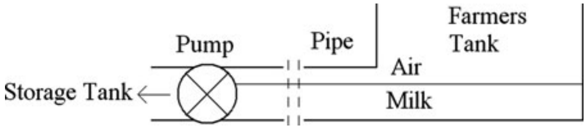

Fig. 1 Generic farmer's tank and pump schematic

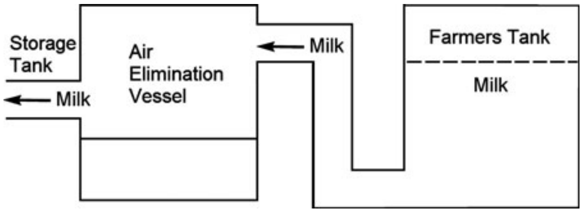

Fig. 2 Proposed design

The proposed solution to this problem is the use of an air elimination vessel (AEV) which will remove any air sucked in with the milk using a second pump.

The problem consists of a number of parts: the simulation of the entire system to show that the design can empty the milk from the farmer's tank, analysis on the flow of milk inside the AEV, and the formation of air bubbles inside the AEV and their removal.

1.1 Proposed Solution

The design for the AEV is confidential so Fig. 2 is a simplified schematic for the proposed solution. The AEV functions by allowing a pool of milk to accumulate which will allow any air bubbles trapped in the milk to rise and escape. This is achieved by a number of independent pumps and valves controlling the flow of milk and air from the AEV. The vessel contains sensors for measuring the pressure of air and the level of milk.

It has been verified that this design will successfully pump milk from the farmer's tank, through the AEV and into the storage tank. This analysis will not be included in this paper. We will instead focus on the flow of milk inside the AEV and the formation of bubbles.

2 Flow of Milk Inside AEV

We now consider the flow of milk inside the AEV as it enters from the pipe at the top. Figure 3 shows the pipe attached tangentially to the top of the AEV. Hence the milk circulates around the inside wall of the AEV and forms a shallow layer of milk. The flow is modelled by a continuity equation (1), a linear momentum equation (2) and an angular momentum equation (3):

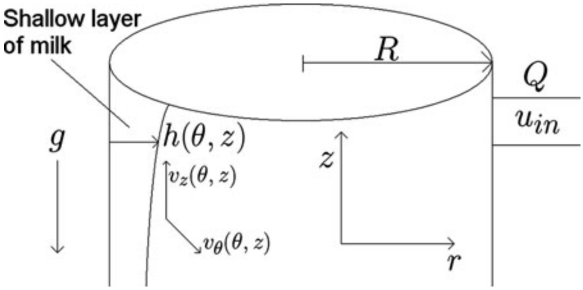

Fig. 3 Milk flow inside the AEV. R is the radius of the AEV, Q is the volumetric flow rate of milk, u_{in} is the inflow velocity, g is acceleration due to gravity, h is the thickness of the layer of milk on the inside wall of the AEV, v_θ and v_z are the azimuthal and axial components of the milk velocity

$$\frac{\partial h}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial \theta}(h v_\theta) - \frac{\partial}{\partial z}(h v_z) + q(z, \theta), \tag{1}$$

$$\frac{\partial}{\partial t}(h v_z) = -\frac{1}{r}\frac{\partial (h v_\theta\, v_z)}{\partial \theta} - \frac{\partial (h v_z^2)}{\partial z} - g\, h - C\sqrt{v_\theta^2 + v_z^2}\, v_z - \frac{1}{2r}\frac{\partial (v_\theta^2 h^2)}{\partial z}, \tag{2}$$

$$\frac{\partial}{\partial t}(h v_\theta) = -\frac{1}{R}\frac{\partial (h v_\theta^2)}{\partial \theta} - \frac{\partial h v_\theta v_z}{\partial z} - C\sqrt{v_\theta^2 + v_z^2}\, v_\theta - \frac{1}{2R^2}\frac{\partial (v_\theta^2 h^2)}{\partial \theta} + q(z, \theta) u_{in}, \tag{3}$$

where C is the surface friction coefficient for the fluid on the inside of the AEV and $q(z, \theta)$ is the flux of milk into the AEV. The case of $h \ll R$ is used in deriving (3). These equations are based on the St Venant equations [2] but specialised to a cylindrical geometry. The main difference is the inclusion of a centrifugal force term.

