# Contents

1 **Introduction** ................................................................. 1  
1.1 Scattering of Seismic Waves ........................................... 1  
1.2 Lithospheric Heterogeneity ............................................. 6  
1.3 Chapter Structure ..................................................... 9  
1.4 Mathematical Symbols ............................................... 10  
1.5 Further Reading ...................................................... 11  

2 **Heterogeneity in the Lithosphere** ...................................... 13  
2.1 Geological Evidence .................................................. 13  
2.2 Birch’s Law .............................................................. 16  
2.3 Random Inhomogeneity ............................................... 18  
  2.3.1 Velocity Inhomogeneity Revealed from Well-Logs ........ 18  
  2.3.2 Mathematical Description of Random Media ............... 19  
  2.3.3 ACF of Velocity Inhomogeneity Revealed 
              from Well-Logs and Rock Samples ...................... 27  
2.4 Deterministic Imaging Using Seismological Methods ............ 28  
  2.4.1 Refraction Surveys ............................................. 29  
  2.4.2 Reflection Surveys ............................................ 30  
  2.4.3 Receiver Function Method .................................... 34  
  2.4.4 Velocity Tomography .......................................... 34  
2.5 Scattering of High-Frequency Seismic Waves .................... 40  
  2.5.1 Seismogram Envelopes ........................................ 40  
  2.5.2 S-Coda Waves .................................................. 42  
  2.5.3 Three-Component Seismogram Envelopes ................. 49  
  2.5.4 Broadening of Seismogram Envelopes 
              and Excitation of the Orthogonal-Component 
              of Motion .................................................. 50  
  2.5.5 Scattering of Ultrasonic Waves in Rock Samples ....... 57  
  2.5.6 Cross-Correlation Function of Ambient Noise ............ 59
3 Phenomenological Study of Coda Waves

