Modelling and Analysis of the Nonlinear Dynamics of the Transrapid and Its Guideway

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Abstract In the development and optimization of magnetic levitation trains, realistic simulation models of the mechanic, electromagnetic and electronic subsystems both onboard and in the guideway are crucial factors. In this contribution we present coupled mechanical-electromagnetic models of the control subsystems, magnet subsystems, a lateral cross-section and a vertical dynamics model, modeled by the multibody systems method. The models are verified using simulations, eigenmode analysis and displacement measurements from train passages on a test track. The models are suitable e.g. for simulating the effects of train passages on the ground and they are applied to the analysis of a novel guideway support. It is shown that ground vibrations caused by the vehicle can be significantly reduced by a flexible spring-mass system as support for the girders.

1 Introduction

The Maglev vehicle Transrapid as described in [7] and [9] is levitated by magnetic forces which pull the vehicle's levitation chassis towards the guideway from below, as shown in Fig. 1. The levitation magnets are distributed equally along the vehicle

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Fig. 1 Vehicle TR09 on the TVE test facility (picture: Fritz Stoiber Productions, 2008)

and possess poles with alternating fluxes which are part of the synchronous long stator linear motor. Furthermore, guidance magnets lead the vehicle along the guideway. The guideway usually consists of a series of individual concrete girders equipped with stator packs as reaction surface for the levitation magnets and steel rails as reaction surface for the guidance magnets. Within the stator packs the long stator motor winding is placed for the propulsion and braking of the vehicle. An overview of the design of magnets and guideway is given in [10] and a detailed comparison between the system properties of the Transrapid and the system properties of conventional high-speed trains can be found in [11].

Dynamically, the Transrapid represents a typical mechatronic system comprising mechanical, electrical, magnetic, electronic and control subsystems. As the magnetic levitation is inherently instable a well performing and reliable control system is of paramount importance. In order to analyze the dynamic behavior and to optimize the control parameters systematically simulation models can be applied. Such simulations have long been described by various authors, e.g. [2, 6, 8]. While only small models could be handled by computer systems in the early development phase of the Transrapid, today state-of-the-art IT-systems permit the analysis of detailed models, containing the interaction between the subsystems which requires the consideration of many state variables. Lately, a multibody model of a complete Transrapid vehicle consisting of three sections (coaches) has been described [3]. In this model, the controlled forces include effects from the geometric shape of the magnetic field but the nominal magnet forces are calculated by a simplified PD control law for each magnet. The work presented here augments this model by a detailed representation of the electro-magnetic subsystem and the control system yielding the full mechatronic model. In order to validate and analyze the stability of the controlled system in detail, further mechanical representations of the vehicle and of a magnet test bench are generated.

2 Modelling

The models for analyzing the lateral and vertical dynamics are constructed using the same basic structure: The main model is divided into submodels for the mechanical subsystem, the electromagnetic subsystem and the electronic subsystem. Figure 2

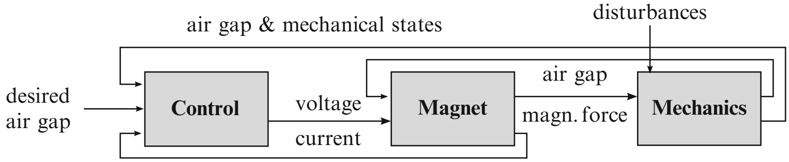

Fig. 2 Schematic representation of the basic model structure

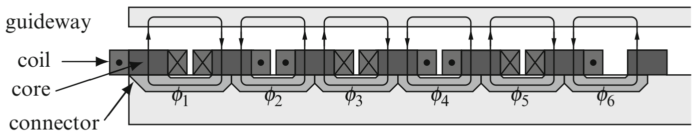

Fig. 3 Image of the left half of a levitation magnet, consisting of six interconnected poles

shows a schematic representation of the submodels' input-output-structure. In Sect. 2.1 the model of the electronic and electromagnetic subsystem is described. Sections 2.2 and 2.3 explain the mechanical models of the vertical and lateral cross section and how these are coupled with the magnetic model. For more details on the magnetic model see [12].

