Basics of Valuation

Introduction

All companies deal with valuation from time to time. Capital budgeting, company and asset valuation, or value based management rely on valuation.

Two approaches are the foundation of valuation, discounted cash flow valuation and relative valuation. The first one is a bottom-up approach where the present value of an asset's future cash flows is calculated, the second determines the value of an asset by comparing it to similar other assets.

While relative valuation is well applicable by common sense, DCF needs considerable understanding of the relevant input parameters. As DCF is a vital approach to valuation in life sciences and the basis of decision tree analysis and real options valuation, it is worthwhile to discuss in detail how the method is properly applied.

We discuss in the following chapters the reasoning behind DCF and how to define the input parameters to value an asset. We also discuss the current problems to this valuation approach, such as the problem of risk and uncertainty, and some methods that try to overcome these problems.

Fundamentals

Cash Flows

The cash flow is, as its name describes, money that flows in or out of a company. Cash flows can be classified into:

- Cash flows from operations: Cash flows from day-to-day, income-producing activities.
- Cash flows from investing activities: Net cash flow from investing activities, defined as divestments minus investments.
- Cash flows from financing activities: Net cash flow from financing activities. Issue of new debt and equity minus repayment of debt, minus equity return, minus payment of dividends.

B. Bogdan, R. Villiger, *Valuation in Life Sciences*, 3rd ed., DOI 10.1007/978-3-642-10820-4_2,

- Change in liquidity: Liquidity at the end of a period minus the liquidity at the beginning the same period.

The cash flows of a company are presented either in the statement of cash flows or in the accounting statement of cash flows. The latter is useful to see the change in accounting cash and differs from the statement of cash flows mainly in the interest expense. Below is Virtual Corp.'s statement of cash flows for one accounting period:

Table 2.1. Virtual corporation statement of cash flows

Cash Flow of the Firm	('000s)
Operating Cash Flow	$200
Capital Spending	(100)
Additions to net working capital	(50)
Total	$50
Cash Flow to Investors of the Firm	
Debt	$45
Equity	$5
Total	$50

Extracting the valuation relevant cash flows from these statements is not always straightforward. For the purpose of the valuation theory outlined in this book, we will therefore not refer to accounting cash flows such as depreciation or to terms like additions to net working capital. We assume that all investments are immediately depreciated. Nevertheless, the final valuation result remains with both approaches the same.

The essential input for any valuation are the cash flows. We have to identify and estimate all relevant cash flows in terms of:

- Size
- Time
- Probability

Cash flow size. A cash flow is either positive, i.e. a revenue, or negative and therefore an expense, with an absolute size. A negative cash flow is either noted with a negative sign or between brackets, i.e. $ 50 expense is $ –50 or $ (50).

Time of cash flows. We need to define the time when a cash flow occurs as the time influences the value of money. The discount rate accounts for this.

Probability of cash flows. Once we know the size and the time of a cash flow, we have to estimate the probability of it. A cash flow that is certain has more value than a cash flow that is uncertain.

We will learn in the following chapters, that valuation is the process of defining the cash flows size, time, and probability, the risk adjusting and netting of all relevant cash flows and finally calculating the net present value by discounting. Once we have exactly identified and described all cash flows for the valuation, the major part of the work is done.

Discounting

In valuation we compare cash flows that:

- occur at different points in time,
- are not accurately predictable in their size,
- occur with different likelihood.

The first point alludes to the time value of money, the second to the uncertainty of the estimate, and the third to the risk that the cash flow occurs at all. Discounting takes full account of time value and uncertainty. Probabilities (or success rates in the life sciences context) cover the shortfall risk.

Time value of money. The concept of time value bases on the fact that people prefer a dollar today to a dollar tomorrow. A dollar today has more value than dollar in the future. To keep its value, money must accrete. This value increment is called interest. Somebody investing a dollar in a project wants to get more than the dollar he invested after one year, because he could also have put his money on the bank and earn interest on it.

Interests are generally compounded annually. An interest rate of 5% means that the investor of \$ 1,000 earns after one year \$ 50 of interest. After another year we earn 5% of the meanwhile \$ 1,050, ending up with \$ 1,102.50. This example shows that compounding of interests has an impact on the final amount. In the second year, the investor receives not only 5% of the initial amount, but also 5% on the interests he earned in the first year.

We determine the risk free interest rate by taking the yields of treasury bonds of the US or the EURO zone. The risk free interest rates depend on the maturity of bonds and vary from country to country. Usually, interest rates rise with increasing maturity. In Japan, interest rates are at the moment low (0%-2%), while they are higher in Europe (1%-5%) and in the United States (3%-7%). The numbers depend on the state of the economy in these countries and are subject to fluctuations. For valuation purposes, we will take interest rates of bonds with long maturities, i.e. 10 to 30 years.

Risk. Next to the time value, discounting reproduces the risk of the investment and the willingness of the investor to take risk. We estimate future costs and revenues to a certain degree, but the real cash flow can be higher or lower than our initial estimate. This uncertainty is typical for predictions. However, people tend to dislike under-performance or loss more than they do like over-performance or gain. They are willing to pay a fee in order to avoid any shortfalls. Insurances base their business on this asymmetric attitude called risk aversion.

Discounting must now compensate not only for the loss of value over time, but also for the impending difference between the expected and the actual return. Consequently, uncertain investments should reward the investor at a higher rate than safe investments, as it is more likely that the actual return is closer to the expected return E(R).

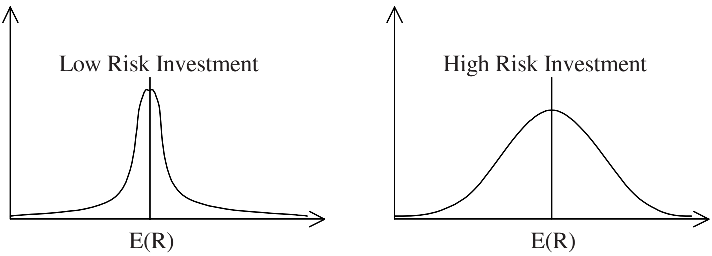

Fig. 2.1. Expected return of assets with different risk profile

The valuation can reflect this increased return expectancy by adding a spread on top of the rate that displays the above-described value loss over time. This spread depends on the uncertainty of the cash flow estimates and can range from 0% up to 20%. Typically, companies use a spread between 5% and 8%.

Discrete and continuous compounding. Discounting is one of the most technical parts of valuation. It is worthwhile to spend some time to get the technicalities right. We differentiate between discrete compounding and continuous compounding. Discrete compounding pays the interests accrued over a period $T–t$ at the end of the same period. With r being the annualized interest rate an amount of money S_T becomes:

$$S_T = S_t\left(1 + r_{dis}\right)^{(T-t)} \tag{2.1}$$

Continuous compounding assumes that interests are paid out and reinvested continuously, i.e. the investor earns at every moment already interests on the interests earned in the previous glimpse of time. Continuous compounding yields therefore a higher interest. After a time period t with interest rate r the amount S becomes, if continuously compounded:

$$S_T = S_t \exp\left(r_{con}\left(T - t\right)\right) \tag{2.2}$$

Unlike interest calculations where we have to calculate an amount we will receive in the future, discounting is used to determine the present value of a future amount. Instead of moving forward in time, we now move backward in time. This leads to the following discrete and continuous discounting methods, r being the discount rate:

$$S_t = S_T\left(1 + r_{dis}\right)^{-(T-t)} \tag{2.3}$$

$$S_t = S_T \exp\left(-r_{con}\left(T - t\right)\right) \tag{2.4}$$

Usually interest and discount rates refer to the discrete method, using one year as reference period. The continuous rate r_{con} that corresponds to the discrete rate r_{dis} can be found with the following relationship:

$$r_{con} = \ln\left(1 + r_{dis}\right) \tag{2.5}$$

The Cost of Capital

The discount rate of a firm corresponds to the average rate at which the stakeholders want their capital to accrete. A company has two major classes of stakeholders, debtholders or bondholders, and shareholders. Both require compensation for the risk of their investment into the company. Bondholders typically receive the principal of the bond. Only in the case of default of the company, they can lose up to the full amount invested. The spread of the bond that comes on top of the risk free rate must offset this risk. The spread depends on the likelihood of the company defaulting. Rating agencies like Standard & Poor's, Moody's, or Fitch give their opinions on the trustworthiness of companies. According to these ratings, the return rates of the bonds are determined. These bond returns then correspond to the cost of debt, i.e. the minimum rate a company must achieve to satisfy the demands of their bondholders.

The rate required by the shareholders is called cost of equity. Unfortunately, this parameter is not observable. Several theories have evolved to determine the cost of equity. After this section, we will present the capital asset pricing model and the market-derived capital pricing model as two representatives.

Once we have determined cost of debt and cost of equity, we can compute the average cost of capital of a company. The debt part of the company must accrete by the cost of debt, the equity part by the cost of equity. An important property of debt is its tax-deductibility. This makes debt even cheaper capital. Taking account of the ratio between bond- and shareholders and the tax advantages of debt the cost of capital, or in this case the weight-adjusted cost of capital (WACC), becomes (with D being the market value of debt, E equity, and T the tax rate):

$$WACC = \frac{D}{D+E} r_D (1-T) + \frac{E}{D+E} r_E \tag{2.6}$$

The capital invested in a project must appreciate at least at the discount rate, otherwise the company cannot satisfy the expectations of its stakeholders. This is the reason why the discount rate is often called hurdle rate.

In practice we rarely use the wacc, because usually we do not want to know the value of the company but rather the value of equity, i.e. the value represented by the shares. In this case we apply the cost of equity in the valuation. And if we estimate future cash flows based on today's assumptions, which we always do, we must adjust the valuation for inflation. Estimating future cash flows we would have to consider inflation as well, i.e. in reality the cash flows would be higher than estimated. But instead of doing so for every cash flow, we can also perform this adjustment in the discount rate. For this sake we reduce the discount rate (more precisely: the cost of equity) by the inflation rate. So, if the yearly inflation is expected to be of 2%, we simply subtract 2% from the discount rate and calculate with this. The following example explains this method.

Assume that an artist is going to sell a painting for US$ 10,000. But you are only going to sell it in three years, and the expected inflation rate amounts to 2%. So the actual selling price won't be US$ 10,000 but rather US$ 10,612 (=10,000*$(1+2\%)^3$). The discount rate you apply is 13%. So the value of this painting to the artist is US$ 7,355 (=10,612*$(1+13\%)^{-3}$). This value was calculated by first adjusting US$ 10,000 for the inflation over three years and then discounting it over the same period. We have multiplied a cash flow by the fraction (1+2%)/(1+13%) or more general by the fraction of inflation adjustment (1+ρ) over discount (1+r), where ρ is the expected inflation rate and r is the discount rate. If ρ is much smaller than r then this becomes approximately:

$$\left(\frac{1+\rho}{1+r}\right)^t = (1+\rho)^t (1+r)^{-t} \cong (1+r-\rho)^{-t} \tag{2.7}$$

This means, that instead of adjusting for inflation and then discounting, we could directly discount at a rate reduced by the inflation. Of course, this is not 100% correct, but if ρ is relatively small the error is negligible. In our case the artist must discount the US$ 10,000 at a rate of 13%–2%=11%. This gives US$ 7,312. We see that it is a shortcut method, because we have a difference of US$ 43 to the exact solution. We can see that if applying continuous discounting this method becomes exact:

$$e^{\rho_c t} e^{-r_c t} = e^{\rho_c t - r_c t} = e^{-(r_c - \rho_c)t} \tag{2.8}$$

Terminal Value

Sometimes we want to know the value of series of yearly cash flows from now to infinity. The bank of England for instance issued a bond that pays out each year a certain fixed coupon, let's say £ 10. If we want to know the value of this bond we have to calculate the sum of all future discounted cash flows. This can be easily calculated with the following identity:

$$\sum_{i=1}^{\infty} CF \frac{1}{(1+r)^i} = \frac{CF}{r} \tag{2.9}$$

Using a discount rate of 4% the value of the perpetual bond of the Bank of England therefore becomes £ 10/4%=£ 250.

If we assume that the cash flow will happen each year, but will also become larger with a constant growth rate μ, then the formula becomes:

$$\sum_{i=1}^{\infty} CF (1+\mu)^i \frac{1}{(1+r)^i} \cong \frac{CF}{r-\mu} \tag{2.10}$$

Again, the same imprecision applies like for the inflation adjustment. But the formula above is the one commonly used for terminal value calculations.

Capital Asset Pricing Model

One way to determine the cost of equity is the capital asset pricing model. This model differentiates between diversifiable and non-diversifiable risks. The non-diversifiable risks are generally denoted as market risk, the diversifiable risks as asset specific risk. We assume that there exists a portfolio that is only exposed to market risk, all diversifiable risks cancel each other out. This portfolio is defined by the modern portfolio theory by Harry Markowitz (Markowitz 1952) and optimises the risk-return ratio (Sharpe ratio). The portfolio is called market portfolio. The return of an asset can then be described as follows:

$$r_{asset} = \beta\, r_{market} + \underbrace{\left(r_{asset} - \beta\, r_{market}\right)}_{\varepsilon} \tag{2.11}$$

r_{market} is the return of the market portfolio that is solely exposed to the non-diversifiable market risk. The factor β indicates by what extent the asset is exposed to the market risk. ε then represents the diversifiable part of the asset's risk. Note that β is chosen such that ε is completely independent of r_{market}, i.e. there is no correlation between ε and r_{market}. Note that r_{market} and ε are random variables and are only predictable to a certain degree.

