The idea for writing this book came up when the authors met at the University of Valencia in 2005. While comparing our experiences with regard to various aspects of the linear ordering problem (LOP), we realized that most of the optimization technologies had been successfully applied to solve this problem. We also found that there were only a small number of books covering all state-of-the-art optimization methods for hard optimization problems (especially considering both exact methods and heuristics together). We thought that the LOP would make an ideal example to survey these methods applied to one problem and felt the time was ripe to embark on the project of writing this monograph.

Faced with the challenge of solving hard optimization problems that abound in the real world, classical methods often encounter serious difficulties. Important applications in business, engineering or economics cannot be tackled by the solution methods that have been the predominant focus of academic research throughout the past three decades. Exact and heuristic approaches are dramatically changing our ability to solve problems of practical significance and are extending the frontier of problems that can be handled effectively. In this text we describe state-of-the-art optimization methods, both exact and heuristic, for the LOP. We actually employ the LOP to illustrate current optimization technologies and the design of successful implementations of exact and heuristic procedures. Therefore, we do not limit the scope of this book to the LOP but, on the contrary, we provide the reader with the background and strategies in optimization to tackle different combinatorial problems.

This monograph is devoted to the LOP, its origins, applications, instances and especially to methods for its effective approximate or exact solution. Our intention is to provide basic principles and fundamental ideas and reflect the state-of-the-art of heuristic and exact methods, thus allowing the reader to create his or her personal successful applications of the solution methods. The book is meant to be of interest for researchers and practitioners in computer science, mathematics, operations research, management science, industrial engineering, and economics. It can be used as a textbook on issues of practical optimization in a master’s course or as a reference resource for engineering optimization algorithms.
To make the book accessible to a wider audience, it is to a large extent self-contained, providing the reader with the basic definitions and concepts in optimization. However, in order to limit the size of this monograph we have not included extensive introductions. Readers interested in further details are referred to appropriate textbooks such as [4, 84, 102, 117, 118, 124].

The structure of this book is as follows. Chapter 1 provides an introduction to the problem and its applications and describes the set of benchmark instances which we are using for our computational experiments and which have been made publically available. Chapter 2 describes such basic heuristic methods such as construction and local searches. Chapter 3 expands on Chapter 2 and covers meta-heuristics in which the simple methods are now embedded in complex solution algorithms based on different paradigms, such as evolution or learning strategies. Chapter 4 discusses branch-and-bound, the principal approach for solving difficult problems to optimality. A special version based on polyhedral combinatorics, branch-and-cut, is presented in Chapter 5. Chapter 6 deals in more detail with the linear ordering polytope which is at the core of branch-and-cut algorithms. The book concludes with Chapter 7, where a number of further aspects of the LOP and potential issues for further research are described.

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