The system of equations is simplified by neglecting time and angular dependance to give

$$-\frac{d}{dz}(h\, v_z) + q(z) = 0, \tag{4}$$

$$-\frac{d}{dz}(h\, v_z^2) - gh + C v_z^2 - \frac{1}{2r}\frac{d}{dz}(u_{in}^2\, h^2) = 0. \tag{5}$$

This simplification can be made as it is expected that a steady state thin film of milk circulating around the inside walls of the AEV will occur for a continuous stream of milk from the farmer's tank. Solving (4) for $h(z)$, and assuming $\int q(z)\, dz = \frac{Q}{2\pi r}$ where Q is the steady flux from the pipe and r the radius of the pipe, results in an expression for $h(z)$ which can be substituted into (5) to produce a single ODE

$$\frac{Q}{2\pi r}\frac{dv_z}{dz} + \frac{Q}{2\pi r}\frac{g}{v_z} + C v_z^2 + \frac{Q^2 u_{in}^2}{4\pi^2 r^3}\frac{1}{v_z^3}\frac{dv_z}{dz} = 0. \tag{6}$$

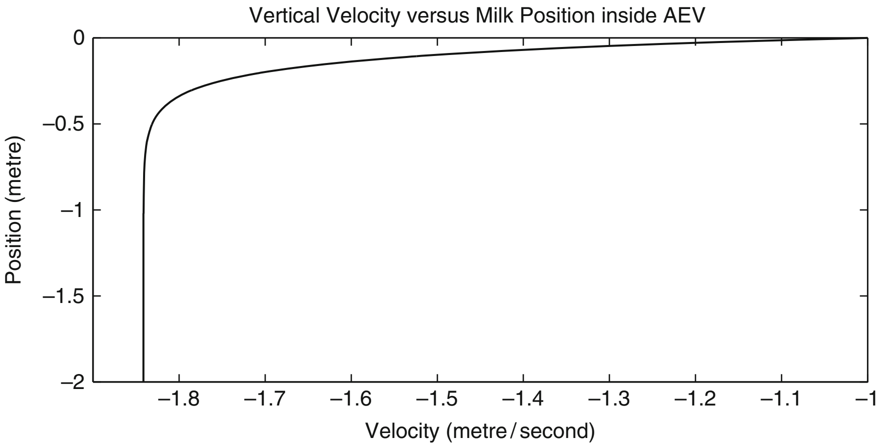

Fig. 4 Solution of (6) for velocity of milk as it flows down side of AEV

Figure 4 is a plot of the numerical solution of (6). A nonzero initial condition is used as the assumption of the shallow layer of milk swirling around the inside of the AEV breaks down due to the rapid flow of milk from the pipe from the farmer's tank. The plot shows that the milk flowing down the sides reaches a terminal velocity. This is due to the gravity and friction terms in (6) balancing.

3 Bubble Formation

In the previous section we obtained a terminal velocity for the milk flowing down the inside walls of the AEV. We will now use this to calculate how deep bubbles will be entrained into the pool of milk inside the AEV. From analysis done on the pumping system, it was found that a steady state occurs inside the AEV when the depth of milk is 60 cm. The depth these bubbles travel into the pool of milk is required to determine if they will have time to rise and be removed from the milk or sink all the way to the bottom and be pumped out with the milk.