While an investor can reduce or even avoid his exposure to diversifiable risks by holding a large and well-diversified portfolio, he cannot reduce the non-diversifiable risks. He only must be rewarded for the non-diversifiable part of the asset's risk. The average return the investor therefore claims, i.e. the cost of equity if the asset is a share, is the risk free rate plus a spread on top, proportional to the non-diversifiable risk he accepts to bear. The reward for taking the market risk is called market risk premium. Unfortunately, this measure is not observable and might even be perceived differently by each investor. However, people generally accord that the market risk premium corresponds to the excess of the market portfolio over the risk free rate, $\hat{r}_{market} - r_f$, where $\hat{r}_{market}$ denotes the average return of the market portfolio. $\hat{r}_{market} - r_f$ is exactly the reward the market pays for bearing the market risk. Consequently, the cost of equity an investor claims for an equity investment is:

$$r_E = r_f + \beta\, \underbrace{\left(\hat{r}_{market} - r_f\right)}_{market\ risk\ premium} \tag{2.12}$$

It remains to define the parameters r_f, β, and the market risk premium. The risk free rate should not pose major problems. For the market risk premium one can use the historic excess performance of the market portfolio compared to risk free investments. Unfortunately, the theoretic market portfolio is not known, therefore we don't know $\hat{r}_{market}$ either. A generally accepted bypass is to assume an index as the S&P 500 or FTSE100. $\hat{r}_{market}$ then becomes the historical performance of the chosen index. Finally, we have to determine β. This measure is defined by:

$$\beta = Corr\left(r_{asset}, r_{market}\right)\frac{\sigma_{asset}}{\sigma_{market}} = \frac{Cov(r_{asset}, r_{market})}{\sigma^2_{market}} \tag{2.13}$$

Hence, β equals the correlation between the returns of asset and market multiplied with the ratio of their volatilities. A correlation of zero means that the asset is completely independent of the market portfolio and has no market risk component, consequently the β is zero as well. A correlation of one on the other hand means that the asset fluctuates in exactly the same way as the market portfolio, a correlation of minus one means that the asset moves exactly opposite to the market.

Drawbacks of CAPM. The CAPM method is the most widely used way to determine the cost of equity. However, it has major flaws. First, it relies to a great extent on the knowledge of the market portfolio. The market portfolio is the portfolio that maximises Sharpe's risk-return ratio in the universe of all possible investment opportunities, including not only equity and debt markets, but also commodities and illiquid and not transparent markets such as real estate, private equity, art, or even wine. Nobody knows the market portfolio; therefore, we cannot use it as a reference for calculation purposes.

Second, all measures in the CAPM are prospective. The β and the risk premium relate to the future. The β of an asset is a notoriously unstable parameter. Historic and future β do not have to be identical. Even historical β depend strongly on the chosen observation period. This instability renders the determination of the cost of capital arbitrary. Often it is possible to achieve any cost of capital simply by choosing an adequate observation period for β and the market risk premium.

Third, β intentionally captures solely the non-diversifiable part of risk. The argument goes that the investor can diversify and is therefore not rewarded for diversifiable risks. However, very few investors would agree with this, and especially within the private equity universe, investors have large parts of their fortunes invested in one single company, effectively impeding a sufficient diversification. Furthermore, the company or asset specific risks have an impact on the business of the company. It is little consolation for a company and its investors facing financial distress that the risks causing this situation are diversifiable and therefore should not have been included in any valuation anyway. Diversifiable risks can have significant impact and should therefore be included in the valuation. This is particularly true for life sciences companies as we will see further down.

Market-Derived Capital Pricing Model

The market-derived capital pricing model (McNulty et al. 2002) tries to overcome the major flaws of CAPM. First, it does not rely on an unobservable risk factor like the market portfolio. Second, MCPM uses explicitly prospective and observable parameters, and avoids reliance on arbitrarily measured parameters like β. Third, and most importantly, MCPM accounts for all risks of a company, whether they are diversifiable or not, trying to reflect better the risk perception of managers and investors.

The fundamental idea of MCPM is that shareholders should at least earn the cost of debt. We assume that shareholders insure this rate of return with a put option. The cost of equity then equals the cost of debt plus the annualised cost of the put option. As a consequence of this definition, the cost of equity depends on the time horizon. The put option has the forward price of the share at the cost of debt as strike price.

$$r_E(T) = r_D + \frac{put\left(1, (1+r_D)^T, T\right)}{\dfrac{1}{r_D} - \dfrac{1}{r_D(1+r_D)^T}} \tag{2.14}$$

The put option uses 1 as the value of the underlying, the forward price $(1+r_D)^T$ as the strike, and T as maturity.

In many cases, MCPM yields intuitively better results than CAPM. MCPM however is not free of problems. First, the requirement that shareholders should be able to insure a return that is equivalent to the cost of debt lacks a stringent argumentation. Why should shareholders be able to avoid the downside while keeping all the upside? Second, the put option uses a prospective or implied volatility. This is only applicable if options are traded, i.e. when the company has already a respectable market capitalization. For smaller or even unlisted companies the prospective volatility is a very delicate measure. Unfortunately, the cost of equity is relatively sensitive to this parameter. Third, the cost of equity depends on the chosen time horizon. The authors calculate with a time horizon of five years, however, it is not clear how this is determined.

While being an interesting and in many cases plausible alternative to CAPM, MCPM is difficult to apply to small companies. Furthermore, the diversifiable risk is included in the cost of debt and the volatility, leading to some double counting and consequently to a too high cost of equity. As a remedy, one could use the risk free rate instead of the cost of debt.

Uncertainty

Cash flows are linked to two different natures of uncertainty. The first uncertainty concerns the accuracy of the estimate. We do not know in advance, how large a cash flow will be. Many factors like competition or regulation influence the potential size of the cash flow. This uncertainty can have a negative impact, if for instance a competitor launches a better product. But the uncertainty has also an upside; imagine that the product sells better than anticipated, or costs are lower than expected.

The second nature of uncertainty is the technical uncertainty. We do not know in advance, whether a cash flow will occur at all. The reason for uncertain cash flows is usually caused by failure risk, e.g. failure in clinical trials, and quantified by attrition rates. Attrition rates are purely value destructive. If a cash flow has only a probability p to happen, then you cannot count with the full cash flow, but only with its expectation, i.e. p times the cash flow. For valuation purposes, a cash flow must always be multiplied with its probability to happen. This requirement becomes clear when playing heads or tails. Assume you win one dollar with heads and you do not win anything with tails. Obviously, you will win half a dollar on average, which is exactly 50% of the dollar, i.e. the probability times the cash flow. However, if you play just once, you either win or you do not. But if playing already two times, you have 25% chance to win two dollars, 50% chance to win one dollar, and 25% to win nothing at all. You are already more likely to win the average sum. If playing ten times, you have a chance of more than 65% to win 4, 5, or 6 dollars, i.e. close to the average. The more often you play, the more likely you win exactly the average value of the game and the better your are placed to predict the average outcome of the game.

Technology projects like in life sciences have the same property. The more projects a company has, the more likely it will end up with the number predicted by the success rates. The figure displays how many projects will succeed (in percents), assuming a one product company (Biotech), a company with a pipeline of ten projects (MidPharma), and a company with a pipeline of 100 projects (Pharma). Each project has a success rate of

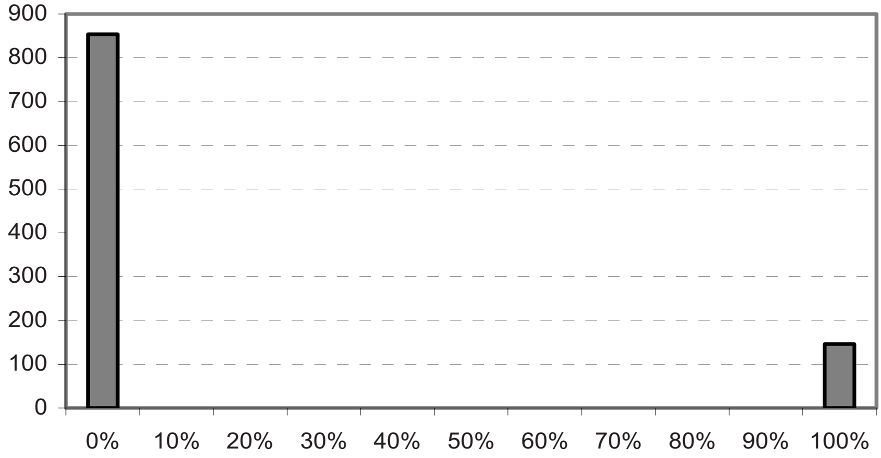

Fig. 2.2. 1,000 simulations of a one-project pipeline (Biotech)

15% to reach market. The outcome for the biotech company is binary, either complete success or complete failure. According to the success rate, the success scenarios make up 15% of all scenarios. In the case of one product the ultimate value must lie somewhere between the value of the good and the bad scenario. However, after learning the result (success or failure) we notice that the project jumps considerably in value. Either it looses all of its value in case of failure, or the value increases significantly, because no uncertainty about the outcome has to be factored in anymore.

For a mid-sized pharma company complete failure would mean that all ten projects fail. This probability is rather low $((100\%–15\%)^{10}=19.6\%)$. With a diversification of ten projects the risk of complete loss could already be lowered by an impressive 65%. Furthermore, the initially expected outcome of either one or two successful projects has a probability of 63%. We see that the diversification not only reduces the downside risk of complete failure, but also makes the outcome more predictable. The value difference between start and end of trials seems more manageable. The bulk part of scenarios will end up with one or two successes, and no scenario contains more than 5 successes.

Looking at the simulations of the pharma pipeline of 100 projects we see that in 83% between 10 and 20 projects succeed. On the other hand, it seems almost impossible that more than 30 projects will make it to the market. The actual numbers of the simulations displayed vary between 5 and 29 successful projects. So the risk of complete failure has been totally removed due to a broader diversification, although theoretically it is still

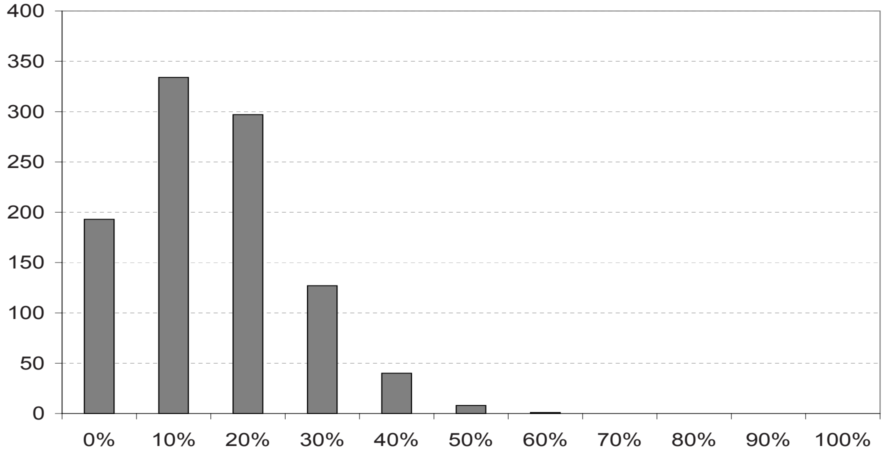

Fig. 2.3. 1,000 simulations of a ten projects pipeline (MidPharma)

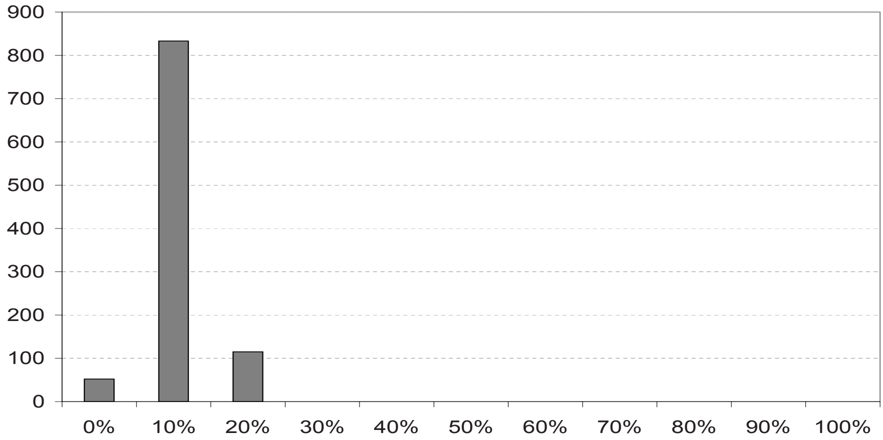

Fig. 2.4. 1,000 simulations of a 100 projects pipeline (Pharma)

possible that no project reaches the market (=(100%-15%)^{100}=0.00001%). But this would correspond to one simulation out of 10 mn simulations.

No company is perfectly diversified. In the contrary, most companies depend on the success of a few projects. In the extreme case we have a one-project company that will either be a success or a complete failure. The value of the company then extremely depends on the success of the project. If the project succeeds, the uncertainty is resolved and its negative effect can be excluded, i.e. we do not have to multiply the cash flows by the success rate linked to that uncertainty. This will lead to a jump in value in the order of *(1-p)/p*. Assume *V* to be the value after the uncertainty has been resolved. Consequently the value just before should be around *p times V*. The value increase is therefore:

$$\frac{V - pV}{pV} = \frac{1 - p}{p} \tag{2.15}$$

Consequently, the value jump is higher the smaller the success rate. If the project turns out to be a failure, the value drops to zero. The value of a company or a project corresponds to the average outcome. After negative results related to one important project, people blame valuation not to have predicted the value drop that has occurred meanwhile. Such statements are based on a misconception of valuation. A sound valuation does not predict winning or losing, but it quantifies the odds. Playing or investing many times, one realises the odds, just like in the casino, where in the long run the bank always wins, i.e. the bank realises the odds. Taking another look at the simulations we see that a pharmaceutical company plays the drug development game many times and comes close to winning exactly the odds. Investing in a pharmaceutical company is like placing bets on hundreds of products and we are sure that the one or other project will make it to the market. The risks of such an investment is relatively small because of the inherent diversification. Value jumps due to success and failure of projects offset each other. In a biotech company there are usually no projects that can compensate for the failure of the lead project. The payoff profile of a biotech investment is therefore much more risky. An investment in a biotech company therefore is more like placing a bet on one project. Investors want to be rewarded for this uncertainty when investing in a biotech company. They ask for a higher discount rate compared to investing in a pharma company as we will see.

