Chapter 2
Weak Interaction Before the Standard Model

All existing present data are in perfect agreement with the unified theory of the electromagnetic and weak interactions (Standard Model). Before this theory was created, there was a long phenomenological period of the development of the theory of the weak interaction. In this introductory chapter we will briefly consider this period.

2.1 Pauli Hypothesis of Neutrino

The only weak process which was known in the twenties and thirties was the β-decay of nuclei. In 1914 Chadwick discovered that the energy spectrum of electrons from β-decay is continuous. If β-decay is a process of the transition of a nucleus (A,Z) into a nucleus (A,Z+1) and the electron (as it was believed at that time), from conservation of energy and momentum follows that the electron must have a fixed kinetic energy approximately equal to $Q \simeq m_{A,Z} - m_{A,Z+1} - m_e$ (where $m_{A,Z}$ ($m_{A,Z+1}$) is the mass of the initial (final) nucleus and m_e is the mass of the electron).

For many years continuous β spectra were interpreted as the result of the loss of energy of electrons in the target. However, in 1927 Ellis and Wooster performed a crucial calorimetric β-decay experiment. They measured the total energy released in a RaE (^{210}Bi) source which was put inside of a calorimeter. For the β-decay of ^{210}Bi the total energy release is $Q = 1.05$ MeV. In the Ellis and Wooster experiment it was found that the average energy per one β-decay is equal to (344 ± 34) KeV which is in an agreement with the average energy of the electrons (390 KeV). Thus, it was demonstrated that continuous β spectra cannot be explained by the energy loss of electrons in a target.

There were two possibilities to explain this experimental data

1. To assume that in β-decay together with the electron a neutral penetrating particle, which is not detected in experiments, is produced. The total released energy is shared between the electron and the new particle. As a result, electrons produced in β-decay, will have a continuous spectrum
2. To assume that in β-decay the energy is not conserved.

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The idea of new particle was proposed by W. Pauli. The second point of view was advocated by N. Bohr.

Pauli wrote about his idea in a letter to Geiger and Meitner who participated in a nuclear conference at Tübingen (December 4, 1930). Pauli asked Geiger and Meitner to inform the participants of the conference on his proposal.

Pauli called the new particle "neutron". He assumed that the "neutron" has spin 1/2, small mass (of the same order of magnitude as the mass of the electron) and large penetration length. Pauli assumed that the "neutron" is emitted together with the electron in the β-decay of nuclei. Later E. Fermi and E. Amaldi proposed to call the Pauli particle neutrino (from Italian, *neutral, small*).

Below there is Pauli's letter translated into English.

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant.

I agree that my remedy could seem incredible because one should have seen those neutrons very earlier if they really exist. But only the one who dare can win and the difficult situation, due to the continuous structure of the beta spectrum, is lighted by a remark of my honored predecessor, Mr Debye, who told me recently in Brussels: "Oh, It's well better not to think to this at all, like new taxes". From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge. Unfortunately, I cannot appear in Tübingen personally since I am indispensable here in Zürich because of a ball on the night of 6/7 December. With my best regards to you, and also to Mr Back.
Your humble servant W. Pauli

At the time when Pauli proposed the idea of the existence of the "neutron", nuclei were considered as bound states of protons and electrons. As it is seen from Pauli's letter he assumed that his new particle "exists in the nuclei". This assumption allowed him to solve the problem of the spin-statistic theorem for some nuclei. Let us consider the nucleus $^7N_{14}$. According to the proton-electron model this nucleus is a bound state of 14 protons and 7 electrons. Because spins of protons and electrons are equal to 1/2 the spin of $^7N_{14}$ must be half-integer. However, from the analysis of the spectrum of $^7N_{14}$ molecules it was found that nucleus $^7N_{14}$ satisfies Bose–Einstein statistics and, according to the spin-statistic theorem, the spin of the this nucleus must be integer. An odd number of "neutrons" in $^7N_{14}$ would make its spin integer.

After the discovery of the neutron (Chadwick, 1932) E. Majorana, W. Heisenberg and D. Ivanenko assumed that the constituents of nuclei are protons and neutrons. This assumption (which, as we know today, is the correct one) explained all nuclear data.

The problem of the spin of $^7N_{14}$ and other nuclei disappeared. What about β-decay and continuous β-spectrum? This problem was solved by quantitatively E. Fermi in 1934 on the basis of Pauli's hypothesis of the neutrino in the framework of the proton-neutron model of nuclei.

2.2 Fermi Theory of β-Decay

Fermi proposed the first Hamiltonian of the β-decay. He assumed that electron-neutrino pair *is produced in the transition of a neutron into a proton*

$$n \to p + e + \nu. \tag{2.1}$$

Fermi built the Hamiltonian of the process (2.1) in analogy with the Hamiltonian of the electromagnetic interaction.

The Hamiltonian of the electromagnetic interaction has the form of a scalar product of the electromagnetic current and the electromagnetic field $A^{\alpha}(x)$. For the Hamiltonian of the electromagnetic interaction of protons we have

$$\mathcal{H}_I^{\mathrm{EM}}(x) = e j_{\alpha}^{\mathrm{EM}} A^{\alpha}(x), \tag{2.2}$$

where e is the electric charge of the proton and the electromagnetic current is given by the expression

$$j_{\alpha}^{\mathrm{EM}}(x) = \bar{p}(x)\gamma_{\alpha} p(x) \tag{2.3}$$

where $p(x)$ is the proton field, $\bar{p}(x) = p^{\dagger}(x)\gamma^0$ and γ_{α} are the Dirac matrices.

Fermi suggested that the Hamiltonian of the process (2.1) is the product of the neutron-proton current

$$j_{\alpha}^{\mathrm{CC}}(x) = \bar{p}(x)\gamma_{\alpha} n(x), \tag{2.4}$$

which provides the transition $n \to p$ and a vector which provides the emission of the electron-neutrino pair. Assuming that there are no derivatives of the fields in the Hamiltonian, Fermi came to the following expression for the Hamiltonian of the β-decay

$$\mathcal{H}_I^{\beta}(x) = G_F\, \bar{p}(x)\gamma_{\alpha} n(x)\; \bar{e}(x)\gamma^{\alpha}\nu(x) + \text{h.c.}, \tag{2.5}$$

where G_F is an interaction constant.