Clanet and Lasheras [1] show that when the velocity of the fluid containing bubbles decreases, due to buoyancy and drag, to $0.22\,\mathrm{m\,s^{-1}}$ or less, the bubbles will start to rise. We need to determine the depth at which the entrained bubbles reach this velocity. We do this by considering conservation of momentum. Figure 5 shows the formation of bubbles by the entrainment of air. The positive direction of z has been reversed for convenience. Milk flows down the inside wall by gravity.

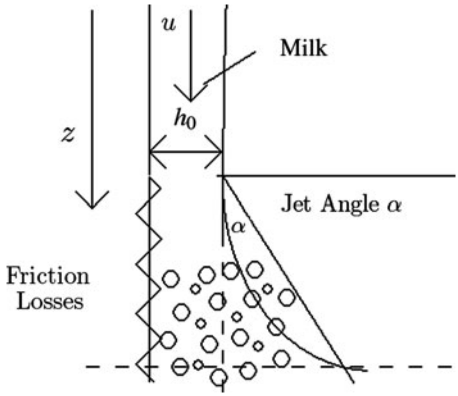

Fig. 5 Bubble formation as milk flows down the inside wall of the AEV

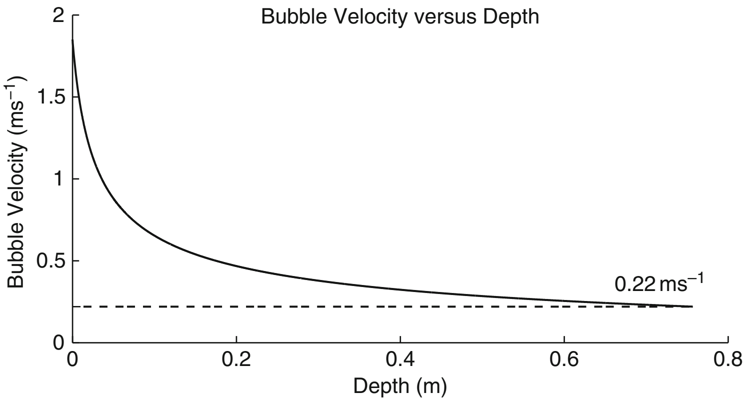

Fig. 6 Bubble velocity versus bubble depth

The bubbly liquid loses momentum to entrained fluid described by the jet angle α, friction with the wall and buoyancy. This is modelled by

$$\frac{d}{dx}(u^2) = \frac{-u^2\left[\frac{C}{2} + \tan(\alpha)\right] - g\beta_0 h_0}{h_0 + x\tan(\alpha)}, \tag{7}$$

where h_0 is the height of the thin film of milk, u is the velocity of the jet of milk and β_0 is the bubble volume fraction. This was derived using similar analysis to the derivation of the model used by [1] but making assumptions appropriate to a planar jet instead of a circular jet. This equation relates the velocity of the bubbly fluid in the pool of milk to its depth. The initial condition used to solve this is the terminal velocity the milk reaches as shown in Fig. 4. This was solved numerically using MATLAB. A plot of the solution is given below in Fig. 6. This shows bubble velocity decreasing with depth as expected. However, it achieves the velocity 0.22 m s^{-1} at a much greater depth than expected. As the height of the milk reaches a steady state at approximately 0.6 m, as predicted by simulation of the entire system, and the bubbles here reach this velocity at approximately 0.8 m, the bubbles may not have time to escape and will instead be sucked out by the pump at the base of the AEV.

4 Conclusions

First we consider the flow of milk in the AEV. A system of three PDEs is derived, simplified and solved to calculate a terminal velocity for the milk flowing down the sides of the AEV.

We use this terminal velocity in the Sect. 3 when we calculate the depth at which these bubble sink to. It was shown depth is approximately 0.8 m. This is greater then the depth of milk in the AEV and would allow air to be pumped with the milk from the AEV.

This suggests a redesign of the AEV is required. Due to the confidential nature of the AEV design, this redesign cannot be outlined here.

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