Human beings feel uncomfortable with probabilities. People have a natural aversion against uncertainty and tend to even pay an amount just to avoid risk. The insurance industry makes its living thanks to this human trait. In order to demonstrate this let us play the following simulation.

Simulation of Risk Aversion

Mr. Human owns US$ 500,000. He has to play one of the three games listed below. In order to avoid playing the game, Mr. Human can pay a certain amount to Mr. Insurance beforehand and can keep the remainder of his fortune.

1. *Mr. Insurance tosses a coin (and it is supposed to be a fair coin with 50% probability for head and 50% probability for tail). If the result is head then Mr. Insurance completes Mr. Human's fortune to one million dollars. If it is tail, then Mr. Human gives his fortune to Mr. Insurance. So basically, they play for Mr. Human's fortune.*
2. *Mr. Insurance tosses 10 coins. For each head Mr. Human receives US$ 50,000. For the coins with tail Mr. Human gives US$ 50,000 to Mr. Insurance.*
3. *Mr. Insurance tosses 100 coins. For each head Mr. Human receives US$ 5,000. For the coins with tail Mr. Human gives US$ 5,000 to Mr. Insurance.*

How much is Mr. Human ready to pay to Mr. Insurance in order to avoid the game?

In gamble 1 Mr. Human faces a potential loss of US$ 500,000, but also a potential gain of US$ 500,000. From the initial fortune of US$ 500,000 he could go to US$ 0 or to US$ 1,000,000. Since each outcome has a probability of 50% the game is fair; on average Mr. Human will still have a fortune of US$ 500,000. Mr. Human, however – if he lives up to his name – will be more worried about the loss of half a million dollars than he would be pleased by the gain of the same amount. Doubling the fortune does not satisfy Mr. Human to the same degree as losing it disappoints him. He is therefore ready to pay something in order to avoid putting his fortune at risk. The amount Mr. Human is ready to pay depends on his personal attitude towards risk – his risk preference.

In gamble 2 Mr. Human only faces gradual losses or gains. Complete loss is rather unlikely with all coins showing tail ($=50\%^{10}=0.1\%$*). The outcome is not binary anymore. In 25% of all possible cases Mr. Human can even keep his US$ 500,000, because exactly 5 coins show head. He could even say that a loss of US$ 100,000 is still bearable, so he actually only cares about the 17% of all cases where he looses more than US$ 100,000. Clearly, Mr. Human thinks that gamble 2 is already less dangerous than gamble 1 and consequently he would spend less money to avoid the gamble.*

In gamble 3 finally, the chances to end up with less than US$ 400,000 are less than 2%, so very unlikely. While in gamble 1 Mr. Human was maybe ready to spend US$ 100,000 in order to keep at least parts of his fortune, in gamble 3 Mr. Human doesn't even face a noticeable risk to loose as much. If Mr. Human still wants to insure his fortune and avoid playing, then obviously to a much lower price than in gamble 1 and even than in gamble 2.

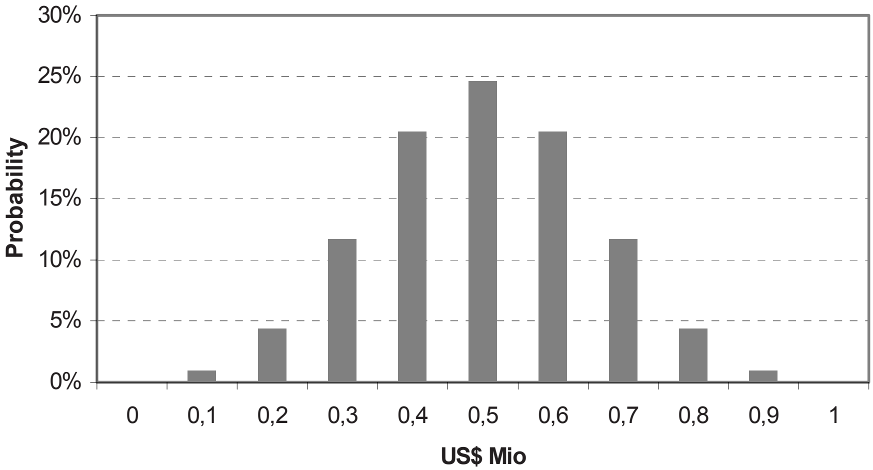

Fig. 2.5. Payoff profile of gamble 2

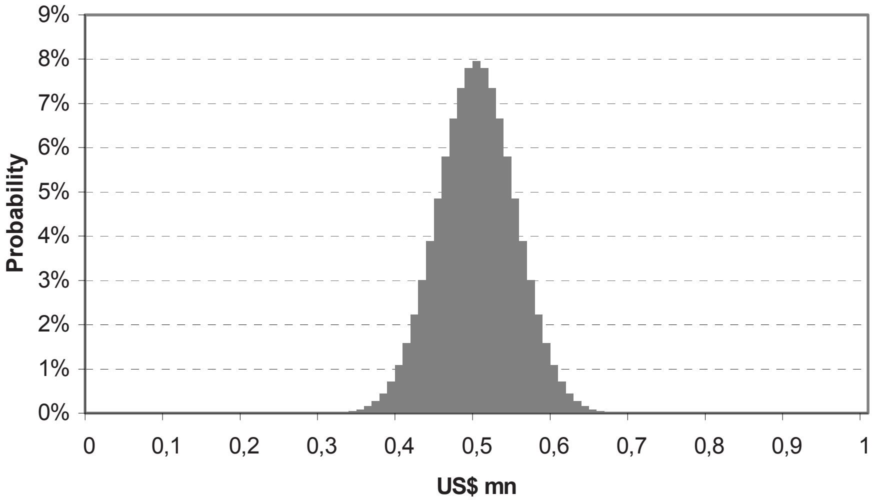

Fig. 2.6. Payoff profile for gamble 3

The three gambles offered by Mr. Insurance show that Mr. Human is sensitive to risk, and that the higher the risk, the less comfortable he feels, although the fair value remains the same (on average Mr. Human keeps his fortune in each gamble).

As a consequence, investors require or expect a higher rate of return for an investment that bears a lot of risk compared to a relatively safe investment. This translates directly into a higher discount rate. Imagine a business plan assumes to realise US$ 100 mn cash in 10 years; and for the sake of the example we do not question this assumption. If the investor will receive these US$ 100 mn with certainty after ten years, then he is ready to invest at nearly the risk free rate, let's say 5%. This would then mean that the venture is worth $100*(1+5\%)^{-10}$=US$ 61 mn, because if one lets US$ 61 mn accrete at 5% they become exactly US$ 100 mn after ten years. However, if the investor is not sure whether he will receive maybe only US$ 80 mn, but maybe also US$ 120 mn after ten years, i.e. there is some uncertainty linked to the investment, then the investor feels uncomfortable with the 5% discount rate. Finally he could put his money on a bank account and earn exactly 5% without running any risk. The investor therefore requires a higher rate of return for the money he puts into the venture. He therefore wants to start from a much lower initial value to have a higher return once achieving the US$ 100 mn. If we start at US$ 55 mn and expect them to become US$ 100 mn in ten years, then they have to accrete at a rate of 6.2%. Alternatively, if we discount the US$ 100 mn over ten years at a rate of 6.2% we get a present value of US$ 55 mn. The main conclusions out of this short example are first that the expected return and the discount rate correspond to each other and Second that more risk leads to a higher expected rate of return/higher discount rate and therefore to a lower initial value.

We can relate the investment opportunity directly to the preceding simulation about risk aversion. Imagine the investor owns US$ 61 mn and has the choice between putting it on a bank account and earning the risk free rate of 5% (playing it safely), or he invests it in the described venture that could turn out to generate US$ 80 mn or US$ 120 mn. Obviously the investor would opt for the first option, because the value of the two options is the same, but he has no downside risk on the bank account. But what if he could keep some of his US$ 61 mn and participate with remainder in the risky venture? The few millions that the investor can keep would correspond to the premium he would pay in order to avoid the investment. Or in other words, the amount of money that makes him indifferent between the two investments. If this amount equals US$ 6 mn, this means that he thinks investing US$ 55 mn in the risky venture corresponds to putting US$ 61 mn to the bank account. Or: if he can avoid the risky investment, he would only do it up to a premium of US$ 6 mn. But if the insurance is more expensive, then he prefers the risky investment. So, in the investor's eyes, the additional risk is worth a 1.2% premium on the discount rate.

As we have seen in the simulation diversification reduces the need of a risk premium in the rate of return. So, if an investor is diversified, then he basically does not have to claim a high rate of return. This is exactly the idea of CAPM. In CAPM we assume that all investors are well diversified. However, as we will see later, this assumption is not reflected in reality. First, most investors are far from being well diversified. And diversification is also a strange animal, because in periods of financial distress, when diversification is needed most dearly, all values tend to fall together, i.e. a previously assumed well-diversified portfolio suddenly behaves as if it were one risky asset. Second, diversification is a value added by the investor to his portfolio and the investor prefers harvesting the fruits of his work – the diversification – on his own, rather than letting others benefit from it. Because if he would apply a lower discount rate to his investments because of his diversification, the co-investors in the same venture profit from his relatively low discount rate by getting less diluted. As we will see in the discussion about discount rates in the life science industry we opt for a theory that takes account of the risk profile of the company rather than the investor's portfolio. Why should the investor when investing in biotech give away his premium he earned by building up a diversified portfolio? The same applies to big pharma when in-licensing a project. The hurdle rate, i.e. the discount rate, is usually higher for in-licensing project candidates than for in-house projects.

If we agree that mainly the risk profile of the company matters for the discount rate we come to the conclusion, that a start-up company certainly

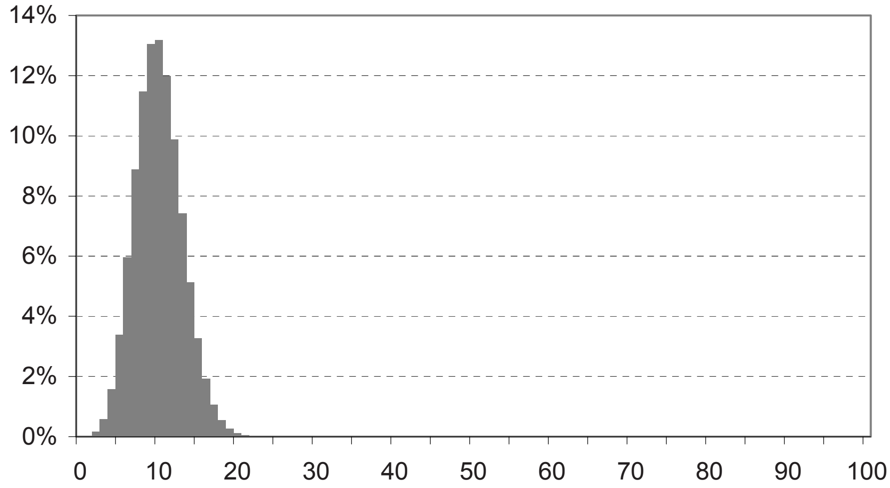

Fig. 2.7. Distribution of success with 100 projects

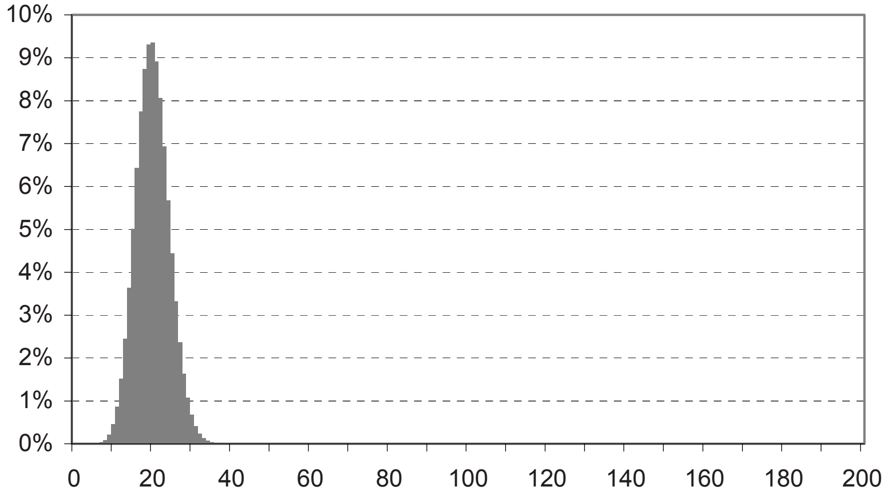

Fig. 2.8. Distribution of success with 200 projects

has to expect to be discounted at a much higher discount rate than a pharmaceutical company, similar to the above-displayed risk profiles of biotech, mid pharma and big pharma. Nevertheless, even for a pharmaceutical company it is impossible to be perfectly diversified, as show the two figures for companies with 100 and 200 projects. Although scientific uncertainty, as represented by success rates, is diversifiable, no company or investor will achieve a degree of diversification where the value is certainly realised. This would be perfect diversification. The 200 projects pipeline, the double of the 100 projects pipeline, still produces outcomes between 10 and 30 commercialised projects. It is not difficult to imagine that the value of 10 and 30 products differ substantially. Consequently, every investor wants to be rewarded for this risk despite of its diversifiable nature. CAPM does not account for this portion of risk, exactly because of its apparently-not-to-be-rewarded diversifiability.