Let us stress an important difference between the Hamiltonians (2.2) and (2.5). The Hamiltonian (2.2) describes the interaction of two fermions and a boson while the Hamiltonian (2.5) describes the interaction of four fermions. As a consequence of that, the Fermi constant G_F and the electromagnetic charge e have *different dimensions*. In the system of the units $\hbar = c = 1$, which we are using, e is a dimensionless quantity whereas the Fermi constant G_F has the dimension $[M]^{-2}$. We will return to a discussion of this point later.

The Fermi Hamiltonian (2.5) allowed to describe only such β-decays, in which spins and parities of the initial and final nuclei are the same (Fermi selection rule)

$$\Delta I = 0 \quad \pi_i = \pi_f$$

However, it was also observed such β-decays of nuclei which satisfy the Gamov-Teller selection rule:

$$\Delta I = \pm 1,\ 0 \quad \pi_i = \pi_f,$$

The observation of such decays meant that in addition to the Fermi Hamiltonian the total Hamiltonian of the β-decay must include additional terms.

2.3 Fermi-Gamov-Teller Hamiltonian of β-Decay

The Fermi Hamiltonian is the product of vector×vector term. If we assume that in the Hamiltonian of the β-decay there are no derivatives of the fields, for *the most general four-fermion Hamiltonian* we obtain the sum of the products of scalar (S), vector (V), tensor (T), pseudovector (A) and pseudoscalar (P) terms:

$$\mathcal{H}_I^{\beta}(x) = \sum_{i=S,V,T,A,P} G_i\ \bar{p}(x)\ O_i n(x)\ \bar{e}(x)\ O^i \nu(x) + \text{h.c.} \tag{2.6}$$

Here

$$O \to 1,\ \gamma_\alpha,\ \sigma_{\alpha\beta},\ \gamma_\alpha\gamma_5,\ \gamma_5\,. \tag{2.7}$$

and G_i are coupling constants, which have the dimensions $[M]^{-2}$. This Hamiltonian was proposed by Gamov and Teller in 1936. The Hamiltonian (2.6) could describe all β-decay data. Transitions, which satisfy the Fermi selection rules, are due to V and S terms and transitions, which satisfy the Gamov-Teller selection rules, are due to A and T terms.

The Hamiltonian (2.6) contains five arbitrary coupling constants G_i. It was, however, general belief that the number of the fundamental constants in the Hamiltonian of the β-decay must be smaller. For many years the aim of experiments on the investigation of the β-decay was to find the dominant terms in the Hamiltonian (2.6). The situation was, however, uncertain until 1957. The data on the measurements of the β-spectra were in favor of the combination of S and T terms or V and A terms. However, the measurements of the electron-neutrino angular correlation, which could distinguish these two possibilities, gave contradictory results. In 1957–1958 understanding of the β-decay and other weak processes drastically changed. This was connected with the discovery of the nonconservation of parity in the weak interaction.

The Fermi and Gamov-Teller Hamiltonians are invariant under space inversion, i.e. these Hamiltonians conserve parity. There was a general belief at that time that parity is conserved in all interactions. However, from the study of the weak decays of kaons in the fifties, there were indications that this assumption was incorrect.

2.4 Violation of Parity in β-Decay

In 1956 Lee and Yang analyzed existing experimental data and came to the conclusion that there were no data contradicting the assumption that parity is not conserved in the weak interaction. They proposed different experiments that would check this possibility.

The first experiment, in which violation of parity was discovered was performed by Wu et al. in 1957. In this experiment the dependence of the probability of the β-decay of the polarized nuclei Co^{60} on the angle between the (pseudo)vector of the polarization and the vector of the momentum of the electron was measured. From the invariance under rotation (conservation of the angular momentum) for the probability of the emission of the electron with momentum $\mathbf{p}$ by a nucleus with the polarization $\mathbf{P}$ we have the following general expression

$$w_{\mathbf{P}}(\mathbf{k}) = w_0(1 + \alpha\, \mathbf{P} \cdot \mathbf{k}) = w_0(1 + \alpha P \cos\theta), \tag{2.8}$$

where $\mathbf{k} = \frac{\mathbf{p}}{p}$ is the unit vector in the direction of the momentum of the electron and α is the asymmetry parameter. If the parity is conserved we have

$$w_{\mathbf{P}}(\mathbf{k}) = w_{\mathbf{P}}(-\mathbf{k}). \tag{2.9}$$

Therefore, in this case the pseudoscalar term in the probability (2.8) must be equal to zero ($\alpha = 0$). In the experiment Wu et al. was found that $|\alpha|$ is not equal to zero and large ($\alpha \simeq -0.7$). Thus, it was proven that in the β-decay parity is not conserved.

From the discovery of the nonconservation of parity followed that the Hamiltonian of the β-decay is the sum of scalar and pseudoscalar terms. The most general four-fermion Hamiltonian which does not conserve parity was proposed by Lee and Yang. It has the form

$$\mathcal{H}_I^{\beta}(x) = \sum_{i=S,V,T,A,P} \bar{p}(x)\; O_i n(x)\; \bar{e}(x)\; O^i (G_i - G_i^{'} \gamma_5)\nu(x) + \text{h.c.} \tag{2.10}$$

The constants G_i and $G_i^{'}$ characterize the scalar and pseudoscalar terms of the Hamiltonian. The Wu et al. experiment suggested that the constants G_i and G_i' are of the same order.

The interaction (2.10) is characterized by 10 (!) coupling constants. *In 1957–1958 there were two fundamental steps which brought us to the modern effective Hamiltonian of the β-decay and other weak processes.*

2.5 Two-Component Neutrino Theory

The first step was the theory of the massless two-component neutrino, proposed by Landau, Lee and Yang and Salam.

