Chapter 1
Introduction

The construction of classical physics started at the beginning of the seventeenth century. By the end of the nineteenth century, the building appeared to have been completed: the mechanics of Galileo Galilei and of Isaac Newton, the electromagnetism of Michael Faraday and James Maxwell, and the thermodynamics of Ludwig Boltzmann and Hermann Helmholtz were by then well established, from both the theoretical and the experimental points of view. The edifice was truly completed when Albert Einstein developed the special and the general theories of relativity, in the early twentieth century.

Classical physics deals with the trajectory of particles (falling bodies, motion of planets around the sun) or with the propagation of waves (light waves, sound waves). The construction of this edifice required intuition to be abandoned in favor of a formalism, i.e., a precise treatment that predicts the evolution of the world through mathematical equations. Classical physics has a deterministic character. The existence of a physical reality, independent of the observer, is an implicitly accepted dogma.

Cracks in this conception appeared around the beginning of the last century. Light waves not only appeared to be absorbed and emitted in lumps (black-body radiation) [2], but turned out to behave completely like particles (photoelectric and Compton effects [3, 4]). Experiments with electrons displayed diffraction patterns that had up to then been seen as characteristic of waves [5]. However, the most disturbing discovery was that an atom consists of a positively charged, small, heavy nucleus, surrounded by negatively charged light electrons [6]. According to classical physics, matter should collapse in a fraction of a second. Nor was it understood why atoms emitted light of certain wavelengths only, similar to an organ pipe that produces sounds at certain well-defined frequencies [7].

In 1913 Niels Bohr was able to explain both the stability of the hydrogen atom and the existence of discrete energy levels by means of a partial rejection of classical mechanics and electromagnetism [8]. However, this model was largely a patch. Bohr himself assumed the role of leader in the quest for an adequate formalism. In 1925 Werner Heisenberg alone [9] and, subsequently, in collaboration with Max Born and Pascual Jordan [10] developed a quantum mechanical formalism

in which the classical variables of position and momentum were represented by (non-commuting) matrices. Also in 1925, Dirac introduced the idea that physical quantities are represented by operators (of which Heisenberg's matrices are just one representation) and physical states by vectors in abstract Hilbert spaces [11]. In 1926 Erwin Schrödinger produced the differential formalism of quantum mechanics, an alternative approach based on the differential equations that bear his name [12].

Since the presentation of the Heisenberg and Schrödinger formulations[1] is at least twice as difficult as the introduction of a single one, a relatively large number of undergraduate quantum mechanics textbooks confine themselves to a discussion of the Schrödinger realization. The present author contends that such a presentation of quantum mechanics is conceptually misleading, since it leads to the impression that quantum mechanics is another branch of classical wave physics. It is not. Let me quote Schwinger's opinion [14].

> I have never thought that this simple wave approach [de Broglie waves and the Schrödinger equation] was acceptable as a general basis for the whole subject.

This approach of presenting both Heisenberg and Schrödinger formulations also pays dividends through the natural appearance of the most quantum of all operators: the spin. In addition to its intrinsic conceptual value, spin allows us to simplify discussions on fundamental quantum phenomena like interference and entanglement, on time dependence (as in nuclear magnetic resonance), and on applications of quantum mechanics in the field of quantum information.

The uncertainty of a presentation may be reduced by increasing the amount of detail, and vice versa. Bohr used to say that accuracy and clarity were complementary concepts (Sect. 15.5.1). Thus, a short and clear statement can never be precise. We may go further and state that the product of the indeterminacy inherent in any message ($\Delta\pi$) times the amount of details ($\Delta\sigma$) is always larger or equal than a constant k ($\Delta\pi \times \Delta\sigma > k$). The quality of textbooks should be measured by how close this product is to k, rather than by their (isolated) clarity or completeness. It is up to the reader to judge how closely we have been able to approach the value k. If we have achieved our aim, a more rigorous and sufficiently simple presentation of quantum mechanics will be available to undergraduate and first-year graduate students.

This book should be accessible to students who are reasonably proficient in linear algebra, calculus, classical mechanics and electromagnetism. Previous exposure to other mathematical and/or physics courses constitutes an advantage, but it is by no means a *sine qua non*.

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[1]There also exist other formulations of quantum mechanics. All of them yield the same result for the same problem, but one of them may be easier to apply or may provide a better insight in a given situation. The list of quantum formalisms includes the path-integral (Feynman), phase space (Wigner), density matrix, second quantization, variational, pilot wave (Bohm) and the Hamilton–Jacobi formulations [13]. In the present text the second quantization formalism is presented in Sect. 7.8†, the density matrix in Sect. 14.4 and the path integral in Sect. 11.3†.

The reader will be confronted in Chap. 2 with a condensed presentation of Hilbert spaces and Hermitian and unitary operators. This early presentation implies the risk that the reader might receive the (erroneous) impression that the book is mathematically oriented, and/or that he or she will be taught mathematics instead of physics. However, Sects. 2.7* and 2.8* include practically all the mathematical tools that are used in the text (outside of elementary linear algebra and calculus, both being prerequisites). Consistent with this "physics" approach, the results are generally starkly presented, with few detailed derivations. It is the author's contention that these derivations do not significantly contribute to filling the gap between merely recognizing quantum mechanical expressions and learning how to "do" and "feel" quantum mechanics. This last process is greatly facilitated by solving the problems at the end of each chapter (with answers provided at the end of the book). The instructor should act as an "answerer" and "motivator" of students' questions and not merely as a "problem solver on the blackboard," to facilitate the filling of the aforementioned gap.

Sections labeled by a dagger display a somewhat more advanced degree of difficulty. The student may leave them for a second reading, unless he or she is specially interested on the subject. An asterisk indicates the mathematical background of material that has been previously presented.

Although the text has been conceived as a whole unit, it also allows for different shorter readings:

- If the aim is to operate with the formalism within some particular branch of microphysics (solid state, molecular, atomic, nuclear, etc.), one can progress straightforwardly from Chap. 2 to Chap. 9, probably leaving Chaps. 10 and 11 for a second reading.
- Readers more interested in recent advances on quantum theory and on applications to quantum information may skip Chaps. 7, 8, 10, and 11, to get to Chaps. 12–14 (concerning entanglement and its consequences).
- A reading with more emphasis on conceptual aspects of quantum mechanics should proceed through Chaps. 2–7, 9, 11, 12 and 14.

A brief history of quantum mechanics is presented to acquaint the newcomer with the development of one of the most spectacular adventures of the human mind to date (Chap. 15). It also intends to convey the feeling that, far from being finished, this enterprise is continuously being updated.