Valuation Methods

Discounted Cash Flows Valuation

We have learned in the chapter on discounting that we can calculate the value of future cash flows back to today by discounting them. We then receive the present value of the cash flows. The discounted cash flow valuation is finally nothing else than netting the present value of all future cash flows. The result is the net present value (NPV) of the cash flows. The terms DCF and NPV are therefore equivalent. If we furthermore adjust the cash flows with the probability (risk) that they occur, we get the risk adjusted net present value (rNPV). In the following we will use the term DCF for the method to calculate the risk adjusted net present value.

The rNPV is calculated with the following formula:

$$rNPV = -I_o + \sum_{t=1}^{T} \frac{rCF_t}{(1+r)^t} \tag{2.16}$$

I_0 = Investment into the project at time 0 (=CF_0)
rCF_t = Risk adjusted cash flow at time t
r = Discount rate
T = Endpoint of the project (if today is t = 0, T = duration of the project)

In order to illustrate how to use the DCF valuation, we now calculate the rNVP of the project *Supersolution*. The project is still in development for one year. The investment for the development is due today and amount to \$ 50,000. After one year, we have a 50% chance that the project passes the development phase and will finally be launched. The launch, including marketing and production expenses, amounts to \$ 500,000. After the launch, the project generates the following revenues: Year 2 \$ 100,000, year 3 \$ 200,000, year 4 \$ 300,000, year 5 \$ 200,000, and in the final year 6 \$ 100,000. Once the project is on the market, the expenses are 10% of the revenues. The company developing the project applies a discount rate of 15% to value all its investments:

Table 2.2. rNPV calculation of project Supersolution

Year	0	1	2	3	4	5	6
Expenses	(\$ 50,000)	(\$ 500,000)	(\$ 10,000)	(\$ 20,000)	(\$ 30,000)	(\$ 20,000)	(\$ 10,000)
Revenues			\$ 100,000	\$ 200,000	\$ 300,000	\$ 200,000	\$ 100,000
Net CF	(\$ 50,000)	(\$ 500,000)	\$ 90,000	\$ 180,000	\$ 270,000	\$ 180,000	\$ 90,000
Probability	100%	50%	50%	50%	50%	50%	50%
Risk adjusted CF	(\$ 50,000)	(\$ 250,000)	\$ 45,000	\$ 90,000	\$ 135,000	\$ 90,000	\$ 45,000
Discount	100%	87%	76%	66%	57%	50%	43%
rpCF	(\$ 50,000)	(\$ 217,391)	\$ 34,026	\$ 59,176	\$ 77,187	\$ 44,746	\$ 19,455
rNPV	(\$ 32,801)						

To calculate the risk adjusted net present value of the project we:

1. List the expenses (negative cash flows) for each year.
2. List the revenues (positive cash flows) for each year.
3. Net the cash flows.
4. Risk adjust the discounted cash flows with the success rate.
5. Discount the cash flows by multiplying the risk adjusted net cash flows with the discount factor. The result is the risk adjusted present cash flows (rpCF).
6. Sum the risk adjusted present values of all cash flows. The result is the rNPV, the risk adjusted net present value of all cash flows.

The discount factor is calculated in the following way:

$$DF_t = \frac{1}{(1+r)^t} \tag{2.17}$$

The risk adjusted net present value of the project is ($ 32,801). The company should therefore not invest in the project. In general, projects that yield a positive value should be continued, those with a negative value abandoned. We will discuss in the chapter on portfolio management how to decide which projects to halt and which not.

Internal rate of return (IRR). The project above yields a negative value if we value it with a discount rate of 15%. The rate of 15% is the hurdle rate the company applies to decide if projects should be continued or abandoned. Projects that give a positive rNPV when discounted with 15% have a return that is higher than the demanded 15%. These projects pass the hurdle rate and are continued. We now calculate for our example the discount rate that yields exactly a project value of zero. The resulting discount rate is the internal rate of return of the project.

$$rNPV = -I_0 + \sum_{t=0}^{T} \frac{rCF_t}{(1+IRR)^t} = 0 \tag{2.18}$$

The IRR of our project is 10.13%. At this discount rate the project yields a value of zero. The figure below displays the relation between the discount rate and the rNPV:

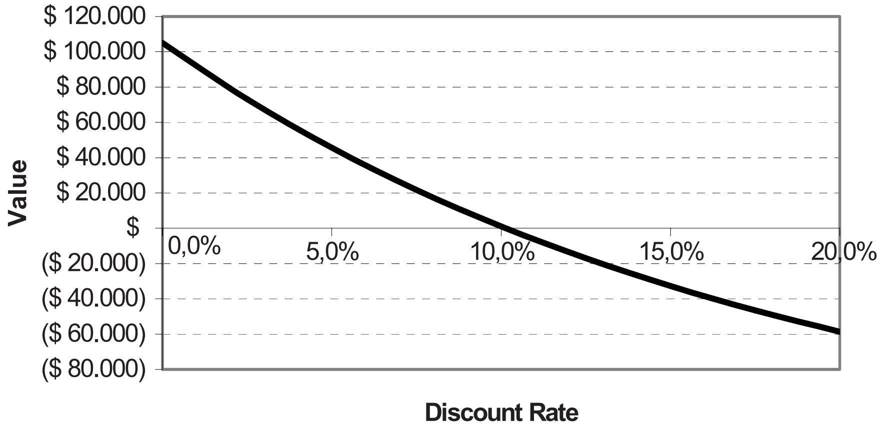

Fig. 2.9. Influence of discount rate on project value

The internal rate of return is therefore the exact calculation of the return of a project. The money invested into the project above therefore accretes with 10.13%. As long as the IRR is below the discount rate, the project has a negative value.

Decision Tree

Some projects have not a predefined development path or strategy. At some points in the life cycle of the project the original plan might be altered, depending on an event like own trial results or the entry of a new competitor. Some authors already consider a risk adjusted net present value calculation as a decision tree, because at the end of each trial we decide whether to continue or to abandon the project, as displayed in the figure.

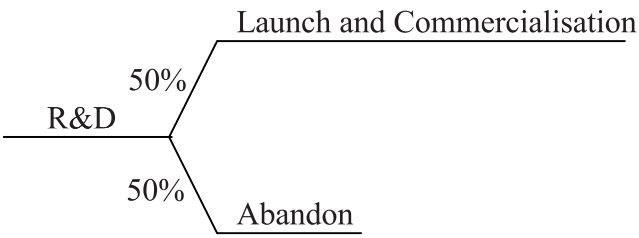

Fig. 2.10. Plan of project Supersolution as decision tree

In the rNPV calculation we did not have to bother too much about the abandoned branch of the decision tree, because no cash flows were linked to that branch. We could therefore focus on the successful branch, but adjusting all cash flows for launch and commercialisation with the probability of that branch.

In reality, project plans can involve more decision points. In project Supersolution it is for instance possible, that the R&D phase can produce not only two possible results like success or failure, but three: Either the product does not work, or it does work, but not to the extent as originally expected, or it works even better than expected. In the second case the company is required to add another R&D phase of one year at US$ 50,000 to improve the product such that it then meets the original expectations. This corresponds to the following figure.

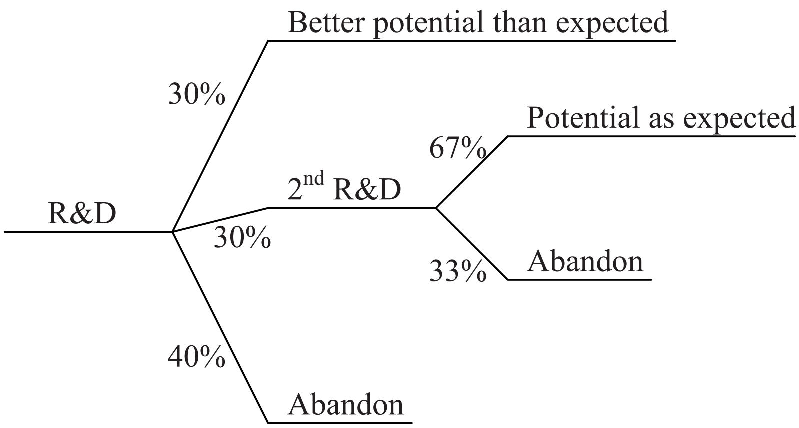

Fig. 2.11. More detailed decision tree for project Supersolution

We see that the overall success rate of the project remains 50% (30%+30%*67%=50%). When constructing the decision tree it is important that at each decision point the probabilities of all subsequent options add up to 100%. At the first decision point the project is abandoned with a probability of 40%, further developed with 30% and launched with 30%, adding up to 100%. At the second decision point we abandon the project with 33% and launch it with the remaining 67%.

When calculating the value of the project we first have to calculate the value for each end-branch. These are the immediately abandoned scenario, the abandoned scenario after the 2nd R&D phase, the immediate launch, and the launch after the 2nd R&D phase. The abandoned scenarios have no cash flows anymore and their values are US$ 0. The immediate launch scenario assumes launch costs of US$ 500,000 and then sales that are 33% higher than originally expected. The launch scenario after the 2nd R&D phase then corresponds to the original launch scenario.

Year	1	2	3	4	5	6
Expenses	($ 500.000)	($ 13.300)	($ 26.700)	($ 40.000)	($ 26.700)	($ 13.300)
Revenues		$ 133.000	$ 267.000	$ 400.000	$ 267.000	$ 133.000
Net CF	($ 500.000)	$ 119.700	$ 240.300	$ 360.000	$ 240.300	$ 119.700
Discount	100%	87%	76%	66%	57%	50%
pCF	($ 500.000)	$ 104.087	$ 181.701	$ 236.706	$ 137.392	$ 59.512
NPV	$ 219.398					
Discount rate	15%					

Fig. 2.12. NPV of immediate launch scenario

Year	2	3	4	5	6	7
Expenses	($ 500.000)	($ 10.000)	($ 20.000)	($ 30.000)	($ 20.000)	($ 10.000)
Revenues		$ 100.000	$ 200.000	$ 300.000	$ 200.000	$ 100.000
Net CF	($ 500.000)	$ 90.000	$ 180.000	$ 270.000	$ 180.000	$ 90.000
Discount	100%	87%	76%	66%	57%	50%
pCF	($ 500.000)	$ 78.261	$ 136.106	$ 177.529	$ 102.916	$ 44.746
NPV	$ 39.558					
Discount rate	15%					

Fig. 2.13. NPV of launch scenario after

With this we get the provisional values at the end branches of the decision tree as displayed in the figure.

From here we must first calculate the middle option, i.e. when a second R&D phase is necessary. The outcome of this second R&D phase is either US$ 39,558 with a probability of 67% or US$ 0 with probability of 33%, on average US$ 26,504. Since this is only the average value after the second R&D phase we have to discount this value for one year and subtract the R&D costs from it to get the value of the middle branch.

With this we have all necessary values for the decision point right after the first R&D phase.

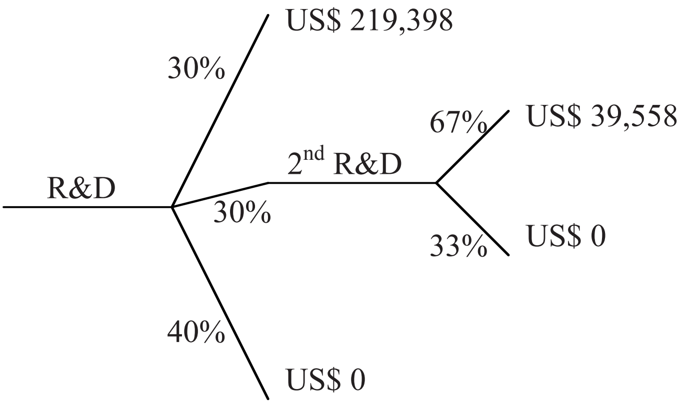

Fig. 2.14. Provisional values for project Supersolution

Year	1	2
Expenses	($ 50.000)	
Value		$ 39.558
Net CF	($ 50.000)	$ 39.558
Probability	100%	67%
rCF	($ 50.000)	$ 26.504
Discount	100%	87%
pCF	($ 50.000)	$ 23.047
NPV	($ 26.953)	

Fig. 2.15. Calculation of middle branch

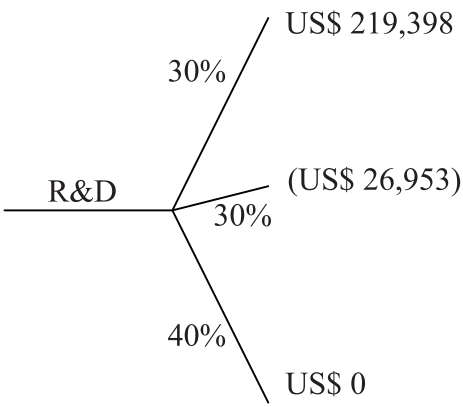

Fig. 2.16. Continuation values after first R&D phase

If the first R&D phase reveals that a second phase is necessary, then the company abandons the project, because it would have to invest US$ 50,000 in a project with negative rNPV at that time. We therefore assume that reaching the middle branch is equivalent to abandoning the project, but this time not because of technical reasons, but because of economic considerations.

The final value is then calculated in a similar way like the value of the middle branch. The first R&D phase leads after one year to an average value of US$ 65,819 (=30%*US$ 219,398). This has to be discounted by for one year and reduced by the R&D costs, giving us a final value of US$ 7,234.

If we would not include the option to abandon the project in the middle branch, so if we calculated with the actual value of (US$ 26,953), we would receive a final value of US$ 203. The assumption that the company abandons negative projects increased the project value therefore by US$ 7,031. We will see that real options valuation focuses particularly on this value added by management.

Decision tree is a very useful method that encourages thinking through a project plan in more detail. It has however the problem that mostly the used probabilities are not known to the degree of detail that is required in the decision tree. It is common to know the success rate of projects, but we find rarely data quantifying the probability of success after the first or only after the second R&D phase. The decision tree method is therefore a prone to subjective assumptions.