A method of the measurement of the neutrino mass was proposed by Fermi and Perrin in 1934 . This method is based on the measurement of the high-energy part of β-spectrum in which the neutrino has a small energy. At the time of the discovery of the violation of parity, from experiments on the measurement of the neutrino mass m was found the following upper bound: $m \lesssim 200$ eV. Thus, it was found that neutrino mass is much smaller than the mass of the electron. The authors of the two-component neutrino theory assumed that neutrino mass is equal to zero.

Any fermion field can be presented in the form of the sum of left-handed and right-handed components. We have

$$\nu(x) = \nu_L(x) + \nu_R(x), \tag{2.11}$$

where

$$\nu_{L,R}(x) = \frac{1 \mp \gamma_5}{2} \nu(x) \tag{2.12}$$

are left-handed and right-handed components of the field $\nu(x)$. Notice that $\nu_L(x)$ and $\nu_R(x)$ have the same Lorenz-transformation properties as $\nu(x)$.

The authors of the two-component neutrino theory assumed that

$$\text{neutrino field is } \ \nu_L(x) \ \ (\text{or } \ \nu_R(x)).$$

This is possible if the neutrino mass is equal to zero. In fact, the mass term of the neutrino with mass m has the form

$$\mathcal{L}^m(x) = -m \ \bar{\nu}(x)\nu(x) = -m \ (\bar{\nu}_L(x)\nu_R(x) + \bar{\nu}_R(x)\nu_L(x)) \tag{2.13}$$

If the neutrino field is $\nu_L(x)$ (or $\nu_R(x)$) in this case the mass term (2.13) cannot be built (and, consequently, $m = 0$)[1]

If we assume that the neutrino field is $\nu_L(x)$ (or $\nu_R(x)$) in this case

[1] This is correct for the Dirac neutrino. As we will see later for the Majorana neutrino the mass term can be built also in the case of a $\nu_L(x)$ (or $\nu_R(x)$) field.

1. $G_i = G'_i$ (or $G_i = -G'_i$) and the parity in the β-decay is violated maximally. This corresponds to the results of the Wu et al. experiment.
2. The neutrino helicity is equal to -1 $(+1)$ and the antineutrino helicity is equal to $+1$ (-1).

In fact, for the neutrino field $\nu(x)$ we have the following expansion

$$\nu(x) = \int N_p \left(\sum_{r=\pm 1} u^r(p) c_r(p)\, e^{-ipx} + \sum_{r=\pm 1} u^r(-p) d_r^\dagger(p)\, e^{ipx} \right) d^3 p. \quad (2.14)$$

Here $c_r(p)$ $(d_r^\dagger(p))$ is the operator of the absorption of a neutrino (creation of an antineutrino) with momentum p and helicity r and $N_p = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\,p^0}}$ is the normalization factor.

The Dirac equation for the massless neutrino has the form

$$\not{p}\, u^r(p) = 0, \quad (2.15)$$

where $\not{p} = \gamma_\alpha p^\alpha$. The spinor $u^r(p)$ describes a particle with helicity equal to $r (r = \pm 1)$. We have

$$\Sigma \cdot \mathbf{k}\, u^r(p) = r\ u^r(p), \quad (2.16)$$

where Σ is the operator of the spin and $\mathbf{k}$ is the unit vector in the direction of the momentum $\mathbf{p}$. For the operator of the spin we have

$$\Sigma = \gamma_5 \alpha = \gamma_5 \gamma^0 \gamma. \quad (2.17)$$

From (2.15) and (2.17) we have

$$\Sigma \cdot \mathbf{k}\, u^r(p) = \gamma_5\, u^r(p). \quad (2.18)$$

Thus, for a massless particle operator γ_5 is the operator of the helicity. From (2.16) we find

$$\gamma_5\, u^r(p) = r u^r(p). \quad (2.19)$$

Similarly, for the spinor $u^r(-p)$ which describes the state with negative energy $-p^0$ and momentum $-\mathbf{p}$ we have

$$\gamma_5\ u^r(-p) = -r\ u^r(-p). \quad (2.20)$$

From (2.19) we find that $\frac{1-\gamma_5}{2}$ is the projection operator:

$$\frac{1-\gamma_5}{2}\,u^{-1}(p)=u^{-1}(p),\qquad \frac{1-\gamma_5}{2}\,u^{1}(p)=0. \tag{2.21}$$

From (2.20) we have

$$\frac{1-\gamma_5}{2}\,u^{1}(-p)=u^{1}(-p),\qquad \frac{1-\gamma_5}{2}\,u^{-1}(-p)=0. \tag{2.22}$$

From these relations for the left-handed neutrino field we find

$$\nu_L(x)=\int N_p\left(u^{-1}(p)\,c_{-1}(p)\,e^{-ipx}+u^{1}(-p)\,d_1^{\dagger}(p)\,e^{ipx}\right)d^3p. \tag{2.23}$$

Analogously, for the right-handed neutrino field we have

$$\nu_R(x)=\int N_p\left(u^{1}(p)\,c_{1}(p)\,e^{-ipx}+u^{-1}(-p)\,d_{-1}^{\dagger}(p)\,e^{ipx}\right)d^3p. \tag{2.24}$$

The neutrino helicity was measured in 1958 in a spectacular experiment by Goldhaber, Grodzins and Sunyar. In this experiment the helicity of the neutrino was determined from the measurement of the circular polarization of γ-quanta in the chain of reactions

$$\begin{array}{l} e^- + \mathrm{Eu} \to \nu + \mathrm{Sm}^* \\ \qquad\qquad\qquad\quad \downarrow \\ \qquad\qquad\qquad\quad \mathrm{Sm} + \gamma \end{array} \tag{2.25}$$

It was found that the helicity of neutrino is negative:

$$h=-1\pm 0.3 \tag{2.26}$$

The experiment by Goldhaber et al. confirmed the theory of the two-component neutrino. It was established that the neutrino is the left-handed particle and the neutrino field is $\nu_L(x)$.[2]

2.6 μ-e Universal Charged Current. Current × Current Theory

The next decisive step in the construction of the Hamiltonian of the β-decay and other weak processes was done by Feynman and Gell-Mann, Marshak and Sudarshan in 1957–1958. Generalizing the theory of the two-component neutrino,

[2] Let us stress that the experiment by Goldhaber et al. does not exclude that the neutrino has a small mass. In fact, if in the Hamiltonian of the β-decay enters $\nu_L(x)$ and the neutrino mass is not equal to zero in this case the longitudinal polarization of the neutrino for $m \ll E$ is equal to $P_{\parallel} \simeq -1+\frac{m^2}{2E^2} \simeq -1$.

