

Contents

Part I A Cultural Heritage

1	Early Beginnings	3
1.1	Prehistory.....	3
1.2	Geometry in the New Stone Age.....	4
1.3	Early Mathematics and Ethnomathematics.....	6
2	The Great River Civilizations	7
2.1	Civilizations Long Dead: And Yet Alive.....	7
2.2	Birth of Geometry as We Know It.....	11
2.3	Geometry in the Land of the Pharaoh.....	12
2.4	Babylonian Geometry.....	15
2.5	The (u,v) Explanation of Plimpton 322.....	22
2.6	Regular Reciprocal Pairs, Babylonian Number-Work and Plimpton 322.....	23
2.7	Parametrization of Pythagorean Triples.....	25
3	Greek and Hellenic Geometry	31
3.1	Early Greek Geometry: Thales of Miletus.....	31
3.2	The Story of Pythagoras and the Pythagoreans.....	34
3.3	The Geometry of the Pythagoreans.....	44
3.4	The Discovery of Irrational Numbers.....	46
3.5	Origin of the Classical Problems.....	50
3.6	Constructions by Compass and Straightedge.....	54
3.7	Squaring the Circle.....	56
3.8	Doubling the Cube.....	56
3.9	Trisecting Any Angle.....	60
3.10	Plato and the Platonic Solids.....	61
3.11	Archytas and Doubling the Cube.....	64
4	Geometry in the Hellenistic Era	75
4.1	Euclid and Euclid's Elements.....	75
4.2	The Books of Euclid's Elements.....	77
4.2.1	Euclid's Definitions.....	77
4.2.2	Euclid's Postulates.....	78

4.2.3	Alternative Versions of Euclid's Fifth Postulate	80
4.2.4	Euclid's Common Notions or Axioms	80
4.3	The Roman Empire	85
4.4	Archimedes	88
4.5	Eratosthenes and Doubling the Cube	106
4.6	Nicomedes and His Conchoid	109
4.7	Apollonius of Perga and the Conic Sections	116
4.8	The End of the Republic in Rome	120
4.9	The First Emperors	134
4.10	Heron of Alexandria	135
4.11	Nero and the Year of the Four Emperors	138
4.12	From Vespasian to Marcus Aurelius	139
4.13	Menelaus of Alexandria	143
4.14	Claudius Ptolemy	144
4.15	The Rule of Sines and the Law of Cosines	148
4.16	From Commodus to the End of the Crisis of the Third Century	149
4.17	Diophantus of Alexandria	151
4.18	Pappus of Alexandria	153
4.19	The Late Roman Empire	155
4.20	The Murder of Hypatia	158
4.21	Fall of the Roman Empire	165
4.22	Byzantium	165
4.23	Preservation of a Heritage	167
5	Arabic Mathematics and Geometry	173
5.1	The Arab Expansion	173
5.2	Arab Science and Culture	177
5.3	The Founder of the House of Wisdom in Baghdad	180
5.4	Abu Ja'far Muhammad Ibn Musa Al Khwarizmi	181
5.5	Ibn Qurra and al-Battani	188
5.6	Muhammad Abu al Wafa al-Buzjani	189
5.7	Abu Sahl Wijan bin Rustam al Quhi	190
5.8	Yusuf al Mutaman ibn Hud and his Library	193
5.9	Omar al-Khayyam	194
5.10	Sharaf al-Din	201
5.11	Nasir al-Din al-Tusi	207
6	The Geometry of Yesterday and Today	211
6.1	The Dark Middle Ages	211
6.2	Geometry Reawakening: A New Dawn in Europe	216
6.3	Elementary Geometry and Higher Geometry	216
6.4	Desargues and the Two Pascals	219
6.5	Descartes and Analytic Geometry	221
6.6	Newton and Leibniz	222