Real Options

General Aspects

DCF convinces by its simplicity. Nevertheless, this simplicity is only achieved at the cost of some strong hypothesis. In the DCF method we assume that the market does not change, i.e. once we have fixed our estimate of the peak sales, we do not question this number anymore. One can argue that the estimated peak sales correspond to the average of what we can expect. Whatever happens, there is about the same probability that the actual outcome lies above or below the estimated number. On average, the sales equal what we have predicted. So why not just calculate with the estimated peak sales?

This reasoning bears two dangers. First by assuming one scenario, i.e. the average, we run the risk to take this number for granted. This can lead to over-reliance on this scenario that is still only a vague estimate. Second, the reduction of the future to just one scenario avoids considering alternatives in case the estimate turns out to be wrong or must be adjusted due to new information. This would correspond to a static management unable to react to a changed marketplace. Of course, this does not mean that managers who use DCF just twiddle their thumbs. Even with DCF, you can revaluate the project once some value drivers change. This is common practice and does not depend on the valuation method. However, DCF does not take into account this practice. DCF values the project as if the management takes a one-time go or no go decision. It would be reasonable to assume that the company alters its plan if one critical measure deteriorates or maybe improves way above expectations. The company has various options at hand: it can scale down the project, put it on hold, halt it completely, or in the good case accelerate or expand it. The management constantly decides what to do in order to increase value. In some scenarios, although not in the most probable ones, management takes actions that differ from the initial project plan. DCF does not consider this managerial flexibility.

The real options approach takes into account that some decisions in a project's or company's life can or even must occur at a later stage depending on future market conditions. These can be fundamentally different from current market conditions; therefore, future decisions can only be anticipated to a certain degree. Consequently, management has to maintain flexibility to react to changes. This flexibility is used either to increase profits or to avoid losses. Either way it generates value. If a project includes such options where the management can influence the ongoing of the project, this should flow into the valuation process. Flexibility is always used to increase the value of a project or company, therefore a valuation method that captures this flexibility returns a superior value than one that does not. This value creation is not captured by DCF valuation.

The novelty of real options valuation compared to DCF is that some future decisions about the ongoing of the project or company are conditioned on the respective market conditions and not vaguely anticipated. In real options theory the development of a project or company is depending on the changing market conditions, while in DCF the course of the project is predefined, no matter what happens.

In order to correctly model the decisions, it is necessary to model the conditions, or parameters, they depend on. For this, real options theory makes use of the extensive research in the field of financial options that deal with the same issues. The market fluctuates and it is not sure from the beginning what payoff a holder of a derivative will incur at expiry.

While in financial derivatives the different ways to execute are predefined, management has to recognise the different ways how to adapt the company's business to the changed environment. Finally, it does not suffice to recognise that the company can still adapt its decisions at a later moment in time. We also need to quantify this flexibility. Some projects might have several probable alternatives and consequently, flexibility is important and represents a considerable part of its value. Other projects are unlikely to be redirected, mostly clearly profitable projects; their inherent flexibility is almost worthless. We must model the different ways to react to a changed market environment. These different reaction types define the different embedded real options in a project or a company. The existing literature provides six categories of real options based upon the types of managerial flexibility:

- Option to defer
- Option to expand or contract
- Option to abandon or license
- Option to switch
- Option to stage investments
- Option to grow

Option to defer. An investment cannot be recuperated anymore once it is triggered. Hence it should only be triggered if the subsequent revenues are expected to be higher. The option to defer is finally a trade-off between risk and return. Investing immediately gives the company earlier, less discounted, revenues. Waiting might resolve some uncertainty about the market. For some projects it might be advisable to wait although they have a positive NPV due to their risk profile that might better be cleared prior to investing. An option to defer is specially valuable if the project outcome is highly uncertain and the investment irreversible. But waiting involves also the risk of loosing the first mover advantage. Therefore, in a competitive environment the option to defer might be worth less. Costs to keep the option alive, e.g. renting costs, have a negative impact on its value as well.

Option to expand or contract. Options to expand or contract incorporate the possibility to change the existing scale of the project according to the market movements. A company may build production facilities in excess of the expected demand. The company may incur additional revenues in times of high demand. On the other hand the company should engineer their plants in a way that they can be partially shut down in periods of low demand, reducing the fix costs. Testing a new market with a scout product and expanding if the test run proves successful is another example of this option type.

Option to abandon or license. If a project fails to meet the expectations and is even deficitary, the company's management should consider abandoning the project, possibly avoiding losses. The technology of the project might be licensed and the infrastructure sold. The company can recuperate a salvation value. The more reversible the investments, the higher the salvation value.

Option to switch. A company can reconsider its production location and switch to a cheaper place. It is also imaginable that a production plant will be modified in order to fabricate another product. In these cases the company switches either the production costs or the sales revenues.

Option to stage investments. Some projects require a staged investment. The firm resolves in every subsequent stage further uncertainties. Based on the learned facts, the project is revaluated and consequently continued or abandoned after each phase. These staged investments are modeled as compound options. After each period the company has the option to continue or not. Continuing corresponds to the acquisition of a new option. A staged investment process is common to R&D projects and to start-ups.

Option to grow. Growth options are the most common and most widely cited real options. A company is supposed to have the possibility to expand its activities to other countries, to other clients, or to other products, if the initial activity proves to be successful. In a first stage the company resolves uncertainty about the market conditions, such as product acceptance, demand, and price politics. Once these doubts are removed, the company decides whether it is worth to expand – to grow – or not. Finally growth options are nothing else than staged investments. But normally growth options are considered to be just a one-stage option, while R&D real options are multi-stage options.

As we will see later, real options valuation in life sciences mainly requires the option to abandon a project once it is not profitable anymore. Other options, e.g. options to grow or options to switch, are useful in certain instances but not for everyday valuation.

We have already mentioned that it is the duty of the management to recognize the multiple options that the company holds in its hands and react to them. Different strategies offer different options. Real options valuation must then be used to compare the different strategies. As we see later, real options are not only a strategic tool for value aspects, but they also provide important information about risk and liquidity management.

Nevertheless, there are also some dangers attached to the real options approach. So-called options and later opportunities should not be misused to justify any kind of investments. Real options still demand a thorough valuation process; they can be worth less than the initial investment to acquire the option. Furthermore, projects tend to create their proper dynamics and might be difficult to abandon. It is not sufficient to change the valuation method from DCF to real options; it is necessary to adapt the entire decision taking process to this new method. Real options assume that the management takes a decision in the future based on the best available information according to rational decision criteria. The valuation loses its justification if the management does not stick to this practice anymore. Eventually real options valuation is more a strategic than a valuation tool; it is a new way of management.

We have mentioned above that the options – or the flexibility linked to them – represent a value that does not go into the DCF valuation. Usually real options valuation returns a higher value than DCF. This does not mean that the project is suddenly worth more if valued with real options. The project is still the same. And even when the management uses DCF, it revaluates a project periodically and adjusts its hypothesis. But the DCF valuation does not take this periodical revaluation into account; a revaluation that might trigger new investments or the abandonment of the project. DCF simply omits an aspect of the project life. In some cases this leads to a missed opportunity. We discuss this aspect in further detail in the following chapters. Finally, we can oppose DCF and real options valuation in the following manner:

Table 2.3. Comparison of DCF and real options

	Advantages	Disadvantages
DCF	Easy to implement and to understand Standard in all sectors of the economy	Misses the value of flexibility and market uncertainty Not suitable for risk management
Real Options	Captures market uncertainty and the management's ability to react Suitable for risk management Improves strategic thinking if properly understood	Relies on more hypothesis and requires more data Technical

Resolution Methods

Financial mathematics offers four different methods to value an option:

1. Formula (closed form solution)
2. Trees
3. Simulations
4. Finite differences

All methods yield the same result, but the ways to get there are different. We discuss the methods with focus on their applicability to the valuation of real options. Some methods have been developed for special purposes and do not necessarily suit the requirements of real options valuation. First, the assumptions of real options are relatively modest compared to financial options. Second, the users of real options valuation most often do not have the required mathematical knowledge to apply the approach in practice. The complexity of the valuation method should therefore be as simple as possible, but not any simpler. No rocket science is required. Above all, the method should show the particularities that are considered in real options valuation and should not hide them in complicated algorithms. The decision takers need an efficient, explanatory and understandable tool, not a scientific high-end black box. The managers must be able to defend and explain their decisions without a long monologue about assumptions and technicalities. The valuations should support them in the communication of value and risk of a certain investment. We will now investigate the four valuation techniques with these thoughts in mind.

Formulae. Formulae are well suited for the valuation of simple call and put options determined by five input parameters (underlying, strike, maturity, interest rate, and volatility). Unfortunately, it is virtually impossible to capture the complexity of real options in project valuation with a closed form formula. The complexity quickly surmounts the mathematical knowledge of most users and ends up being a black box. Finally, it is not helpful to use a formula, as the real option framework should not only serve to generate a number, but also as a managerial instrument to visualise possible scenarios and the impact of revaluations and decisions. We will further discuss the applicability of formulae to real options in the chapter about the differences between real options and financial options.

Trees. Trees are a simplistic model of future market movements and are broadly used to value financial options. Binomial trees subdivide the time to maturity in small time steps and assume that in each time step the market, or in what we will see later, the peak sales, can go both up or down, each scenario with a certain probability.

Binomial trees are best spanned in a way that after a step up and a step down the market is in the same state again. We call this type of tree a recombinant binomial tree. Usually the tree is spanned until the time of option expiry, the time steps can be chosen to satisfy the accuracy requirements, yearly, quarterly, monthly, or even daily. Normally, the smaller the time steps, the more accurate the final value. The option value is obtained by calculating the tree back from the final leaves to the root. At expiry, the option value is clear if the market state is known, and the tree precisely indicates the market state for every leave. After that, we place ourselves in a node one time step before expiry. From there, we have to deal with a

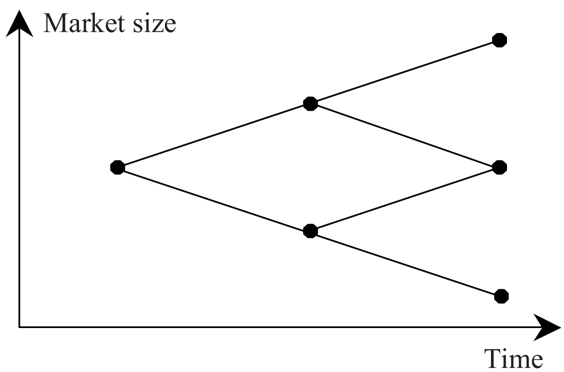

Fig. 2.17. Recombinant binomial tree

simplified situation: We have just two possibilities, whereof we know the probability. Being able to deal with such simplified situations, we can value the option for each possible situation at every time step before expiry. Working back the tree time step by time step, we finally reach the root node of the tree. By subdividing the initial problem into clearly arranged situations, we can calculate a value for the option.

Trees are easy to understand. They visualize what can happen to the market. While a formula yields a value in a magic way, the link to the value drivers is missing. A tree allows comprehending how and when a value driver impacts the value. Everyone resolving a tree has no problems to say how the final value is built up.

Simulations. Simulations follow an even simpler concept than trees. We determine the future beforehand, e.g. we assume certain peak sales numbers, and then attribute a value to the option given that future. This is repeated many times with an algorithm that randomly chooses the outcome. The simulated scenarios must on average correspond to the assumed probability distribution. Once all scenarios are simulated, there is no uncertainty anymore. Every manager can tell how much an investment is worth if he already knows the future. The simulations return not only an average value of the option; they also give a good impression about the uncertainty of the value and are therefore an excellent risk measurement tool.

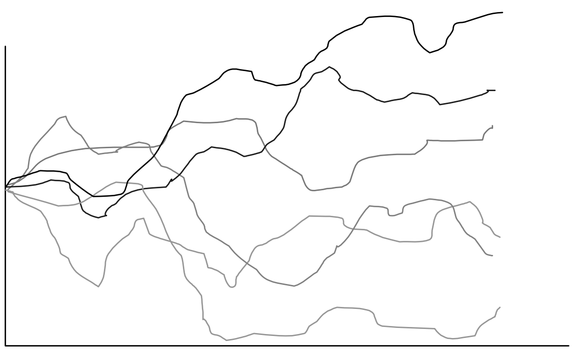

Fig. 2.18. Simulation paths

Simulation techniques can very well deal with more complicated assumptions, but struggle with some special option structures called path-dependency. Nevertheless, simulations enjoy great popularity in the financial world and are well suited for real options valuation. Unfortunately, simulations require programming expertise and are thus often too complex. We will give some guidelines on how to use simulations in a separate chapter on simulations.

Finite differences. Option valuation has first been approached using a mathematical concept called partial differential equations (PDEs). PDEs are a very flexible concept that can deal with many additional hypotheses. They however only display characteristics and properties of the option value, e.g. how it behaves by modifying some parameters, but do not provide the value. For simple hypothesis it is possible, to derive the option value out of a PDE; most of the times however this is impossible. Similar to binomial trees, finite differences subdivide the problem into small concise subproblems. While it is impossible to predict the value development on a large scale, finite differences use PDEs to forecast the value development on a small scale. Finite differences are widely used in engineering, biology, or even meteorology; they are extremely powerful to make forecasts, but require a high degree of proficiency with mathematical concepts and programming.

Table 2.4. Comparison of resolution methods

Method	Advantages	Disadvantages
Formula	Easy to use	Calculation process not visible
	Fast	Only for simple option structures
	Sensitivities	Simple assumptions
Trees	Easy to understand	Rigid
	Visualisation	
	can deal with more complicated options	
Simulations	Easy to understand	Time consuming
	Visualisation	Problems with path dependency
	Can deal with more complicated assumptions	
Finite differences	Can deal with more complicated assumptions	Calculation process not visible, hard to understand
	Can deal with more complicated options	Technically demanding

For real options valuation of biopharmaceutical projects trees are most suitable. A simple formula cannot cope with the project structure. We will also discuss simulations but omit finite differences because of their complexity.