Feynman and Gell-Mann, Marshak and Sudarshan assumed that *in the Hamiltonian of the weak interaction enter only left-handed components of fields*. In this case the most general four-fermion Hamiltonian of the β-decay has the form

$$\mathcal{H}_I^{\beta} = \sum_{i=S,V,T,A,P} G_i \, \bar{p}_L O_i n_L \ \bar{e}_L O^i \nu_L + \text{h.c.}, \tag{2.27}$$

where O_i are Dirac matrices (see (2.7)).

We have

$$\bar{e}_L O_i \nu_L = \bar{e} \frac{1+\gamma_5}{2} O_i \frac{1-\gamma_5}{2} \nu. \tag{2.28}$$

It is obvious that

$$\frac{1+\gamma_5}{2} \left(1;\ \sigma_{\alpha\beta};\ \gamma_5\right) \frac{1-\gamma_5}{2} = 0\,. \tag{2.29}$$

Therefore, S, T and P terms do not enter into the Hamiltonian (2.27). Moreover A and V terms are connected by the relation:

$$\frac{1+\gamma_5}{2} \gamma_\alpha \gamma_5 \frac{1-\gamma_5}{2} = -\frac{1+\gamma_5}{2} \gamma_\alpha \frac{1-\gamma_5}{2}. \tag{2.30}$$

Thus, if we assume that only left-handed components of the fields enter into the four-fermion Hamiltonian , we come to the unique expression for the Hamiltonian of the β-decay

$$\begin{aligned} \mathcal{H}_I^{\beta} &= \frac{G_F}{\sqrt{2}} 4\, \bar{p}_L \gamma_\alpha n_L \ \bar{e}_L \gamma^\alpha \nu_L + \text{h.c.} \\ &= \frac{G_F}{\sqrt{2}}\, \bar{p} \gamma_\alpha (1-\gamma_5) n \, \bar{e} \gamma^\alpha (1-\gamma_5) \nu + \text{h.c.} \end{aligned} \tag{2.31}$$

The Hamiltonian (2.31) is the simplest possible four-fermion Hamiltonian of the β-decay which takes into account large violation of parity. Like the Fermi Hamiltonian (2.5), it is characterized by only one interaction constant.[3]

The theory proposed by Feynman and Gell-Mann, Marshak and Sudarshan was a very successful one: the Hamiltonian (2.31) allowed to describe all existing β-decay data. We know today that (2.31) is the correct effective Hamiltonian of the β-decay, of the process $\bar{\nu} + p \to n + e^+$, and other connected processes.

Until now we have only considered the Hamiltonian of the β-decay. At the time when parity violation was discovered other weak processes involving a muon-neutrino pair were known:

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[3] In order to keep the numerical value of the Fermi constant the coefficient $\frac{1}{\sqrt{2}}$ was introduced in (2.31).

$$\mu^- + (A, Z) \to \nu + (A, Z-1) \quad (\mu - \text{capture}) \tag{2.32}$$

$$\mu^+ \to e^+ + \nu + \bar{\nu} \quad (\mu - \text{decay}). \tag{2.33}$$

In 1947 B. Pontecorvo suggested the existence of a $\mu - e$ *universal weak interaction*, which is characterized by the same Fermi constant G_F. He compared the probability of the μ-capture (2.32) with the probability of the K-capture

$$e^- + (A, Z) \to \nu + (A, Z-1) \tag{2.34}$$

and found that the constant of the interaction of the muon-neutrino pair with nucleons is of the same order as the Fermi constant G_F. The idea of a $\mu - e$ universal weak interaction was also proposed by Puppi, Klein, Tiomno and Wheeler.

In order to build a $\mu - e$ universal theory of the weak interaction, Feynman and Gell-Mann introduced the notion of *the charged weak current*[4]

$$j^\alpha = 2\,(\bar{p}_L \gamma^\alpha n_L \; + \; \bar{\nu}_{\mu L} \gamma^\alpha e_L \; + \; \bar{\nu}_{eL} \gamma^\alpha \mu_L) \tag{2.35}$$

and assumed that the Hamiltonian of the weak interaction has the current×current form[5]

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}}\, j^\alpha \, j^+_\alpha, \tag{2.36}$$

where G_F is the Fermi constant.

There are two types of terms in the Hamiltonian (2.36): nondiagonal and diagonal. The nondiagonal terms are given by

$$\begin{aligned} \mathcal{H}_I^{nd} = \frac{G_F}{\sqrt{2}} 4\,\{ & [(\bar{p}_L \gamma^\alpha n_L)(\bar{e}_L \gamma_\alpha \nu_{eL}) \; + \text{h.c.}] \; + \\ & [(\bar{p}_L \gamma^\alpha n_L)(\bar{\mu}_L \gamma_\alpha \nu_{\mu L}) \; + \text{h.c.}] \; + \\ & [(\bar{e}_L \gamma^\alpha \nu_{eL})(\bar{\nu}_{\mu L} \gamma_\alpha \mu_L) \; + \text{h.c.}]\} \end{aligned} \tag{2.37}$$

The first term of this expression is the Hamiltonian of β-decay of the neutron (2.1), of the process $\bar{\nu}_e + p \to e^+ + n$ and other connected processes. The second term of (2.37) is the Hamiltonian of the process $\mu^- + p \to \nu_\mu + n$ and other connected processes. Finally the third term of (2.37) is the Hamiltonian of the μ-decay (2.33) and other processes.