3.1 Coda Excitation Models

3.1.1 Scattering Characteristics

3.1.2 Single Scattering Models

3.1.3 Diffusion Model

3.1.4 Energy-Flux Model

3.1.5 Simulations of Wave Scattering

3.2 Coda Analysis

3.2.1 Measurements of Total Scattering Coefficient

3.2.2 Measurements of Coda Attenuation

3.2.3 Duration Magnitude

3.2.4 Lg Coda

3.2.5 Coda Amplitude Decay for a Long Lapse-Time Range

3.2.6 Rayleigh-Wave Coda at Long Periods

3.3 Coda Normalization Method

3.3.1 Site Amplification Measurements

3.3.2 Source Radiation Measurements

3.3.3 Attenuation Measurements

3.4 Spatial Variation of Medium Heterogeneities

3.4.1 Spatial Variation of Scattering Characteristics

3.4.2 Spatial Variation of Intrinsic Absorption

3.4.3 Reflection from a Subducting Oceanic Slab

3.5 Temporal Change in Medium Characteristics

3.5.1 Temporal Change in Coda Attenuation and Scattering

3.5.2 Temporal Change in Velocity

3.5.3 Temporal Change in Site Factors

3.6 Related Seismogram Envelope Studies

3.6.1 Precursor and Coda Associated with Core Phase

3.6.2 Back Scattering of T-Waves by Seamounts

3.6.3 Envelope Correlation Method for Locating Low-Frequency Events

3.7 Further Reading

4 Born Approximation for Wave Scattering in Inhomogeneous Media

4.1 Scalar Waves

4.1.1 Born Approximation for a Localized Velocity Inhomogeneity

4.1.2 Scattering by Random Velocity Inhomogeneities

4.2 Elastic Vector Waves

4.2.1 Born Approximation for a Localized Elastic Inhomogeneity

4.2.2 Reduction of Independent Parameters by Using Birch’s Law
### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.3</td>
<td>Scattering by Random Elastic Inhomogeneities</td>
<td>145</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Conversion Scattering Between Body and Rayleigh Waves</td>
<td>148</td>
</tr>
<tr>
<td>5</td>
<td>Attenuation of High-Frequency Seismic Waves</td>
<td>153</td>
</tr>
<tr>
<td>5.1</td>
<td>Measurements of Attenuation in the Lithosphere</td>
<td>153</td>
</tr>
<tr>
<td>5.2</td>
<td>Intrinsic Attenuation Mechanisms</td>
<td>154</td>
</tr>
<tr>
<td>5.3</td>
<td>Scattering Attenuation in Random Inhomogeneities</td>
<td>160</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Travel-Time Corrected Born Approximation for Scalar Waves</td>
<td>162</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Travel-Time Corrected Born Approximation for Vector Waves</td>
<td>169</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Evaluation of Cutoff Scattering Angle</td>
<td>178</td>
</tr>
<tr>
<td>5.3.4</td>
<td>Diffraction Effects</td>
<td>179</td>
</tr>
<tr>
<td>5.4</td>
<td>Scattering Attenuation Due to Distributed Cracks and Cavities</td>
<td>180</td>
</tr>
<tr>
<td>5.5</td>
<td>Further Reading</td>
<td>184</td>
</tr>
<tr>
<td>6</td>
<td>Synthesis of Three-Component Seismogram Envelopes of a Small Earthquake</td>
<td>185</td>
</tr>
<tr>
<td>6.1</td>
<td>Earthquake Source</td>
<td>186</td>
</tr>
<tr>
<td>6.1.1</td>
<td>Point Shear-Dislocation Source</td>
<td>186</td>
</tr>
<tr>
<td>6.1.2</td>
<td>Omega-Square Model for the Source Spectrum</td>
<td>188</td>
</tr>
<tr>
<td>6.2</td>
<td>Envelope Synthesis in an Infinite Space</td>
<td>189</td>
</tr>
<tr>
<td>6.2.1</td>
<td>Geometry of Source and Receiver</td>
<td>189</td>
</tr>
<tr>
<td>6.2.2</td>
<td>Power Spectral Density of Velocity Wavefield</td>
<td>191</td>
</tr>
<tr>
<td>6.2.3</td>
<td>Numerical Simulations</td>
<td>198</td>
</tr>
<tr>
<td>6.3</td>
<td>Envelope Synthesis on the Free Surface of a Random Elastic Medium</td>
<td>202</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Body-Wave Reflection at the Free Surface</td>
<td>203</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Conversion Between Body Waves and Rayleigh Waves</td>
<td>208</td>
</tr>
<tr>
<td>6.4</td>
<td>Further Reading</td>
<td>210</td>
</tr>
<tr>
<td>7</td>
<td>Wave Propagation in Random Media and the Radiative Transfer Theory</td>
<td>211</td>
</tr>
<tr>
<td>7.1</td>
<td>Scalar Waves in Random Media</td>
<td>212</td>
</tr>
<tr>
<td>7.1.1</td>
<td>Scalar Wave Equation</td>
<td>212</td>
</tr>
<tr>
<td>7.1.2</td>
<td>Green’s Function for a Homogeneous Medium</td>
<td>213</td>
</tr>
<tr>
<td>7.1.3</td>
<td>Random Media</td>
<td>213</td>
</tr>
<tr>
<td>7.2</td>
<td>First Order Smoothing Method</td>
<td>214</td>
</tr>
<tr>
<td>7.2.1</td>
<td>Equation for the Mean Wave</td>
<td>214</td>
</tr>
<tr>
<td>7.2.2</td>
<td>Mass Operator</td>
<td>215</td>
</tr>
<tr>
<td>7.2.3</td>
<td>Mean Green’s Function</td>
<td>216</td>
</tr>
<tr>
<td>7.3</td>
<td>Radiative Transfer Equation</td>
<td>221</td>
</tr>
<tr>
<td>7.3.1</td>
<td>Multi-Scale Analysis</td>
<td>222</td>
</tr>
<tr>
<td>7.3.2</td>
<td>Radiative Transfer Equation and Scattering Coefficient</td>
<td>230</td>
</tr>
</tbody>
</table>
## Contents

7.4 Radiative Transfer Equation in Integral Form .......................... 232
  7.4.1 Scalar Wave Case ........................................... 232
  7.4.2 Elastic Wave Case .......................................... 235
7.5 Diffusion Approximation ........................................... 237
  7.5.1 Scalar Wave Case ........................................... 238
  7.5.2 Elastic Wave Case .......................................... 240
7.6 Further Reading ...................................................... 243