Binomial Tree Valuation in Real Options

We will show in the following chapters in detail how to apply binomial trees to real options valuation. It is essential that the reader understand how to construct the tree and how to use it in the valuation. It will be the basis in all subsequent chapters dealing with real options. The application of the Black-Scholes formula to real options valuation will be discussed in a separate chapter.

Using a tree, we model the market development, i.e. possible changes of the sales revenues. While we know the market state today, we are already uncertain about the market state after one time step, let's say next month.

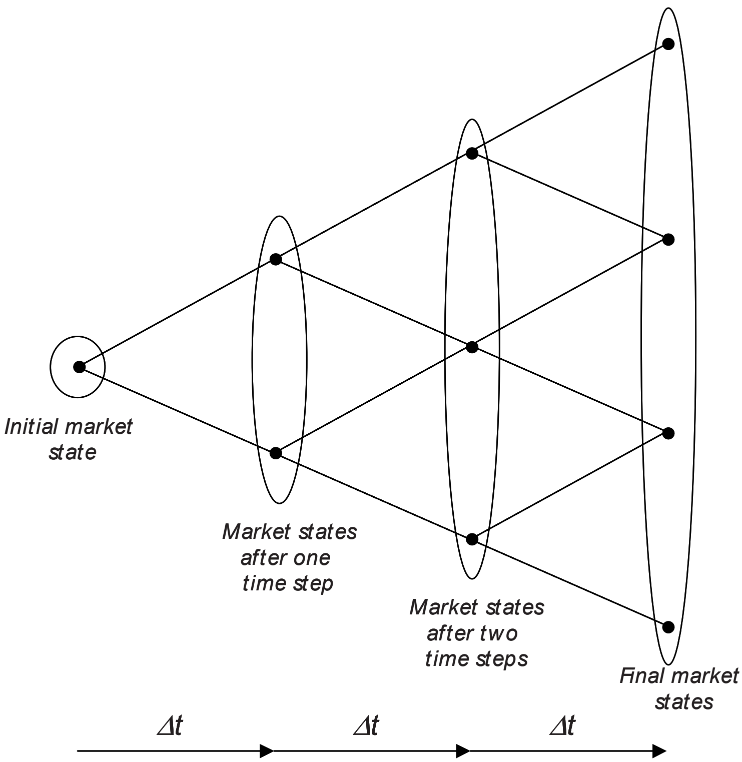

Fig. 2.19. Binomial tree

The market can either improve or deteriorate. The up- or down-steps of the tree reflect this. The further we go into the future, the less certain we are about the forecast. The tree therefore widens the farther we move in time. Each node represents another market state for the corresponding time. The end nodes represent market states at the final time point of the tree.

In real options valuation we model the market up to the last decision point of the project we want to value. We then place ourselves in each of the end nodes, i.e. at the time of the last decision and assume a market that corresponds to the node. The market state in the end node gives us all future cash flows, revenues and expenses. This situation is no different than if we just have to decide without any further uncertainty and can be calculated using standard DCF valuation for all future cash flows of the project after the last node of the tree. In some end nodes the market state might be so bad that the DCF value is negative, i.e. the expected revenues do not provide enough compensation for the necessary investments. In these states, we abandon the project. Consequently, we put the value of the project to zero by abandoning it, because there are no future cash flows anymore. Putting the value to zero instead of keeping the negative NPV corresponds to the option to abandon. In these states, the project value is increased from a negative value to zero, while in the other states where the project is profitable, the values are kept the same. The average value of all scenarios is therefore higher than without putting the negative values to zero. This difference in value is directly linked to the option to abandon, which avoids losses and is the source for the different values in DCF and real options valuation.

In a next step we place ourselves in the nodes one time step before the nodes we have just solved. The uncertainty in these states is not too complex. There are just two possibilities, either we reach after one time step the upper state or the lower state, in our case the upper or lower end node. For both scenarios we know the probability and the value of the project in these states – we have just calculated them. We can therefore take the expectation of the project values, discount it and account for the cash flows that are due in this very time step. Assessing the value for each node at that time step, we have already calculated the values for the final two layers of the tree. Applying the same procedure to each previous layer we can work back the tree to its root. Finally, the value of the project corresponds to the value calculated for the root node, because this is exactly the time and market state of today.

We will now explain each step of the tree valuation outlined above in detail and illustrate it with an example.

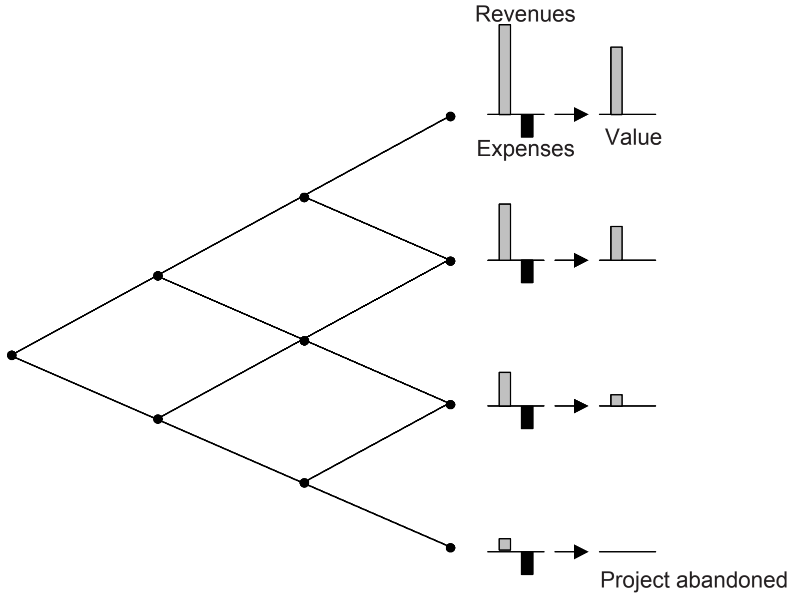

Fig. 2.20. Calculation of end nodes

Step 1: Determine the project parameters, i.e. the underlying S, growth rate μ, volatility σ, the time step size Δt, the expenditures, the success rates, and the duration of the project. Before constructing the tree, we have to fix the input parameters of the project for the valuation. Not all are already relevant for the tree; but it is better to define the parameters before starting to set up the Excel sheet.

We first define the underlying of the binomial tree, *S*. In most cases the underlying of the tree is the peak sales or the market state. Once we have the peak sales, we estimate their growth rate and their uncertainty, i.e. their volatility. If we know all characteristics of a project, i.e. all test results of the development; we can more or less accurately determine the peak sales at the time of launch. Before, i.e. during development, many factors have an impact on the estimated peak sales. New knowledge on application, safety and efficacy, a change in the competitive environment, e.g. another company launches a similar product, or an economic downturn, influence the estimate of the project's peak sales *S*. We do not know how all these risk factors will evolve, but we assume that first, the general market for the medication is going to grow at the growth rate μ and second, the volatility of the peak sales estimate *S* is σ. The volatility is a measure of uncertainty, i.e. the exactness of our peak sales estimate today. The more uncertain the peak sales estimate today is, the higher is the volatility of the estimate. We will further discuss volatility in the chapter on life sciences valuation. Finally, we determine the project related parameters, i.e. the duration of the different phases, the success rates and costs of the phases.

In our example, we assume that a company develops a product. The product first has to pass two development phases, where it is tested on feasibility and other concerns like compatibility with other products, compliance with regulation or safety. We estimate the peak sales to be $ 150,000. The volatility of our estimate is 30%. The project is at the start of the first development phase that lasts 2 years. We now decide whether to go ahead or not, depending on the project value. After two years, the project will finish phase 1 testing with a success rate of 75%. Again, depending on the outcome of phase 1 testing and the market conditions at that time, we will perform another valuation; we will only start phase 2 testing if the project has a positive value. Phase 2 testing lasts 1 year and will be successfully completed in 80% of the cases. The costs for phase 1 and phase 2 testing are $ 25,000, and $ 30,000, respectively. If the project passes phase 2 testing successfully, we will value the project again. Depending on the valuation, we will launch the project. The costs to launch the project are $ 150,000 and are due at the beginning of the fourth year. Our company discounts the project at a rate of 15%. We are looking at 5 years of sales. The operating margin is 50%.

Table 2.5. Project characteristics

Estimated peak sales	$ 150,000
Volatility	30%
Margin	50%
Discount rate	15%
Time on market	5 years
Phase 1 testing	
Duration	2 years
Success rate	75%
Costs	$ 25,000
Phase 2 testing	1 year
Success rate	80%
Costs	$ 30,000
Launch costs	$ 150,000

Step 2: Spanning the tree: With the assumptions made in step 1 we can proceed to spanning the tree. The underlying, in our case the peak sales estimate, move either up or down during the next time step Δt. But on average the estimate move to $Se^{\mu \Delta t}$ (or to $S(1+\mu)^{\Delta t}$ if μ is discretely compounded). As another condition we want the tree to recombine, i.e. the market gets to the same state after a down- and then an up step as after first an up- and then a down-step.

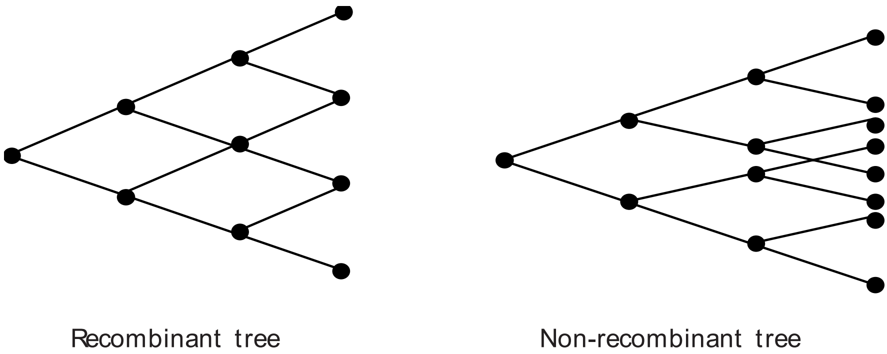

Fig. 2.21. Recombinant versus non-recombinant tree

As we see in the figure, this feature reduces the complexity of our tree considerably. The size of the up and down step is determined by the volatility and by the time interval. The higher the volatility is, the larger is the angle between the branches. The formulae are displayed in the box below. If we design the tree in the way shown in the box, all the conditions are fulfilled. The average value after the time step Δt is $pS_u + (1-p)S_d = Se^{\mu\Delta t}$. Due to the condition $d=1/u$, the tree is recombinant and the uncertainty σ is correctly implemented in the tree. Therefore, we can build a tree with these relationships from now until the last decision point.

The choice of the time step influences the accuracy of the valuation. The shorter the time step, the more accurate the valuation, but the more fastidious the calculation. For real option purposes time steps of one, three or six months, in some cases even of one year are sufficient.

$$\text{step up: } u = e^{\sigma\sqrt{\Delta t}} \tag{2.19}$$

$$\text{step down: } d = e^{-\sigma\sqrt{\Delta t}} \tag{2.20}$$

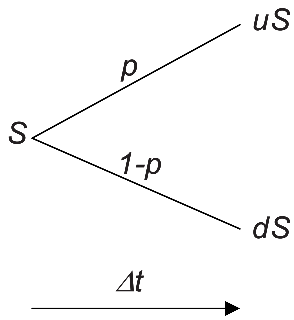

Fig. 2.22. Step up and step down

$$\text{probability up: } p = \frac{e^{\mu \Delta t} - d}{u - d} \tag{2.21}$$

$$\text{probability down: } 1 - p \tag{2.22}$$

We now span the tree for our example. For simplicity, we use time steps of one year. The estimated peak sales today are \$ 150’000, the growth rate is assumed to be zero, and the volatility of the estimate is 30%. Using the formula of the box below, we now calculate the step size for each year.

$$u = e^{\sigma\sqrt{\Delta t}} = e^{30\%\sqrt{1}} = 1.35$$

$$d = \frac{1}{u} = \frac{1}{1.35} = 0.74$$

$$p = \frac{e^{\mu \Delta t} - d}{u - d} = \frac{e^{0 \cdot 1} - 0.74}{1.35 - 0.74} = 43\%$$

In year two we expect the peak sales estimate to change with the probability p to 1.35 times the initial sales estimate of \$ 150,000, resulting in \$ 202,479. With a probability of $1–p$, the estimate goes down by 0.74 to \$ 111,123. The probabilities for the step up, p, and down, $1–p$, are again calculated according to the formula in the box. We receive for p 43%, and for $1–p$ 57%. This means that in every year we expect our sales estimate to increase by the factor 1.35 with a probability of 43%, and to decrease by the factor 0.74 with a probability of 57%. As we have already outlined, the changes in the peak sales estimate derive from the changes of the market, e.g. new information about the project in terms of applications, safety, or customer acceptance. This flexibility of the estimated sales figure is an important difference to

DCF, where we assume that the estimate once fixed, stays constant during the entire project life. Obviously, it is more realistic to assume that an estimate of future sales figures is subject to change and fluctuation. We later see how this is applied to life sciences projects.

The figure below displays the tree spanned to the market entry of the product. The figures in the top of the boxes are the estimated peak sales, and below the probabilities to reach that state.

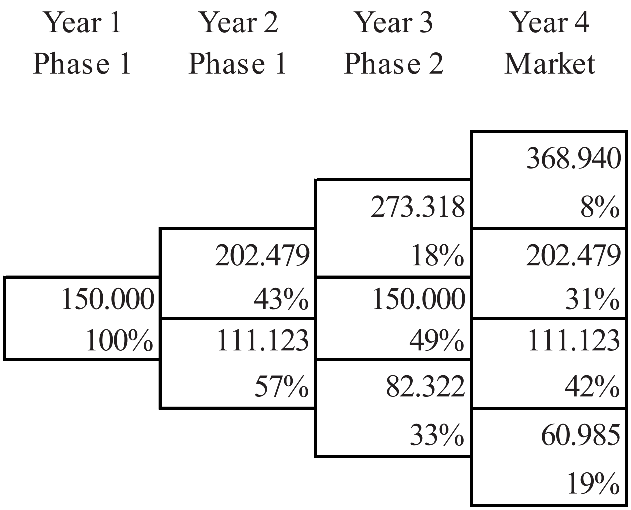

Fig. 2.23. Binomial tree displaying market states and their probabilities

We see that in the best case our peak sales estimate is \$ 368,940, but we expect to reach that number at the time of market entry only in 8% of all cases. The worst case, \$ 60,985, is reached with a probability of 19%.