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[4] We denoted the fields of neutrinos which enter into the current together with the fields of electron and muon, correspondingly, by ν_e and ν_μ. At this point, this is simply a notation. We will see later that in fact ν_e and ν_μ are different particles.

[5] The current j^α changes the charge by one. This is the reason, why this current is called a charged current. Notice also that the hadron current has the form $j^\alpha = v^\alpha - a^\alpha$, where $v^\alpha = \bar{p}\gamma^\alpha n$ and $a^\alpha = \bar{p}\gamma^\alpha \gamma_5 n$ are vector and axial currents. The Feynman-Gell-Mann theory is called the $V - A$ theory.

The diagonal terms of the Hamiltonian (2.36) are given by

$$\mathcal{H}^d = \frac{G_F}{\sqrt{2}} 4[(\bar{\nu}_{eL}\gamma^\alpha e_L)(\bar{e}_L\gamma_\alpha \nu_{eL}) + (\bar{\nu}_{\mu L}\gamma^\alpha \mu_L)(\bar{\mu}_L\gamma_\alpha \nu_{\mu L}) + (\bar{p}_L\gamma^\alpha n_L)(\bar{n}_L\gamma_\alpha p_L)] \tag{2.38}$$

The first term of the (2.38) is the Hamiltonian of the processes of elastic scattering of neutrino and antineutrino on an electron

$$\nu_e + e \to \nu_e + e \tag{2.39}$$

and

$$\bar{\nu}_e + e \to \bar{\nu}_e + e, \tag{2.40}$$

of the process $e^+ + e^- \to \bar{\nu}_e + \nu_e$ and other processes. Such processes were not known in the fifties. Their existence was predicted by the current×current theory.

The cross sections of the processes (2.39) and (2.40) are very small. The observation of such processes was a challenge. After many years of efforts, the cross section of the process (2.40) was measured by F. Reines et al. in an experiment with antineutrinos from a reactor. At that time the Standard Model already existed. According to the Standard Model, to the matrix elements of the processes (2.39) and (2.40) contributes also an additional (neutral current) Hamiltonian. The result of the experiment by F. Reines et al. was in agreement with the Standard Model.

2.7 Theory with Vector *W* Boson

Feynman and Gell-Mann considered a possible origin of the current×current interaction (2.36). Let us assume that a charged vector boson $W^\pm$ exists and the Lagrangian of the weak interaction (analogously to the Lagrangian of the electromagnetic interaction) has the form of a scalar product of the current and the vector field

$$\mathcal{L}_\mathcal{I} = -\frac{g}{2\sqrt{2}}\, j_\alpha\, W^\alpha + \text{h.c.} \tag{2.41}$$

where g is a dimensionless constant and the current j_α is given by Eq. (2.35).

In the theory with W-boson Feynman diagram of the β-decay of the neutron is presented in Fig. 2.1. If

$$Q^2 \ll m_W^2 \tag{2.42}$$

where Q is the momentum of the virtual W-boson and m_W is the mass of W-boson, the matrix element of the β-decay of the neutron can be obtained from the Hamiltonian (2.36) in which the Fermi constant is given by the relation

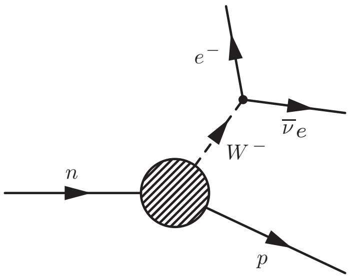

Fig. 2.1 Feynman diagram of the process $n \to p + e^- + \bar{\nu}$ in the theory with the $W^{\pm}$-boson

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\, m_W^2}. \tag{2.43}$$

It is easy to verify that in the theory with W-boson the effective Hamiltonian of all weak processes with the virtual W-boson and $Q^2 \ll m_W^2$ has current×current form (2.36), in which the Fermi constant is given by the relation (2.43).

Thus, the theory with a charged vector W-boson could explain the current×current structure of the weak interaction Hamiltonian and the fact that the Fermi constant has the dimension $[M]^{-2}$. As we will see later, (2.41) is a part of the total Lagrangian of the electroweak interaction of the Standard Model.[6]

2.8 First Observation of Neutrinos. Lepton Number Conservation

The first proof of the existence of neutrino was obtained in the mid-fifties in the experiment by F. Reines and C.L. Cowan. In this experiment (anti)neutrinos from the Savannah River reactor were detected through the observation of the process

$$\bar{\nu}_e + p \to e^+ + n. \tag{2.44}$$

Antineutrinos are produced in a reactor in a chain of β-decays of neutron-rich nuclei, products of the fission of uranium and plutonium. The energies of antineutrinos from a reactor are less than approximately 10 MeV. About $2.3 \cdot 10^{20}$ antineutrinos per second were emitted by the Savannah River reactor. The flux of $\bar{\nu}_e$'s in the Reines and Cowan experiment was about $10^{13}\ \mathrm{cm}^{-2}\, s^{-1}$.

In the theory of the two-component neutrino, the cross section of the process (2.44) is connected with the life-time τ_n of the neutron by the relation

[6] It is interesting to note that the idea of the charged vector W-boson was proposed by O. Klein at the end of the thirties.

$$\sigma(\bar{\nu}_e p \to e^+ n) = \frac{2\pi^2}{m_e^5 f \tau_n} p_e E_e, \tag{2.45}$$

where $E_e \simeq E_{\bar{\nu}} - (m_n - m_p)$ is the energy of the positron, p_e is the positron momenta, f=1.686 is the phase-space factor, m_n, m_p, m_e are masses of the neutron, proton and electron, respectively. From (2.45) for the cross section of the process (2.44), averaged over antineutrino spectrum, the value

$$\bar{\sigma}(\bar{\nu}_e p \to e^+ n) \simeq 9.5 \cdot 10^{-44}\ \text{cm}^2 \tag{2.46}$$

was found.

