Preface

This is a book on numerical methods for singular perturbation problems – in particular, stationary reaction-convection-diffusion problems exhibiting layer behaviour. More precisely, it is devoted to the construction and analysis of layer-adapted meshes underlying these numerical methods.

Numerical methods for singularly perturbed differential equations have been studied since the early 1970s and the research frontier has been constantly expanding since. A comprehensive exposition of the state of the art in the analysis of numerical methods for singular perturbation problems is [141] which was published in 2008. As that monograph covers a big variety of numerical methods, it only contains a rather short introduction to layer-adapted meshes, while the present book is exclusively dedicated to that subject.

An early important contribution towards the optimisation of numerical methods by means of special meshes was made by N.S. Bakhvalov [18] in 1969. His paper spawned a lively discussion in the literature with a number of further meshes being proposed and applied to various singular perturbation problems. However, in the mid 1980s, this development stalled, but was enlivened again by G.I. Shishkin’s proposal of piecewise-equidistant meshes in the early 1990s [121, 150]. Because of their very simple structure, they are often much easier to analyse than other meshes, although they give numerical approximations that are inferior to solutions on competing meshes. Shishkin meshes for numerous problems and numerical methods have been studied since and they are still very much in vogue.

With this contribution we try to counter this development and lay the emphasis on more general meshes that – apart from performing better than piecewise-uniform meshes – provide a deeper insight in the course of their analysis.

In this monograph, a classification and a survey are given of layer-adapted meshes for reaction-convection-diffusion problems. The monograph aims at giving a structured and comprehensive account of current ideas in the numerical analysis for various methods on layer-adapted meshes. Both finite differences, finite elements and finite volumes will be covered.

While for finite difference schemes applied to one-dimensional problems, a rather complete convergence theory for arbitrary meshes is developed, the theory is more fragmentary for other methods and problems. They still require the restriction to certain classes of meshes.
The roots of this monograph are a survey lecture presented at the Oberwolfach seminar *Numerical Methods for Singular Perturbation Problems*, 8–14 April 2001 organised by Pieter W. Hemker, Hans-Görg Roos and Martin Stynes, and a review article [91] invited by Thomas J.R. Hughes. I am indebted to their invitations and their continued encouragement.

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