The following figure exhibits a tree modelling a market of initial 100 units, an annual growth rate of 0% and an annual volatility of 40%. We chose 6 months time increments. After 5 years the tree allows 11 different states of the market, the range lasts from 6 to 1,692. At first sight this seems to overweigh the high values and therefore a positive market development. If we take a closer look at the tree, the numbers keep their justification.

If an ant walks along the tree, always from the left to the right, then there are 1,024 different paths at the ant's choice (there are 10 points where the ant has to chose to go up or down – between two possibilities – hence there are 2^{10}=1,024 different paths). But only one path ends at the very top – if the ant each time chooses to go upwards. On the other hand, there are many more

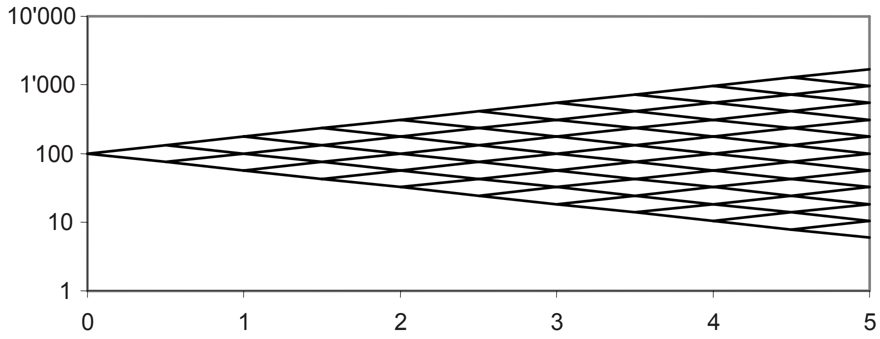

Fig. 2.24. Tree for a five years' period

possibilities to reach one of the middle states. The ant is supposed to go upwards with a probability of 49% and downwards with a probability of 51% – these are the tree parameters that correspond to a growth rate of zero, a volatility of 40%, and a time step of 0.5 years. The different number of paths and their probability induces different weights (probabilities) for each end state. The following graph displays this relationship. The fact that it is slightly more probable that the market goes down at each node (51%) shifts the probability curve on the graph a bit to the left.

The possibility that a medication, whose peak sales are estimated today at $ 100 mn, would reach peak sales of $ 1,692 mn in five years seems not realistic. Nevertheless one can argue that it is possible, though not

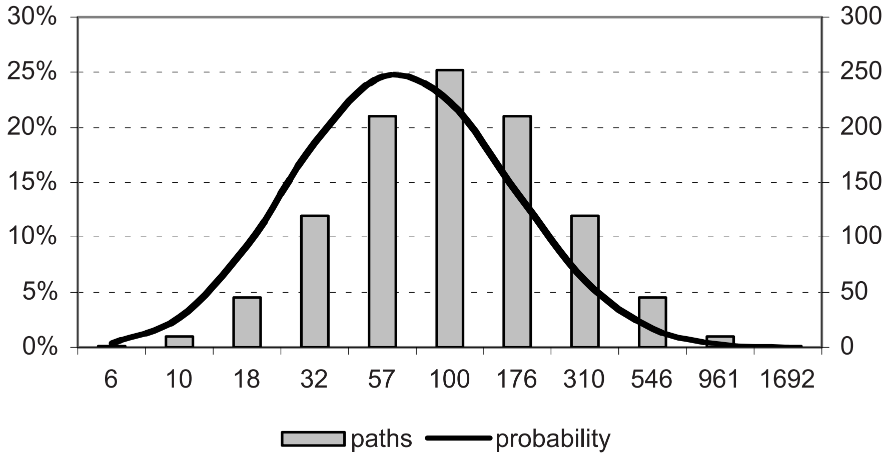

Fig. 2.25. Frequency and probability of different end states

realistic, that competitive products suddenly deal with severe problems; the applications of the drug is consequently expanded to other diseases or the efficacy exceeds anything ever seen and a major demand for this kind of drug arises. This way the medication could become a blockbuster. Roche experienced a similar scenario with the threat of the avian flu pandemic. This sudden change of market state let sales of Tamiflu rise from CHF 330 mn in 2004 to CHF 1,558 mn in 2005. Still this scenario is comparable to winning the lottery, but someone finally wins it. The tree considers that by attributing a minimal probability to this end state. The most probable end states are 57, 100 and 176 with more than 65% of probability all together. On first sight some end states of the tree might seem unrealistic, but the tree captures exactly the probabilistic hypotheses. The more unrealistic a state is, the lower the probability attributed to it by the tree.

Step 3: Solving the end states. After having set up the tree we now solve the tree from the end where no uncertainty is left, i.e. where the decisions can be taken according to clear criteria. The tree puts you into hypothetical states at the final decision points, i.e. at the end nodes of the tree. In each state, the tree gives you a best estimate of the potential of the project, if this state were actually achieved. With this input, we are able to calculate the value of the project. We know the future payoff of the project, as the end node tells us the size of the peak sales and there are no more decision points ahead. Using these peak sales along with the expenditures to launch and market the project; we can calculate the net present value of the project at the end node. Next we decide whether to invest or not according to the value of the DCF calculation: Invest if the discounted (and probability-adjusted) future revenues exceed the investments, abandon otherwise. This way the value of the project is necessarily positive, as all projects with a negative rNPV are not continued. The project either is profitable, i.e. the revenues are higher than the expenses, or the project is abandoned, in which case the value is zero. This is the first implementation of the option idea. Nobody would voluntarily make losses; one rather exercises the option to abandon the project and floors the future losses to zero. The value of the DCF calculation is now attributed to all end nodes in the tree. In all cases where we have a negative value we put the end node value to zero. Resetting the value to zero is only done when we are at a decision point, i.e. start of a new phase, here the market phase. In our example we start with solving the most upper node, with expected peak sales of $ 368’940. The sales will be constant for 5 years and then stop. We now have to find the value for this end node by multiplying the yearly sales with the margin of 50%, and then discount them back to the date of launch, i.e. beginning of year 4.

Table 2.6. Calculation of NPV in the end nodes (in $)

	Year 4	Year 5	Year 6	Year 7	Year 8
	Market	Market	Market	Market	Market
Discount	100%	87%	76%	66%	57%
Sales	368,940	368,940	368,940	368,940	368,940
Operating Profit	184,470	184,470	184,470	184,470	184,470
DCF	184,470	160,409	139,486	121,292	105,471
NPV	**711,129**				
Sales	202,479	202,479	202,479	202,479	202,479
Operating Profit	101,239	101,239	101,239	101,239	101,239
DCF	101,239	88,034	76,552	66,567	57,884
NPV	**390,276**				
Sales	111,123	111,123	111,123	111,123	111,123
Operating Profit	55,561	55,561	55,561	55,561	55,561
DCF	55,561	48,314	42,012	36,533	31,767
NPV	**214,188**				
Sales	60,985	60,985	60,985	60,985	60,985
Operating Profit	30,493	30,493	30,493	30,493	30,493
DCF	30,493	26,515	23,057	20,049	17,434
NPV	**117,549**				

The table above represents the calculation of the net present value of the project's operating profit at the time of launch for the end nodes. We first calculate the operating profit from the sales figures for each year. In a next step we discount these operating profits with the discount factor corresponding to the respective year. The discount factors are calculated with the formula (2.13). We then sum up the discounted yearly operating profits. The results for the four different states are displayed as the NPV. Now we still have to take the decision if we launch the product or not. We therefore subtract the expenses from the net present value of the operating income and receive the following numbers, displayed in bold for each scenario (see Table 2.6).

We see that only in the case of the lowest end node we do not launch the project. The future income would not payoff for the launch expenses and the project would not be profitable. For all further calculations we set the

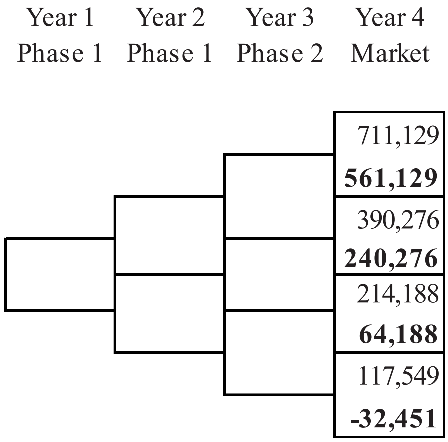

Fig. 2.26. Project value in the endnodes

value of this box to zero, as we do not proceed in this node, avoiding the expected losses.

Step 4: Working back the tree to the previous phase. We now place ourselves in one node right before an end state. This node represents a state Δt before the last decision point. What happens between then and the decision point, i.e. the end node of the tree where we already have the value? First, the results of phase 2 will be known. This is crucial for the ongoing of the project; a negative result corresponds to the end of the project. Second, the market potential is going to fluctuate, either a step up or a step down in the tree. Third, the time simply advances by Δt, this forces us to discount future cash flows by this time difference.

We consider these three points for the calculation of the project value at our specific node:

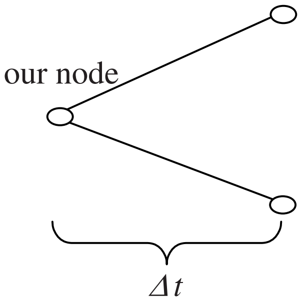

Fig. 2.27. One time step in the binomial tree

1. The results are part of a project's technical uncertainty. The outcome is not known in advance. We can only make use of statistical success rates. We then adjust the project value by this probability.
2. The tree reflects the market uncertainty, there are two end states following our node. The expected value of the project in our state is therefore the average of the project value at the two following end states.
3. Finally the whole value must be discounted for the time interval *Δt*.

If V_T^{up} and V_T^{down} are the project values of the two consecutive end states, then the value $V_{T-\Delta t}$ of the project at our node can be expressed in the following way:

$$V_{T-\Delta t} = \underbrace{P}_{1}\underbrace{\left(pV_T^{up} + (1-p)V_T^{down}\right)}_{2}\underbrace{\frac{1}{(1+r)^{\Delta t}}}_{3} \tag{2.23}$$

P is the statistical success rate of the phase, *p* the probability of the step up. Now we calculate the project value for all nodes at *T-Δt* with the formula above. Then we have to go another time step back to the nodes at *T-2Δt*. Note that if between *T-Δt* and *T* there is no end of a phase, i.e. if we are in the middle of a phase, we do not have to account for a failure of a phase. This must only be done at the very last time step of a phase. The value $V_{T-\Delta t}$ is therefore:

$$V_{T-\Delta t} = \underbrace{\left(pV_T^{up} + (1-p)V_T^{down}\right)}_{2}\underbrace{\frac{1}{(1+r)^{\Delta t}}}_{3} \tag{2.24}$$

Step (1) drops out, only the expectation (2) and the discounting (3) enters into the calculation. Applying this scheme we work back the tree to the root, carefully considering the difference in the calculations at nodes where we take decisions, i.e. after test phases, and where not. We calculate the value for the tree, starting with the most upper node for year three. According to the formula, we get:

$$V_3^{273} = \left[80\%\left(43\% \cdot 561{,}129 + 57\% \cdot 240{,}276\right)\frac{1}{1+15\%}\right] - 30{,}000$$

$$V_3^{273} = \underline{\underline{232{,}134}}$$

We repeat the same for the other two nodes:

$$V_3^{150} = \left[80\% \left(43\% \cdot 240{,}276 + 57\% \cdot 64{,}188 \right) \frac{1}{1+15\%} \right] - 30{,}000$$

$$V_3^{150} = \underline{\underline{66{,}781}}$$

$$V_3^{82} = \left[80\% \left(43\% \cdot 64{,}188 + 57\% \cdot 0 \right) \frac{1}{1+15\%} \right] - 30{,}000$$

$$V_3^{82} = \underline{\underline{-10{,}998}}$$

As we are again at a decision point, the negative value in the lowest node is set to zero, because we do not continue the project in this case and therefore avoid the losses. This actually means that the lowest point at launch is never reached, because the project would already be abandoned beforehand.