A liquid scintillator ($1.4 \cdot 10^3$ l) loaded with $CdCl_2$ was used as a target in the experiment. Positron, produced in the process (2.44), slowed down in the scintillator and annihilated with electron, producing two γ- quanta with energies $\simeq$ 0.51 MeV and opposite momenta. A neutron, produced in the process was captured by Cd within about 5 μs, producing γ-quantum. The γ-quanta were detected by 110 photomultipliers. Thus, the signature of the $\bar{\nu}$-event in the Reines and Cowan experiment was two γ-quanta from the $e^+ - e^-$-annihilation in coincidence with a delayed γ-quantum from the neutron capture by cadmium. For the cross section of the process (2.44) the value

$$\sigma_\nu = (11 \pm 2.6)\ 10^{-44}\ \text{cm}^2 \tag{2.47}$$

was obtained in the experiment. This value is in agreement with the predicted value (2.46).

The particle which is produced in the β-decay together with electron is called antineutrino. It is a direct consequence of the quantum field theory that antineutrino can produce a positron in the inverse β-decay (2.44) and other similar processes. Can antineutrinos produce electrons in weak processes of interaction with nucleons? The answer to this question was obtained from an experiment which was performed in 1956 by Davis et al. with antineutrinos from the Savannah River reactor. In this experiment ^{37}Ar from the process

$$\bar{\nu} +^{37} \text{Cl} \to e^- +^{37} \text{Ar} \tag{2.48}$$

was searched for. The process (2.48) was not observed in the experiment. It was shown that the ^{37}Ar production rate was about five times smaller than the rate expected if antineutrinos could produce electrons via the weak interaction.

Thus, it was established that antineutrinos from a reactor can produce positrons (the Reines-Cowan experiment) but cannot produce electrons. In order to explain this fact we assume that exist *conserving lepton number (charge)* L, the same for $\bar{\nu}$ and e^+. Let us put $L(\bar{\nu}) = L(e^+) = -1$. According to the quantum field theory the lepton charges of the corresponding antiparticles are opposite: $L(\nu) = L(e^-) = 1$. We also assume that the lepton numbers of proton, neutron and other hadrons are equal to zero. Conservation of the lepton number explain the negative result of the

Davis experiment. According to the law of conservation of the lepton number a neutrino is produced together with e^+ in the β^+-decay

$$(A, Z) \to (A, Z-1) + e^+ + \nu \tag{2.49}$$

2.9 Discovery of Muon Neutrino. Electron and Muon Lepton Numbers

In the expression (2.35) the fields of neutrinos, which enter into the charged current together with electron and muon fields, were denoted by ν_e and ν_μ, correspondingly. *Are ν_e and ν_μ the same or different particles?* The answer to this fundamental question was obtained in the famous Brookhaven neutrino experiment in 1962.

The first indication that ν_e and ν_μ are different particles was obtained from an analysis of the $\mu \to e\gamma$ data. The probability of the decay $\mu \to e\gamma$ was calculated by Feinberg in the theory with W-boson and a cutoff. It was found that if ν_e and ν_μ are identical particles and the cut-off is given by the mass of the W-boson the ratio R of the probability of the decay $\mu \to e\gamma$ to the probability of the decay $\mu \to e\,\nu\,\bar{\nu}$ is given by

$$R \simeq \frac{\alpha}{24\pi} \simeq 10^{-4} \tag{2.50}$$

The decay $\mu \to e\gamma$ was not observed in experiment. At the time of the Brookhaven experiment, for the upper bound of the ratio R was found the value

$$R < 10^{-8}, \tag{2.51}$$

which is much smaller than (2.50).

A direct proof of the existence of the second (muon) type of neutrino was obtained by L.M. Lederman, M. Schwartz, J. Steinberger et al. in the first experiment with accelerator neutrinos in 1962. The idea of the experiment was proposed by B.Pontecorvo in 1959.

A beam of π^+'s in the Brookhaven experiment was obtained by the bombardment of Be target by protons with an average energy of about 15 GeV. In the decay channel (about 21 m long) practically all π^+'s decay. After the channel there was shielding (13.5 m of iron), in which charged particles were absorbed. After the shielding there was the neutrino detector (aluminium spark chamber, 10 tons) in which the production of charged leptons was observed.

The dominant decay channel of the π^+-meson is

$$\pi^+ \to \mu^+ + \nu_\mu. \tag{2.52}$$

According to the universal $V - A$ theory, the ratio R of the width of the decay

$$\pi^+ \to e^+ + \nu_e \tag{2.53}$$

to the width of the decay (2.52) is equal to

$$R = \frac{m_e^2}{m_\mu^2} \frac{(1 - \frac{m_e^2}{m_\pi^2})^2}{(1 - \frac{m_\mu^2}{m_\pi^2})^2} \simeq 1.2 \cdot 10^{-4}. \tag{2.54}$$

Thus, the decay $\pi^+ \to e^+ \nu_e$ is strongly suppressed with respect to the decay $\pi^+ \to \mu^+ + \nu_\mu$.[7] From (2.54) follows that the neutrino beam in the Brookhaven experiment was practically a pure ν_μ beam (with a small about 1% admixture of ν_e from decays of muons and kaons).

Neutrinos, emitted in the decay (2.52), produce μ^- in the process

$$\nu_\mu + N \to \mu^- + X. \tag{2.55}$$

If ν_μ and ν_e would be the same particles, neutrinos from the decay (2.52) would produce also e^- in the reaction

$$\nu_\mu + N \to e^- + X. \tag{2.56}$$

Due to the $\mu - e$ universality of the weak interaction one could expect to observe in the detector practically equal number of muons and electrons.

In the Brookhaven experiment 29 muon events were detected. The observed six electron candidates could be explained by the background. The measured cross section was in agreement with the $V - A$ theory. Thus, it was proved that *ν_μ and ν_e are different particles.*[8]

The results of the Brookhaven and other experiments suggested that the total electron L_e and muon L_μ lepton numbers are conserved:

$$\sum_i L_e^{(i)} = \text{const}; \quad \sum_i L_\mu^{(i)} = \text{const} \tag{2.57}$$

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[7] The reason for this suppression can be easily understood. Indeed, let us consider the decay (2.53) in the rest frame of the pion. The helicity of the neutrino is equal to -1. If we neglect the mass of the e^+, the helicity of the positron will be equal to $+1$ (the helicity of the positron will be the same in this case as the helicity of the antineutrino). Thus, the projection of the total angular momentum on the neutrino momentum will be equal to -1. The spin of the pion is equal to zero and consequently the process (2.53) in the limit $m_e \to 0$ is forbidden. These arguments explain the appearance of the small factor $(\frac{m_e}{m_\mu})^2$ in (2.54).