2.1 *Guidance and Levitation Magnets*

The guidance and levitation magnets (see Fig. 6, *left*) are very similar in structure and design, thus it is sufficient to construct one model and to adjust its parameters to account for guidance or levitation magnets. Each levitation magnet consists of six poles which are wired in series. The poles are designed such that the fluxes flow mostly through material with high permeability. See Fig. 3 for an illustration. The guidance magnets do not need alternating fluxes because they do not interact with the motor and can be modeled as one pole systems along the x-direction.

Maxwell's equations for quasi-stationary electric and magnetic fields are used to set up the differential equations for the flux computation. We obtain the following equation from the magnetic system (see Fig. 4, *left*)

$$R_{mag}\phi = \theta \ , \quad R_{mag} \in \mathbb{R}^{6\times 6} \ , \quad \phi = (\phi_1 \dots \phi_6) \ , \quad \theta = (\theta_1 \dots \theta_6) \in \mathbb{R}^6 \qquad (1)$$

with R_{mag} representing the resistances in the magnetic network, ϕ the fluxes and θ the magnetic voltages. For each pole its magnetic voltage can be computed as $\theta_k = n_k I_k, k = 1, \dots, 6$ (n_k: number of windings, and I_k: electric current through the coil).

Similarly, using Kirchhoff's laws, the governing equations of the electric substitute system (Fig. 4, *right*) can be computed to be $U_M = \sum_{i=1}^{6} (U_{ind,i} + R_i I_M)$.

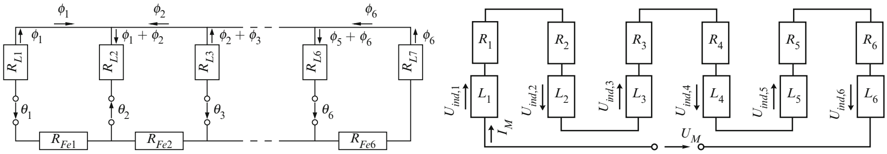

Fig. 4 Substitute systems for the magnetic network (*left*) and electric network (*right*)

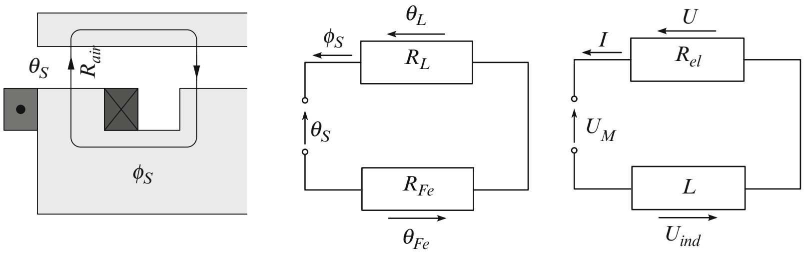

Fig. 5 Smaller magnet model (*left*) and its substitute magnetic (*center*) and electric structure (*right*)

This equation can be coupled with (1) using $U_{ind} = n\dot{\phi}$ which describes the time-dependent behavior of the coil, with U_{ind} being the induced voltage, n the number of coil windings and ϕ the magnetic flux generated by the coil, resulting in a differential equation for the flux. For a given flux, the magnetic force is proportional to the flux squared: $F_{mag} = K_{mag}\phi^2$. The factor K_{mag} as well as all other resistances were calculated taking into account material properties and exact geometries. This leads to a model which is suitable for very exact computations but at the cost of higher simulation effort. To overcome this drawback, a substitute model consisting only of a single pole is constructed (see Fig. 5). A parameter fitting is performed such that its input-output-behavior, i.e. electronic and magnetic voltage and current, resembles the behavior of the original model.

