Preface

There is a universe of mathematics lying in between
the complete differentiations and integrations.

— O. Heaviside

This book is devoted to some questions in Fractional Calculus, that is, the theory of
differential and integral operators of non-integer order, and in particular to differ-
ential equations containing such operators. Even though the first steps of the theory
itself date back to the first half of the nineteenth century, the subject only really came
to life over the last few decades. A particular feature is that engineers and scien-
tists have developed new models that involve fractional differential equations. These
models have been applied successfully, e.g., in mechanics (theory of viscoelasticity
and viscoplasticity), (bio-)chemistry (modelling of polymers and proteins), electri-
cal engineering (transmission of ultrasound waves), medicine (modelling of human
tissue under mechanical loads), etc. The mathematical theory seems to be lagging
behind the needs of those applications but the wealth of applications indeed indi-
cates the truth of the above quote from Heaviside [93, §437]. There are some books
dealing with the aspects that can be summarized as the “pure mathematical” side
of the problems without taking into consideration those questions that arise in the
applications mentioned above, and some that the engineer’s point of view without
a rigorous mathematical justification of the ideas. This book attempts to fill the gap
between these two approaches: We try to establish a mathematically sound theory
of the differential equations that have been shown to be relevant in practice and pro-
vide a thorough mathematical analysis. In order to be self-contained, we repeat the
fundamentals of fractional calculus before coming to the main topic. A particular
goal of this book is to provide a solid foundation that may later be used for the
construction of efficient and reliable numerical methods for fractional differential
equations. The author strongly believes that a successful development and a thor-
ough understanding of such numerical schemes is not possible without such a stable
analytical background.

The reader is assumed to be familiar with classical calculus (differential and
integral calculus and the elementary theory of differential equations). A working
knowledge of Lebesgue integration theory is helpful now and then, but not abso-
lutely essential.
It is my pleasure to thank a number of people for their constant support, interest, encouragement, and many useful discussions, namely Heinz-Wilhelm Alten and Klaus-Jürgen Förster (Universität Hildesheim), Helmut Braß and Marc Weilbeer (Technische Universität Braunschweig), Thomas Hennecke (Universität Kassel) Neville J. Ford (University of Chester), Alan D. Freed (formerly at NASA’s John H. Glenn Research Center in Cleveland, now at Saginaw Valley State University), Paul L. Butzer (RWTH Aachen), Rudolf Gorenflo (Freie Universität Berlin), Francesco Mainardi (Università di Bologna), and André Schmidt (Universität Stuttgart). Moreover I would like to thank Mrs Ute McCrory at Springer-Verlag for her support during the manuscript preparation process.

Parts of the book have been used as a text for a graduate course on fractional differential equations that I taught to students of mathematics, physics and engineering at Technische Universität Braunschweig.

Braunschweig, June 2010

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The Analysis of Fractional Differential Equations
An Application-Oriented Exposition Using Differential Operators of Caputo Type
Diethelm, K.
2010, VIII, 247 p. 10 illus., Softcover
ISBN: 978-3-642-14573-5