[8] In 1963 in the CERN with the invention of the magnetic horn the intensity and purity of neutrino beams were greatly improved. In the more precise 45 tons spark-chamber experiment and in the large bubble chamber experiment the Brookhaven result was confirmed.

Table 2.1 Lepton numbers of particles

Lepton number	ν_e e^-	ν_μ μ^-	Hadrons, γ
L_e	1	0	0
L_μ	0	1	0

The lepton numbers of particles are given in Table 2.1. The lepton numbers of antiparticles are opposite to the lepton numbers of the corresponding particles.

For many years all experimental data were in an agreement with (2.57). At present it is established that (2.57) is an approximate phenomenological rule. It is violated in neutrino oscillations due to small neutrino masses and neutrino mixing. Later we will discuss neutrino oscillations in details.

2.10 Strange Particles. Quarks. Cabibbo Current

The current×current Hamiltonian (2.36) with CC current (2.35) is the effective Hamiltonian of such processes in which leptons, neutrinos and nonstrange hadrons are participating. The first strange particles were discovered in cosmic rays in the fifties. Decays of strange particles were studied in details in accelerator experiments. From the investigation of the semi-leptonic decays

$$K^+ \to \mu^+ + \nu_\mu, \quad \Lambda \to n + e^- + \bar{\nu}_e,$$
$$\Sigma^- \to n + e^- + \bar{\nu}_e, \quad \Xi^- \to \Lambda + \mu^- + \bar{\nu}_\mu$$

and others the following *three phenomenological rules* were formulated.

I. The strangeness S in the decays of strange particles is changed by one

$$|\Delta S| = 1.$$

II. In the decays of the strange particles the rule

$$\Delta Q = \Delta S$$

is satisfied. Here $\Delta Q = Q_f - Q_i$ and $\Delta S = S_f - S_i$, where S_i and S_f are the initial and final total strangeness of the hadrons and Q_i and Q_f are the initial and final total electric charges of hadrons (in the unit of the proton charge).

III. The decays of strange particles are suppressed with respect to the decays of non strange particles.

In 1964 Gell-Mann and Zweig proposed the idea of three quarks u, d , s, constituents of strange and nonstrange hadrons. The quantum numbers of the quarks are presented in Table 2.2 Let us build the hadronic charged currents from the quark

Table 2.2 Quantum numbers of quarks (Q is the charge, S is the strangeness, B is the baryon number)

Quark	Q	S	B
u	2/3	0	1/3
d	$-1/3$	0	1/3
s	$-1/3$	-1	1/3

fields. The current (2.35) changes the charge by one. If we accept the Feynman-Gell-Mann, Marshak-Sudarshan prescription (into the weak current enter only left-handed components of the fermion fields) there are only two possibilities to build such currents from the fields of u, d and s quarks:

$$\bar{u}_L \gamma_\alpha d_L \quad \text{and} \qquad \bar{u}_L \gamma_\alpha s_L. \tag{2.58}$$

The first current changes the charge by one and does not change the strangeness ($\Delta Q = 1$, $\Delta S = 0$). The second current changes the charge by one and the strangeness by one ($\Delta Q = 1$, $\Delta S = 1$). The matrix elements of these currents automatically satisfy rules I and II. Notice that this was one of the first arguments in favor of quark structure of the hadron current.

The weak interaction of the strange particles was included into the current$\times$current theory by N. Cabibbo in 1962. He assumed that the charged current which does not change strangeness and the charged current which changes the strangeness by one are, correspondingly, the $1 + i2$ and $4 + i5$ components of the $SU(3)$ octet current. In order to take into account the suppression of the decays with the change of the strangeness with respect to the decays in which the strangeness is not changed (the rule III) Cabibbo introduced a parameter which is called the Cabibbo angle θ_C. From the analysis of the experimental data on the investigation of the decays of strange particles he found that $\sin\theta_C \simeq 0.2$.

In terms of quark currents (2.58) the Cabibbo current has the form

$$j_\alpha^{\text{Cabibbo}}(x) = 2\,(\cos\theta_C\, \bar{u}_L(x)\gamma_\alpha\, d_L(x) + \sin\theta_C\, \bar{u}_L(x)\gamma_\alpha\, s_L(x) \tag{2.59}$$

The total weak charged current takes the form

$$j_\alpha(x) = 2\,(\bar{\nu}_{eL}(x)\gamma_\alpha\, e_L(x) + \bar{\nu}_{\mu L}(x)\gamma_\alpha\, \mu_L(x) + \bar{u}_L(x)\gamma_\alpha\, d_L^{'}(x), \tag{2.60}$$

where

$$d_L^{'}(x) = \cos\theta_C\, d_L(x) + \sin\theta_C\, s_L(x). \tag{2.61}$$

Notice that there is an asymmetry between quark and lepton terms in (2.60). Namely, in this expression there are two lepton terms and one quark term.

2.11 Charmed Quark. Quark and Neutrino Mixing

It was shown in 1970 by Glashow, Illiopulos and Maiani (GIM) that the charged current (2.59) induces a neutral current which does not change electric charge ($\Delta Q = 0$) and change the strangeness by one ($|\Delta S| = 1$). As a result, the decays like

$$K^+ \to \pi^+ + \nu + \bar{\nu}. \tag{2.62}$$

become possible in such a theory. In the theory with the current (2.60) the width of the decay (2.62) is many orders of magnitude larger than *the upper bound* of the width of the decay obtained in experiments.