The magnet control units regulate the size of the air gaps by adjusting the magnet's input voltages. In each unit the sensor values for air gap, magnet body acceleration and current are filtered, scaled and processed by the actual controller. Afterwards the controller output is transformed into an input voltage for the magnet (cf. Fig. 2).

2.2 *Lateral Cross Section Dynamics*

The lateral dynamics, i.e. vertical, horizontal and rolling motions of the magnetic levitation train are modeled by a multibody system of a lateral cross section of the vehicle. The reference length is one eighth of a Transrapid's section such that two levitation frames and two secondary suspension units are taken into account. We

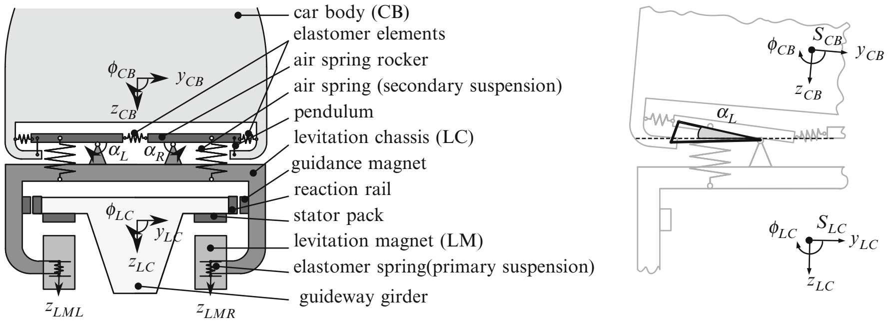

Fig. 6 *Left:* Generalized coordinates ($y_{LC}, z_{CB}, \phi_{LC}, y_{CB}, z_{CB}, \phi_{CB}, z_{LML}, z_{LMR}$) and depending auxiliary variables (α_L, α_R) for the lateral dynamics model. *Right:* The angles of the air spring rockers can be computed by the triangle spanned by body fixed points on rocker, car body and lev. chassis

consider the levitation frames to be equipped with guidance magnets and combine the pair of levitation magnets and the pair of primary suspension units to one object respectively (see Fig. 6). The car body, levitation chassis, levitation magnets and the two air spring rockers are idealized as rigid bodies whereas the primary and the secondary suspensions are modeled as massless connections. The suspensions are reduced to linear spring damper systems acting one-dimensionally. Since deviations of the train from the reference position are known to be small, a linear model of the lateral dynamics is appropriate.

The cross section model exhibits eight degrees of freedom (as depicted in Fig. 6) assuming the levitation magnets translate only vertically. The rotation about the x axis and the translation in y and z direction of the levitation chassis is described with respect to an inertial frame. The coordinates describing the rotational and translational motion of the car body as well as those of the levitation magnets are defined relative to the chassis. At first it is not obvious that this is an admissible choice of generalized coordinates because the topology of the model contains a kinematic chain consisting of the chassis, car body, both air spring rockers and both pendulums. In principle this would lead to a system of differential algebraic equations. However, we obtain a system of ordinary differential equations by substituting the dependent variables by an explicit formulation of the algebraic equations in terms of the independent variables. The dependent auxiliary variables are the rotation angles of the air spring rockers which can be calculated geometrically as sketched in Fig. 6, *right*. For more details see [1].

The modelling is realized using the software package Neweul [4] which derives analytically the equations of motion by the Newton-Euler formalism. The explicitly formulated algebraic equations for the air spring rocker angles are implemented in the computer algebra system Maple such that the partial derivatives for the linearized equations of motion can be derived analytically as well.

2.3 Vertical Dynamics

To examine the interaction of the Transrapid with its guideway in detail, the vertical dynamics have been modeled as well. Our aim was to create a suitable model for studying the ground vibrations induced during train passages. The guideway and the vehicle mechanics are modeled separately such that the resulting entire system has a modular structure and any components can be exchanged comfortably.