8 Envelope Synthesis Based on the Radiative Transfer Theory .... 245
  8.1 Isotropic Scattering and Isotropic Source Radiation ............ 246
    8.1.1 One-Dimensional Case ..................................... 246
    8.1.2 Two-Dimensional Case ..................................... 249
    8.1.3 Three-Dimensional Case ................................... 251
    8.1.4 Multiple Lapse Time Window Analysis (MLTWA) .......... 258
  8.2 Isotropic Scattering with Conversion Between
      P- and S-Wave Modes ........................................ 267
    8.2.1 Seismogram Envelopes of Microearthquakes ............... 267
    8.2.2 Radiative Transfer Equation ............................... 268
    8.2.3 Seismogram Envelopes from an Explosion
          Source in Volcano .......................................... 275
  8.3 Isotropic Scattering and Nonspherical Source Radiation ........ 282
    8.3.1 Radiative Transfer Equation ................................ 282
    8.3.2 Envelopes for a Point Shear-Dislocation Source ........ 288
    8.3.3 Inversion for Energy Radiation from a Large
          Earthquake Fault ............................................ 290
  8.4 Nonisotropic Scattering and Isotropic Source Radiation ....... 294
    8.4.1 Radiative Transfer Equation ................................ 294
    8.4.2 Simulation of Energy Density in Space and Time .......... 299
  8.5 Surface-Wave Envelopes on the Spherical Earth ................. 303
    8.5.1 Single Isotropic Scattering Model .......................... 303
    8.5.2 Multiple Isotropic Scattering Model ........................ 308
    8.5.3 Decay of Late Coda Envelope ................................. 309
  8.6 Monte Carlo Simulation .......................................... 309
    8.6.1 Direct Simulation Monte Carlo Method ....................... 309
    8.6.2 Study of Medium Heterogeneities by Using
          the Monte Carlo Simulation .................................... 312
  8.7 Further Reading ...................................................... 317

9 Parabolic Equation and Envelope Synthesis Based
   on the Markov Approximation ...................................... 319
  9.1 Amplitude and Phase Distortions of Scalar Waves ............... 320
    9.1.1 Parabolic Equation in Random Media ....................... 320
    9.1.2 Transverse Correlations of Amplitude
          and Phase Fluctuations ....................................... 323
    9.1.3 Measurements of Amplitude and Phase Fluctuations .... 328
    9.1.4 Velocity Shift ............................................. 331
9.2 Envelope Synthesis of Scalar Waves Based on the Markov Approximation ...................................... 335
  9.2.1 Envelope of a Plane Wavelet ............................... 336
  9.2.2 Envelope of a Spherical Wavelet ......................... 347
  9.2.3 Monte Carlo Simulation of Envelope ...................... 356
  9.2.4 Comparison with FD Simulations in 2-D Random Media .................................................. 362
  9.2.5 Using Markov Envelope in Radiative Transfer Theory ............................................. 368
  9.2.6 Observation of Envelope Broadening .......................... 369

9.3 Envelope Synthesis of Vector Waves Based on the Markov Approximation ...................................... 378
  9.3.1 Vector-Component Envelopes of a Plane P-Wavelet ...... 380
  9.3.2 Vector-Component Envelopes of a Plane S-Wavelet ...... 386
  9.3.3 Comparison with FD Simulations in 2-D Random Elastic Media ..................................... 390
  9.3.4 Envelopes on the Free Surface ............................. 392
  9.3.5 Observation of Vector-Wave Envelopes ................... 395
  9.3.6 Envelopes of a Spherical P-Wavelet ....................... 396
  9.3.7 Envelopes of a Spherical P-Wavelet in Nonisotropic Random Elastic Media ................... 397

9.4 Further Reading ...................................................... 399

10 Green’s Function Retrieval from the Cross-Correlation Function of Random Waves ................................. 401
  10.1 Green’s Function Retrieval for a Homogeneous Medium ........ 402
      10.1.1 Scalar Waves ........................................ 402
      10.1.2 Elastic Waves ........................................ 418
      10.1.3 White-Spectrum Random Waves in a Bounded Medium ................................. 425
  10.2 Green’s Function Retrieval for a Scattering Medium ........ 429
      10.2.1 Green’s Function for a Scattering Medium .................. 429
      10.2.2 Illumination by Noise Sources Distributed on a Large Spherical Shell .......................... 433
      10.2.3 Illumination by Uniformly Distributed Noise Sources .............................................. 437
      10.2.4 Green’s Function Retrieval from the CCF of Singly-Scattered Coda Waves ................... 440
  10.3 Analyses of CCF of Ambient Noise .............................. 443
      10.3.1 Velocity Analysis and Application to Tomography .................. 444
      10.3.2 Monitoring the Temporal Change in the Crustal Medium Property ............................ 446
  10.4 Further Reading ...................................................... 450
11 Epilogue ................................................................................ 451
  11.1 Developments of Measurement Capability ....................... 451
  11.2 Developments in Theory and New Methods ................. 452
  11.3 Developments in Observations ................................ 453
  11.4 Necessary Developments in Future ....................... 454

A Spherical Harmonic Functions and Wigner 3-j Symbols .......... 457

References ................................................................................ 461

Index ........................................................................................ 489
Seismic Wave Propagation and Scattering in the Heterogeneous Earth : Second Edition
Sato, H.; Fehler, M.C.; Maeda, T.
2012, XVI, 496 p., Hardcover
ISBN: 978-3-642-23028-8