Glashow, Illiopulos and Maiani assumed that there exists a fourth "charmed" quark c with charge 2/3 and that there is an additional term in the weak current into which enters the field of the new quark c_L and the combination of d_L and s_L fields orthogonal to the Cabibbo combination (2.61). The weak currents took the form

$$j_\alpha(x) = 2\,(\bar{\nu}_{eL}(x)\gamma_\alpha\, e_L(x) + \bar{\nu}_{\mu L}(x)\gamma_\alpha\, \mu_L(x) + \bar{u}_L(x)\gamma_\alpha\, d'_L(x) + \bar{c}_L(x)\gamma_\alpha\, s'_L(x)), \tag{2.63}$$

where

$$\begin{aligned} d'_L(x) &= \cos\theta_C d_L(x) + \sin\theta_C s_L(x) \\ s'_L(x) &= -\sin\theta_C d_L(x) + \cos\theta_C s_L(x)\,. \end{aligned} \tag{2.64}$$

As we will see in the next section, in the theory with the charged current (2.63) neutral current which changes the strangeness does not appear.

The relations (2.64) mean that *the fields of d and s quarks enter into the charged current in the mixed form*. The phenomenon of mixing is perfectly confirmed by experiment.

We make the following remark. In the current (2.63) lepton and quark terms enter symmetrically. It will be, however, full lepton-quark symmetry of the current if the neutrino masses are different from zero and the fields of neutrinos with definite masses, like the fields of quarks, enter into the CC in the mixed form

$$\begin{aligned} \nu_{\mu L}(x) &= \cos\theta\ \nu_{1L}(x) + \sin\theta\ \nu_{2L}(x) \\ \nu_{eL}(x) &= -\sin\theta\ \nu_{1L}(x) + \cos\theta\ \nu_{2L}(x), \end{aligned} \tag{2.65}$$

where $\nu_1(x)$ and $\nu_2(x)$ are the fields of the neutrinos with masses m_1 and m_2, correspondingly.

The existence of the c-quark means the existence of a new family of "charmed" particles. This prediction of the theory was perfectly confirmed by experiment. In 1974 the J/Ψ particles, bound states of $c - \bar{c}$, were discovered. In 1976 the $D^{\pm,0}$ mesons, bound states of charmed and nonstrange quarks, were discovered, etc. All data on the investigation of the weak decays and neutrino reactions were in agreement with the current×current theory with the current given by (2.63).

In 1975 the third charged lepton τ was discovered in experiments at $e^+ - e^-$ colliders. In the framework of the Standard Model, which we will consider in the next chapter, the existence of the third charged lepton requires the existence of the corresponding third type of neutrino ν_τ and an additional pair of quarks: the t (top) quark with electric charge 2/3 and the b (bottom) quark with electric charge $-1/3$. All these predictions of the SM were perfectly confirmed by numerous experiments.

The modern charged current has the form

$$j_\alpha^{\mathrm{CC}}(x) = 2\,(\bar{\nu}_{eL}(x)\gamma_\alpha\, e_L(x) + \bar{\nu}_{\mu L}(x)\gamma_\alpha\, \mu_L(x) + \bar{\nu}_{\tau L}(x)\gamma_\alpha\, \tau_L(x) + \bar{u}_L(x)\gamma_\alpha\, d'_L(x) + \bar{c}_L(x)\gamma_\alpha\, s'_L(x) + \bar{t}_L(x)\gamma_\alpha\, b'_L(x)). \quad (2.66)$$

Here

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\ \nu_{iL}(x) \quad l = e.\mu, \tau \quad (2.67)$$

and

$$d'_L(x) = \sum_{q=u,s,b} V_{uq}\, q_L(x),\ \ s'_L(x) = \sum_{q=u,s,b} V_{cq} q_L(x),\ \ b'_L(x) = \sum_{q=u,s,b} V_{tq}\, q_L(x). \quad (2.68)$$

Here U is an unitary 3×3 neutrino mixing matrix and V is an unitary 3×3 quark mixing matrix.

We know today that the vector $W^{\pm}$-boson exists and that the Lagrangian of the CC weak interaction has the form

$$\mathcal{L}_{\mathcal{I}}^{\mathrm{CC}}(x) = -\frac{g}{2\sqrt{2}}\, j_\alpha^{\mathrm{CC}}(x)\ W^\alpha(x) + \mathrm{h.c.} \quad (2.69)$$

2.12 Summary and Outlook

The theory of the weak interaction started with the famous Fermi paper "An attempt of a theory of beta radiation". The Fermi theory was based on (1) The Pauli hypothesis of the existence of the neutrino. (2) The proton-neutron structure of nuclei. (3) The assumption that an electron-neutrino pair is produced in the process of transition of a neutron into a proton. (4) The assumption that in analogy with electromagnetic interaction the weak interaction is the vector one. Later in accordance with experimental data this last assumption was generalized and other terms (scalar, tensor, axial and pseudoscalar) were included.

The discovery of the parity violation in the β-decay and other weak processes played a revolutionary role in the development of the theory of the weak interaction. Soon after this discovery the two-component theory of massless neutrino was proposed. According to this theory in the Hamiltonian of the weak interaction the left-handed (or right-handed) component of the neutrino field enters. In less than one year this theory was confirmed by experiment. It was proved that neutrino is a left-handed particle.

The next fundamental step was the current×current, V-A theory of the weak interaction which was based on the assumption that only left-handed components of the fields enter into charged current.

The electron neutrino was discovered in the fifties in the first reactor neutrino experiment. A few years later in the first accelerator neutrino experiment the muon neutrino was discovered.

After the hypothesis of quarks was proposed, the weak charged current started to be considered as quark and lepton current. One of the fundamental ideas which was put forward in the process of the phenomenological development of the theory was the idea of the quark mixing. At the very early stage of the development of the theory the idea of the existence of the charged heavy vector intermediate $W^{\pm}$ boson was proposed.

It was a long (about 40 years) extremely important period of the development of the physics of the weak interaction with a lot of bright, courageous ideas.[9] The theory which was finally proposed allowed to describe data of a huge number of experiments. The Standard Model of the weak interaction could not appear without the phenomenological V-A theory.

[9] And also many wrong ideas which we did not discussed here.