We model a maglev train consisting of three sections as shown in Fig. 7. In order to reduce the complexity, only one section is modeled as a multibody system comprising 28 mechanical degrees of freedom. The other sections are considered by their nominal magnet forces. To merge the three-dimensional vehicle into a two-dimensional model, the effects of the left and right side are summed up. The resulting translational and rotational degrees of freedom are depicted in Fig. 8, *left*. The equations of motion are derived analytically using Neweul.

The simplified version of the electromagnetic subsystem, see Sect. 2.1, is used to model the levitation magnets. The geometry of the stator packs induces a slight dependence on position of the magnet force. This is accounted for by a position dependent scaling as shown in Fig. 8. Although this position dependency is small compared to the nominal load it is responsible for a significant part of the vibrational load.

It is possible to switch between a dynamic middle and a dynamic front section by simply using a different parameterization as the topologies of both models are equal. For more details of the vertical model, see [5]. The general approach of coupling the vehicle and the guideway model goes back to [3].

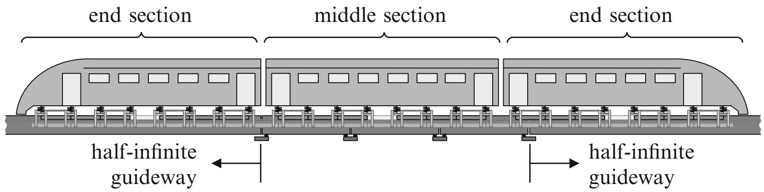

Fig. 7 Maglev train consisting of three sections and a guideway composed of three girders

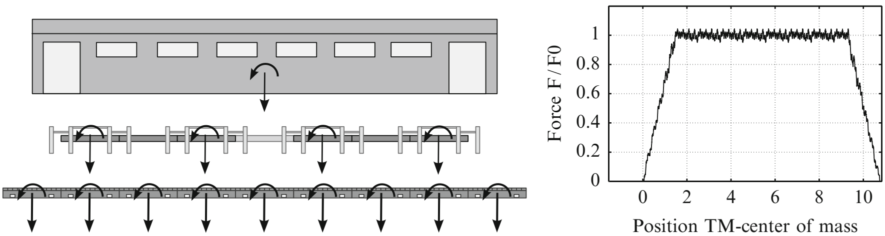

Fig. 8 *Left:* Generalized coord. of a dynamic middle section. *Right:* Magnet force modulation

3 Verification and Analysis

As described in the previous section, the lateral and vertical dynamics models are complex, nonlinear, multiphysics simulation models which have to be verified properly. Subsequently, these models can be used to uncover inherent dynamical properties of the systems.

3.1 *Verification of the Magnet Models*

ThyssenKrupp uses a test bench to test the Transrapid's magnets in various situations. We use a model of the test bench's mechanics to validate the combined magnet-controller models. At first, linearizations of the full and the reduced magnet-controller model are compared. Bode-plots show a similar behavior of the two models over a wide range of frequencies.

To compare our model to the test stand (see Fig. 9, *left*), we generated a Bode diagram for the test stand (see Fig. 9, *right*). The test stand consists of two levitation magnets that follow a forced sinusoidal vertical motion. The frequency of the input motion has been chosen as input and the frequency of the following motion as output for the Bode diagram. The plots show similar but not the same behavior. The differences between the diagrams can be explained by several damping effects that have not been taken into account in the model.

