After the publication of the first edition of this book, stochastic stability of differential equations has become a very popular theme of recent research in mathematics and its applications. It is enough to mention the Lecture Notes in Mathematics, Nos 294, 1186 and 1486, devoted to the stability of stochastic dynamical systems and Lyapunov Exponents, the books of L. Arnold [3], A. Borovkov [35], S. Meyn and R. Tweedie [196], among many others.

Nevertheless I think that this book is still useful for those researchers who would like to learn this subject, to start their research in this area or to study properties of concrete mechanical systems subjected to random perturbations. In particular, the method of Lyapunov functions for the analysis of qualitative behavior of stochastic differential equations (SDEs), the exact formulas for the Lyapunov exponent for linear SDEs, which are presented in this book, provide some very powerful instruments in the study of stability properties for concrete stochastic dynamical systems, conditions of existence the stationary solutions of SDEs and related problems.

The study of exponential stability of the moments (see Sects. 5.7, 6.3, 6.4 here) makes natural the consideration of certain properties of the moment Lyapunov exponents. This very important concept was first proposed by S. Molchanov [204], and was later studied in detail by L. Arnold, E. Oeljeklaus, E. Pardoux [8], P. Baxendale [19] and many other researchers (see, e.g., [136]).

Another important characteristic for stability (or instability) of the stochastic systems is the stability index, studied by Arnold, Baxendale and the author. For the reader’s convenience I decided to include the main results on the moment Lyapunov exponents and the stability index in the Appendix B to this edition. The Appendix B was mainly written by G. Milstein, who is an accomplished researcher in this area. I thank him whole-heartily for his generous help and support.

I have many thanks to the Institute for the Problems Information Transmission, Russian Academy of Sciences, and to the Wayne State University, Detroit, for their support during my work on this edition. I also have many thanks to B.A. Amosov for his essential help in the preparation of this edition.

In conclusion I will enumerate some other changes in this edition.
1. Derivation of the often used in the book Feynman–Kac formula is added to Sect. 3.6.
2. A much improved version of Theorem 4.6 is proven in Chap. 4.
3. The Arcsine Law and its generalization are added in 4.12.
5. New books and papers, related to the content of this book, added to the bibliography.
6. Some footnotes are added and misprints are corrected.

Moscow
March 2011

Rafail Khasminskii
Stochastic Stability of Differential Equations
Khasminskii, R.
2012, XVII, 342 p., Hardcover
ISBN: 978-3-642-23279-4