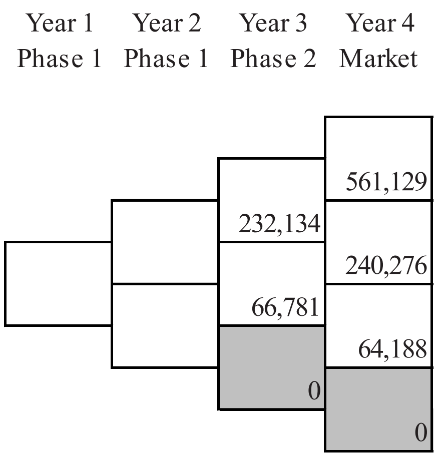

Fig. 2.28. Calculating back the tree to year 2

Step 5: Working back the tree to the root. The resolution of the rest of the tree is now a repetition of the previous steps. At each start of a phase you decide at each node, if it makes sense to continue the project. For this you compare the value of the project you have obtained in the above-described way with the costs of the phase. Replace the previous project value either with the positive difference between value and investment or put it to zero in case the difference is negative and the project consequently abandoned. Again, this is the implementation of an option in the valuation process. Then work back one time step and adjust for the technical uncertainty of the previous phase (1) as well as you take the expectation (2) and discount all values for another Δt (3). After that, you work back to the start of the previous phase by only averaging (2) and discounting (3). Move back to the root of the tree applying these steps for each phase. In our example, for year 2, where we do not have any decisions to take, we calculate the following:

$$V_2^{202} = 75\% \cdot \left(43\% \cdot 232{,}134 + 57\% \cdot 66{,}781\right)\frac{1}{(1+15\%)}$$

$$V_2^{202} = \underline{\underline{89{,}445}}$$

$$V_2^{111} = 75\% \cdot \left(43\% \cdot 66{,}781 + 57\% \cdot 0\right)\frac{1}{(1+15\%)}$$

$$V_2^{111} = \underline{\underline{18{,}534}}$$

The expenses for the phase 1 are triggered at the beginning of year one. We can now calculate the value for the root node the same way as we have calculated the value for year 3.

$$V_1^{150} = (43\% \cdot 89{,}445 + 57\% \cdot 18{,}534)\frac{1}{(1+15\%)} - 25{,}000$$

$$V = \underline{\underline{17{,}357}}$$

The tree then looks the following:

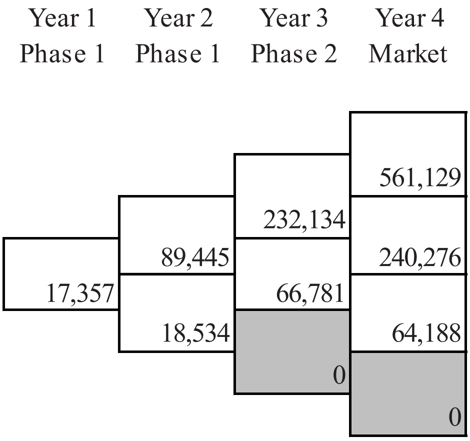

Fig. 2.29. Tree solved up to the root node

The root node value represents the real option value of the project today. The project has a real option value of \$ 17,357.

Difference Between Real Options and Financial Options

In the following chapter we discuss the difference between real options and financial options and the implications on using the Black-Scholes formula for real options. The chapter is intended for readers interested in advanced discussions of real options and is not fundamental for the following chapters of the book.

Real options owe their name to their similarity to financial options. Almost every introduction to real options compares an investment opportunity with a financial call option. This suits to explain the general concept of real options. Unfortunately, virtually all literature about real options only concentrates on the second word of the name, "options". The word "real" is not only a descriptive of the different area of application – real economy vs. financial economy – it is also an important statement about the very nature of these options and has significant impact on quantification.

We start with a short explanation of the fundamentals of financial option pricing, i.e. about hedging. Then we elucidate the difference between real and financial options and explain its influence on valuation.

Financial option valuation. The concept of financial option valuation has been introduced by Fisher Black and Myron Scholes with their seminal paper in 1973 (Black 1973). Since then, the basics haven't changed significantly. They have shown that a financial option is replicated with a portfolio of the underlying and a bond. This allows mitigating all risk. And since the portfolio is without risk, it must earn the risk free rate, otherwise there would be an arbitrage opportunity.

The simplest way to visualise this concept is by means of a binomial tree. First, we model the fluctuations of the underlying of the option. We assume that the value of the underlying can go up or down during the next time step. Assuming today's value of the underlying being S, after one time step it can be either uS or dS, with u denoting the up step, and d the down step. Each scenario, the up or the down step, has a certain probability p or $1-p$.

The parameters u, d, and p should be set in a way such that they model correctly the underlying's fluctuation. In the Black-Scholes concept it is usually assumed that the underlying follows a geometric Brownian motion with growth rate μ (also called drift) and volatility σ. We give two examples of parameter sets u, d, and p that model correctly a geometric Brownian motion with parameters μ and σ.

The table below summarises the parameters to model the fluctuation of the underlying:

Table 2.7. Parameter sets for binomial trees

	Set 1	Set 2
u	$e^{\sigma\sqrt{\Delta t}}$	$e^{\mu\Delta t+\sigma\sqrt{\Delta t}}$
d	$e^{-\sigma\sqrt{\Delta t}}$	$e^{\mu\Delta t-\sigma\sqrt{\Delta t}}$
p	$\frac{e^{\mu\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}$	$\frac{1-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}$

Second, we define the replicating portfolio. The portfolio consists of the option, a portion Δ of the underlying (We use the notation Δ for consistency with the literature. Don't confuse Δ with the time increment Δt). The remaining part is allocated to a risk-free bond, which we can neglect in what follows. Δ is chosen such that, no matter what happens to the underlying, the return of the portfolio is the same. This is achieved with the following reasoning (V, V_{up}, and V_{down} representing the value of the option at the different nodes of the binomial tree):

$$V_{up} + \Delta uS = V_{down} + \Delta dS \tag{2.25}$$

Solving for Δ we get:

$$\Delta = -\frac{V_{up} - V_{down}}{uS - dS} \tag{2.26}$$

Valuation of Financial Options

1. Model the underlying
2. Build a replicating portfolio
3. Calculate the expected risk-free discounted value using risk-neutral probabilities

Third, we have to find a way, how to value the option. From the fact that the portfolio has by construction always the same value after one time step in all nodes of that time, we conclude that it is risk free. A risk free asset or portfolio must earn the risk-free rate r_f, otherwise we would have an arbitrage opportunity.

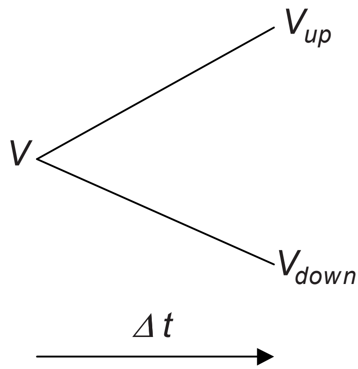

Fig. 2.30. Time step in option valuation

It would now be easy to deduce the value of the option from the values of the option at the leafs of the tree. The value of the option however is not equal to the expected and discounted future value of the option, because we cannot consider the option alone but only in conjunction with the replicating portfolio (otherwise we would not have a risk-free asset). Let us therefore define a fictive number q such that:

$$V = e^{-r_f \Delta t}\left(qV_{up} + (1-q)V_{down}\right) \tag{2.27}$$

The purpose of the introduction of q is a valuation algorithm that can focus on the option alone instead of always having to consider the replication portfolio. We have to stress that the introduction of q is a trick. Using the postulated property of q, the fact that

$$V + \Delta S = e^{-r_f \Delta t}\left(V_{up} + \Delta uS\right) \tag{2.28}$$

and the definition of Δ we can deduce the value of q.

$$V_{up} + \Delta uS - \Delta S e^{r_f \Delta t} = qV_{up} + (1-q)V_{down} \tag{2.29}$$

$$q = \frac{V_{up} - V_{down} + \Delta S\left(u - e^{r_f \Delta t}\right)}{V_{up} - V_{down}} \tag{2.30}$$

$$q = 1 - \frac{u - e^{r_f \Delta t}}{u - d} = \frac{e^{r_f \Delta t} - d}{u - d} \tag{2.31}$$

Using q according to its definition we can now calculate the value of the option V in a simple way:

$$V = e^{-r_f \Delta t}\left(qV_{up} + (1-q)V_{down}\right) \tag{2.32}$$

q is sometimes called risk-neutral probability, because its use corresponds to a discounted expectation, using q as probability within the tree and discounting at the risk-free rate. Note that usually discounting something uncertain at the risk-free rate is not accepted, because investors want to be rewarded with a risk premium. Nonetheless, in the case of financial option valuation this is allowed because it is only a trick. In fact, we discount the whole portfolio at the risk free rate, and the whole portfolio does not fluctuate.

The risk-neutral probabilities for the above mentioned parameter sets are:

Table 2.8. Risk free formulae for binomial tree

	Set 1	Set 2
u	$e^{\sigma\sqrt{\Delta t}}$	$e^{\mu\Delta t+\sigma\sqrt{\Delta t}}$
d	$e^{-\sigma\sqrt{\Delta t}}$	$e^{\mu\Delta t-\sigma\sqrt{\Delta t}}$
q	$\dfrac{e^{r_f\Delta t} - e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}$	$\dfrac{e^{(r_f-\mu)\Delta t} - e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}$

We recognise especially from parameter set 1 that using risk-neutral probabilities corresponds to a change of the underlying's drift parameter μ to r_f. In mathematical terms we have changed the underlying's real-world probability measure to its risk-neutral probability measure. But remember, this is only a trick for the purpose of the valuation, the underlying still follows a geometric Brownian motion with parameters μ and σ, whether there are some options or not.

Real options valuation. We now follow the same steps and quantify real options. First, we model the uncertainty of the underlying. This can be either estimated revenues of a product, the gold or oil price, or another economic metric. For simplicity let us assume that the underlying follows equally a geometric Brownian motion with parameters μ and σ. Second, we build the replicating portfolio. This now leads us to the very difference between real options and financial options. While financial options are a contract that defines rights on a traded asset in a liquid market, real options are business opportunities. The decision to seize such opportunities depends on an economic metric, the underlying of the real option. This underlying however can be a virtual number, an estimate, e.g. the estimated revenues of a not yet developed product, or a share of a specific market the company considers expanding in. Most often the underlying is not tradable, which makes it impossible to build a replicating portfolio. In the case of commodity companies like mines or petrol companies, the underlying is traded in a liquid market. Nonetheless, the question we have to ask is not “Can we build a replicating portfolio?” but rather “Do we build a replicating portfolio?” If we do build a replicating portfolio, then we can use the valuation of financial options as described above. If we do not, then we cannot use the valuation of financial options, because the fundamental concept of replication is not respected. We have to keep in mind that financial options are valued as part of a risk-free portfolio, and only this justifies the use of the risk-free rate in the risk-neutral probabilities. If you valued real options with risk-neutral probabilities you would value something you do not have, namely a business opportunity that is hedged. Even oil companies and mines may only apply risk-neutral valuation if they hedge their real options. Hedging the business opportunities would correspond to pocket in the value of the business opportunity and then hedging it until the end of the life cycle. This would be real arbitrage, a very smart way to run a business. Unfortunately, we have not heard of any company doing so. The reason might be simple: Real options model the uncertainty of some value drivers. But there are still a lot of other factors not built in the model and not hedgeable, and managers cannot trust their models such that this would justify large financial transactions to hedge their business opportunities. For the moment, it seems that the model risk outweighs the business risk.

But, let us return to the standard real options that cannot be replicated anyway. We have to acknowledge that we must not use the elegant valuation method that applies to financial options. So, the second point of financial option valuation cannot be executed. Consequently, the third point that is based on the second cannot be respected either. So what can we do?

Let us go back to the actual meaning of “real options”. Real opportunities are business opportunities that require decisions in the future. Until then, some value relevant parameters can fluctuate. The decision will eventually be based on the parameter set that is encountered at the time of decision. Hence, in contrast to other valuation methods, using real options we just model some additional decisions that influence the course of the project. Typically, these decisions are abandonment or expansion. The decisions and the subsequent scenarios occur with some probability. These probabilities depend on the probability distribution of the decision driving parameters. It is therefore straightforward, first, to model the fluctuation of these decision relevant parameters, second, to anticipate for each possible scenario what management would decide, and third, to calculate the expected value of all these scenarios. In the binomial tree, this means that the value of the real option can be described by:

$$V = e^{-r_a \Delta t}\left(pV_{up} + (1-p)V_{down}\right) \tag{2.33}$$

We notice two differences with respect to financial option valuation. First, we use the real world probability p that models correctly the likelihood that either the one or the other scenario occurs. Second, we use the risk-adjusted discount rate r_a instead of the risk-free rate r. Since the value of the option fluctuates with the underlying, the option holder faces some risk, which he wants to be rewarded for. Again, the financial option was part of a risk-free portfolio, so there was no risk that would have to be compensated.

In this context, real options valuation is nothing else than a risk-adjusted expectation. Risk-neutrality or change of probability measures like for financial options lack any justification. The difference between real options and other valuation methods lies solely in the modelling of future decisions and their underlying drivers.

Some authors try to bypass this apparent inconvenient that avoids the direct application of Black-Scholes by assuming that the real option can be hedged. They suppose that there is a traded asset that could be used to replicate the real option. This hypothesis is called market asset disclaimer (MAD). MAD is about as useful to plan your agenda assuming the week has eight days because you are short in time. The planning might work fine, but at the latest on the eighth day reality will wake you up. In corporate finance the investors' wake-up call uses to be little tender.

Consequences. All formulae from quantitative finance are considered to be applied to financial option valuation. Therefore, we have to adapt them prior to using them for real options valuation. In the case of the binomial tree we have to use the real-world probabilities and use the risk-adjusted discount rate when working back the tree.

Black and Scholes' partial differential equation (Black et al. 1973) has to be modified in the following sense:

$$V_t + \mu S V_S + \frac{1}{2}\sigma^2 S^2 V_{SS} - r_a V = 0 \tag{2.34}$$

For the financial option version of this PDE we have to replace μ and r_a with r. This formula can be derived by using the Hamilton-Jacobi-Bellmann equation.

The well-known and often used option formula, equally derived by Black and Scholes, must read:

$$V = Se^{(\mu - r_a)t} N(d_1) - Ke^{r_a t} N(d_2) \tag{2.35}$$

$$d_1 = \frac{\left(\mu + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}, d_2 = d_1 - \sigma\sqrt{t} \tag{2.36}$$

Conclusion. Black and Scholes derived an elegant method to value financial options as part of a risk free portfolio. Consequently, they could discount at the risk-free rate and use risk-neutral probabilities. In contrast to that, real options cannot be, or, equally important, are not hedged. This implies that real options valuation simply becomes an expectation calculus discounted at the risk-adjusted rate.

If you use risk-neutral valuation for real options and you do not hedge, you value something you do not have – a business opportunity as part of a risk-free portfolio – instead of something you have – a business opportunity with real risks.

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