3.2 *Linear Analysis of the Lateral and Vertical Dynamics*

As a first step we perform an eigenmode analysis of the undamped mechanical subsystems to characterize the shape of the free oscillations. The lateral dynamics

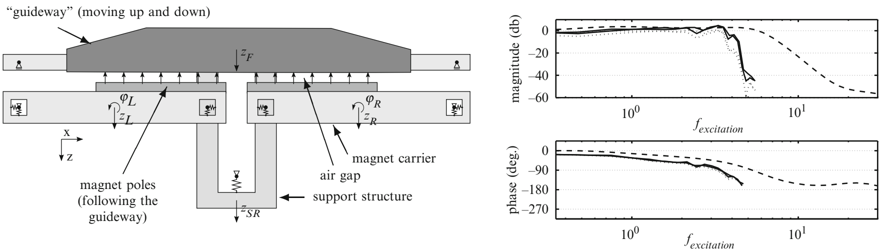

Fig. 9 *Left:* A schematic representation of the magnet test bench. The guideway can be moved up and down and the magnets and the attached carriers and support structure follow its motion. *Right:* This Bode diagram compares measurements of the test bench with simulation results of its model (*dashed line*). As input we chose the up- and downward movement of the guideway and as output the movement of the magnets (*left/right*; *solid* and *dotted lines*)

model has eight eigenfrequencies. Three are zero and belong to a translation or rotation of the entire system because no reset forces have yet been considered. The remaining five eigenmodes are sketched in Fig. 10 and categorized according to the body that is primarily oscillating. The eigenmodes of the vertical dynamics model can be divided into vertical eigenmodes and those resembling rope oscillations in the levitation magnets caused by their chain-like kinematics. When the damping effects of the primary and secondary suspensions are added, natural oscillations similar to the undamped case can be identified.

Next, the mechanical subsystems are connected with the magnetic and controller subsystems. Eigenvalue analyses of the linearized complete systems prove stability for both the lateral and the vertical dynamics systems. We further examine the behavior of the mechanical eigenmodes when adding the other subsystems. A simulation-based analysis shows that some eigenvalues of the mechanical subsystems can still be identified with the former mechanical eigenmodes, e.g. a selection of four eigenmodes of the vertical model can be found in Fig. 11. For a more detailed analysis, we refer to [1] and [5].

These eigenvalues of the complete lateral model can be analyzed with respect to their sensitivity to parameter variations. Among others, we vary the air gap between train and guideway which significantly influences the operating point. The individual eigenvalues react differently to air gap disturbances of the guidance and the levitation magnets. Once more, this verifies the high quality of the model as it reproduces actual behavior realistically.