6.7	Geometry in the Eighteenth Century	222
6.8	Some Features of Modern Geometry	228
6.9	Archimedean Polyhedra and Tessellations	233
7	Geometry and the Real World	241
7.1	Mathematics and Predicting Catastrophes	241
7.2	Catastrophe Theory	243
7.3	Geometric Shapes in Nature	245
7.4	Fractal Structures in Nature	247
 Part II Introduction to Geometry		
8	Axiomatic Geometry	253
8.1	The Postulates of Euclid and Hilbert’s Explanation	253
8.2	Non-Euclidian Geometry	255
8.3	Logic and Intuitive Set Theory	256
8.4	Axioms, Axiomatic Theories and Models	257
8.5	General Theory of Axiomatic Systems	262
9	Axiomatic Projective Geometry	265
9.1	Plane Projective Geometry	265
9.2	An Unsolved Geometric Problem	270
9.3	The Real Projective Plane	273
10	Models for Non-Euclidian Geometry	283
10.1	Three Types of Geometry	283
10.2	Hyperbolic Geometry	283
10.3	Elliptic Geometry	287
10.4	Euclidian and Non-Euclidian Geometry in Space	288
10.5	Riemannian Geometry	292
11	Making Things Precise	299
11.1	Relations and Their Uses	299
11.2	Identification of Points	300
11.3	Our Number System	302
11.4	Complex Numbers and Trigonometry	307
12	Projective Space	313
12.1	Coordinates in the Projective Plane	313
12.2	Projective n-Space	316
12.3	Affine and Projective Coordinate Systems	317

13	Geometry in the Affine and the Projective Plane	325
13.1	The Theorem of Desargues	325
13.2	Duality for $\mathbb{P}^2(\mathbb{R})$	327
13.3	Naive Definition and First Examples of Affine Plane Curves	328
13.4	Straight Lines	328
13.5	Conic Sections in the Affine Plane \mathbb{R}^2	329
13.6	Constructing Points on Conic Sections by Compass and Straightedge	336
13.7	Further Properties of Conic Sections	338
13.8	Conic Sections in the Projective Plane	344
13.9	The Theorems of Pappus and Pascal	347
14	Algebraic Curves of Higher Degrees in the Affine Plane \mathbb{R}^2	351
14.1	Curves of Degree 3 and 4 in \mathbb{R}^2	351
14.2	Affine Algebraic Curves	356
14.3	Singularities and Multiplicities	360
14.4	Tangency	362
15	Higher Geometry in the Projective Plane	367
15.1	Projective Curves	367
15.2	Projective Closure and Affine Restriction	368
15.3	Smooth and Singular Points on Affine and Projective Curves	371
15.4	The Tangent to a Projective Curve	374
15.5	Projective Equivalence	381
15.6	Asymptotes	386
15.7	General Conchoids	387
15.8	The Dual Curve	390
15.9	The Dual of Pappus' Theorem	393
15.10	Pascal's Mysterium Hexagrammicum	394
16	Sharpening the Sword of Algebra	397
16.1	On Rational Polynomials	397
16.2	The Minimal Polynomial	400
16.3	The Euclidian Algorithm	402
16.4	Number Fields and Field Extensions	404
16.5	More on Field Extensions	408
17	Constructions with Straightedge and Compass	413
17.1	Review of Legal Constructions	413
17.2	Constructible Points	414
17.3	What is Possible?	415
17.4	Trisecting Any Angle	420
17.5	Doubling the Cube and Constructing the Regular Heptagon	422
17.6	Squaring the Circle	423

17.7 Regular Polygons424

17.8 Constructions by Folding431

18 Fractal Geometry435

18.1 Fractals and their Dimensions435

18.2 The von Koch Snowflake Curve.....436

18.3 Fractal Shapes in Nature.....437

18.4 The Sierpinski Triangles.....437

18.5 A Cantor Set.....439

19 Catastrophe Theory441

19.1 The Cusp Catastrophe: Geometry of a Cubic Surface441

19.2 Rudiments of Control Theory443

20 General Polyhedra and Tessellations, and Their Groups of Symmetry 445

20.1 Isometries of \mathbb{R}^n 445

20.2 Topological Spaces and Topological Groups447

20.3 Discrete Transformation Groups of Metric Spaces.....449

20.4 Isometries of \mathbb{R}^2 450

20.5 Symmetry of Plane Ornaments451

20.5.1 Rosette Groups451

20.5.2 Frieze Groups.....454

20.5.3 Wallpaper Groups458

20.5.4 The 17 Types of Wallpaper Groups460

20.6 Symmetries in Space.....468

20.6.1 Systems of Rotational Symmetries in Space.....469

20.6.2 Reflection Symmetry472

20.6.3 Prismatic Symmetry Types473

20.6.4 Compound Symmetry and the S_{2n} Symmetry Type.....475

20.6.5 Cubic Symmetry Types475

20.6.6 The Possible Symmetry Types.....478

21 Hints and Solutions to Some of the Exercises483

References.....503

Index507



<http://www.springer.com/978-3-642-14440-0>

Geometry

Our Cultural Heritage

Holme, A.

2010, XVII, 519 p., Hardcover

ISBN: 978-3-642-14440-0