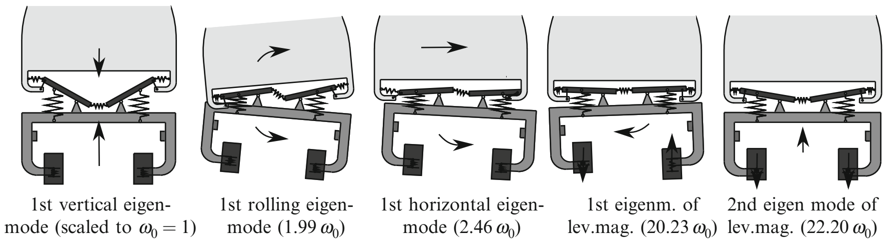

Fig. 10 Eigenmodes of the mechanical subsystem of the lateral cross section

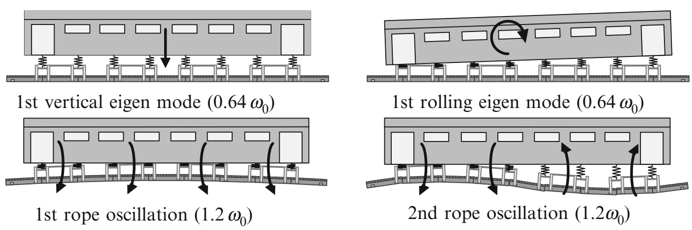

Fig. 11 Sketched eigenmodes of the entire vertical model

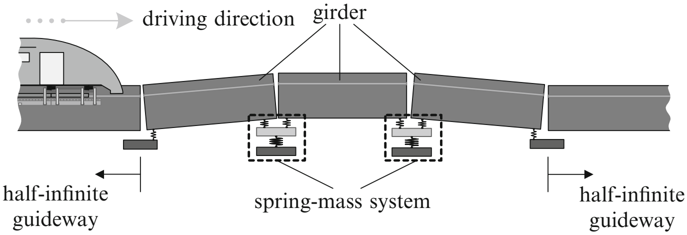

Fig. 12 Setting for simulation of vehicle passages

4 Novel Guideway

As a Transrapid passes a single girder, the interaction of the mechanical and the electromagnetic components induces ground vibrations. New guideway concepts—to be used in sensitive city areas—aim to reduce these vibrations as much as possible. The simulation model of the vertical dynamics is used to approximate these ground vibrations and to study the effects of a new guideway with flexible support.

The guideway consists of individual girders with varying length. They are also modeled as rigid bodies with two degrees of freedom. The displacement of a girder at its support is assumed to be proportional to the joint force. Hence, the displacement relative to the static deviation can be used to determine the ground vibrations and it can be computed by simulations of vehicle passages with constant velocity. The model of the vertical dynamics is suitable for this task as it fits quite well with real measurements, see Fig. 13, *left*.

In the novel guideway concept, the girders have a more flexible support due to an additional body that is located between the girder and its foundation and a soft spring between them. Above this additional mass there are the same elastomer springs as in the non flexible model. This support is modeled as an additional spring-mass system as shown in Fig. 12. It is a simple mechanical system with one degree of freedom for the vertical movement. We used the setting that is shown in Fig. 12 to investigate the flexible support. There are three dynamic girders and two additional spring-mass systems. We also used two half-infinite guideways to obtain well-defined initial and final conditions. The initial conditions of the guideway model are chosen such that the girders are leveled out for the static load of the vehicle. This leads to a deviation in the unloaded, initial state, as can be seen from Fig. 12. We computed several simulations with varying stiffness values of the flexible support. Even for very small stiffness values, i.e. for a great flexibility, train passages are still possible, although this leads to high displacements of the girders. The ground vibrations are significantly reduced in particular for high frequencies as it can be seen in the frequency spectra in Fig. 13, *right*. These spectra again result from simulations of passages with constant velocity. The spectrum with the original support is shown as reference (white line) and three simulations were run with different stiffness values

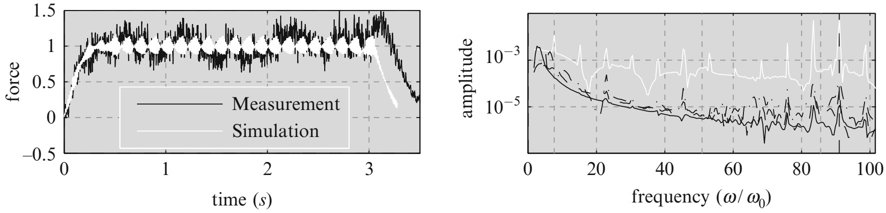

Fig. 13 *Left:* comparison of simulated (white) and measured (*black*) girder displacement. *Right:* frequency spectra of ground vibrations: without flexible support (*white*) and three different stiffness values of the additional spring (*black*)

of the flexible support. We conclude that it is possible and very effective to use this kind of flexible support for the maglev train Transrapid in order to reduce ground vibrations.

Using the lateral dynamics model (cf. Sect. 2.2) and coupling it to a guideway cross section model with varying stiffnesses of the guideway bearing, we also verified the stability of the Transrapid's levitation at zero velocity.

5 Conclusion

Two multidisciplinary models for the Transrapid's dynamics have been introduced, including detailed submodels of the magnets and the controller units. The lateral and vertical dynamics model have been verified using linear analysis tools, simulations and measurements. As an application of our models, we studied two typical scenarios for magnetic levitation trains: the levitation at zero velocity and the interaction of train and guideway during passages at constant speed. The models can contribute to the further development of the system, e.g. by predicting the system's behavior on novel guideways.

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