This book is a collection of papers contributed by some of the greatest names in the areas of chaos and nonlinear dynamics. Each paper examines a research topic at the frontier of the area of dynamical systems. As well as reviewing recent results, each paper also discusses the future perspectives of each topic. The result is an invaluable snapshot of the state of the field by some of the most important researchers in the area.

The first contribution in this book (the section entitled “How did you get into Chaos?”) is actually not a paper, but a collection of personal accounts by a number of participants of the conference held in Aberdeen in September 2007 to honour Celso Grebogi’s 60th birthday. At the instigation of James Yorke, many of the most well-known scientists in the area agreed to share their tales on how they got involved in chaos during a celebratory dinner in Celso’s honour during the conference. This was recorded in video, we felt that these accounts were a valuable historic document for the field. So we decided to transcribe it and include it here as the first section of the book.

The dynamics of maps on the complex plane provide some of the most striking examples of chaos and fractal invariant sets in dynamics, and has been of great importance to the field because they are amenable to rigorous treatment. The first paper in the book is R. Devaney’s investigation of the dynamical properties of singularly-perturbed complex maps. He investigates Julia sets and other related sets which arise in maps with a pole, and classifies their dynamics.

One of the most exciting developments in recent years is the application of dynamical systems techniques to complex networks of interacting components, each having their own internal dynamics, and each being coupled to other nodes. P. Ashwin, G. Orosz and J. Borresen review how complex dynamics can arise even in simple, fully symmetric and globally coupled networks. They make the important point that not only the network topology (which is usually emphasised in the literature), but also the properties of the coupling function are crucial to determine the system’s global dynamics.

Fluid dynamics is an area that has always had a close relation with chaos. The motion of particles advected by time-dependent flows is a prime example of a chaotic system, and chaotic advection has been observed in many beautiful experiments. Most of the existing theoretical work considers advected particles as having
vanishing size, even though it is known that their finite size can have considerable consequences for their dynamics. J. Cartwright et al. review the dynamics of finite-size particles in chaotic flows. This is the first published review of this important subject.

Hamiltonian dynamics occupies a special place in the area of dynamical systems, because of its applications to classical mechanics, celestial mechanics, physics and other areas. In many Hamiltonian systems, there is a clear separation of slow and fast degrees of freedom, and it is common practice to model the effects of the fast variables by noise and damping, which results in a Langevin equation for the slow degrees of freedom. However, the rigorous mathematical foundations for this are not well-established. R. MacKay proposes a rigorous way to derive a Langevin equation for Hamiltonian systems, by assuming that the fast variables have an Anosov mixing dynamics.

In their contribution, A. Politi and A. Torcini review the concepts of stable chaos, that is, the presence of irregular behaviour even though the dynamics is still locally stable. Although the irregular behaviour is transient in these cases, the transient’s lifetime diverges in the thermodynamic limit, which makes stable chaos relevant for out-of-equilibrium statistical mechanics. The authors emphasise the connection between stable chaos and the spatio-temporal chaos shown by a class of cellular automata.

Chaotic transients are also the topic of the next paper, by Y.-C. Lai. He reviews the subject of superpersistent chaotic transients, which refers to the extremely long lifetimes of some systems near bifurcations. He explains the dynamical origins of these long lifetimes, and how they are related to the riddling bifurcation and riddled basins. He also discusses the occurrence of transients in spatially extended systems, and the application of these concepts to the motion of particles in fluid flows.

One of the biggest current topics of research in the field of dynamical systems is synchronisation, and the next three papers are all concerned with this fascinating subject. The paper by P. Read and A. Castrajón-Pita investigates the possibility of synchronisation in spatio-temporal systems, focusing on the Earth’s climate system. They make a good case for the possibility of the existence of synchronised oscillations in weather patterns. Experiments are performed in fluid-dynamical systems which are analogues of major components of our planet’s weather system, and synchronisation is indeed observed.

Many of the systems in which the concept of synchronisation is important are noisy, especially biological systems at the cellular level. It is therefore very important to understand how synchronisation works in the presence of noise. R. Ramaswami et al. give an overview of stochastic synchronisation, giving a number of examples including chemical reactions and gene regulation networks.

Synchronisation was first discovered due to the observation by Christian Huygens that two clocks connected to the same wooden beam eventually synchronise their oscillations. A. Pogromski, D. Rijlaarsdam and H. Nijmeijer present an experimental setup which allows a thorough exploration of the synchronisation of mechanical systems, in the spirit of Huygens’s original observation but in a much more controlled way. Their setup allows the coupling between two mechanical oscillators
to be controlled, and thus makes it possible to study different kinds of synchronisation phenomena, such as the synchronisation of pendula, rotating objects, etc. They describe results of synchronisation experiments with two Duffing oscillators and with two rotating disks.

The control of chaos is one of many areas to which Celso Grebogi has fundamental contributions. G. Riga, S. Lenci and J. M. T. Thompson’s paper reviews the OGY theory of controlling chaos from a historical perspective, and discusses applications in mechanics and related fields. They compare and contrast the OGY strategy based on stabilising a single periodic orbit of the chaotic set with a global method of trying to stabilise the overall system dynamics.

Since the seminal embedding theorems by Takens and others, time-series analysis has been an important area within dynamical systems. Closing the book is R. Stoop and M. Christen’s investigation on methods to extract regular patterns from time-series with noisy background. This is a crucial problem in neuroscience and other fields, and one in which traditional methods, such as power-spectrum and related procedures, usually fail. They propose a method based on staircase-like structures in the correlation plot, and derive a number of analytical results on their method, which suggest that their method is very promising in applications.

The contributions in this book cover a broad range of topics within the large area of dynamical systems and chaos, and they range from pure mathematics to real-world applications. They show that our field is as exciting as ever, and has a brilliant future ahead of it.

In September 2007, a conference was held in Aberdeen, Scotland, to celebrate the 60th birthday of Celso Grebogi. The list of invited speakers, among them many of the contributors to this book, reads like a “who’s who” of the areas of dynamical systems and chaos. This conference was the first time so many great names in the field have been gathered together in a single event in decades, and it presented a unique opportunity to assess the present state of the area and its future directions. It was felt by many of the participants of that event that a book with in-depth surveys of important topics in the field was timely, and this was the driving force for putting together this volume.

Aberdeen, UK
April 2010

György Károlyi, Jürgen Kurths, Alessandro Moura,
Marco Thiel, M. Carmen Romano
Nonlinear Dynamics and Chaos: Advances and Perspectives
Thiel, M.; Kurths, J.; Romano, M.C.; Károlyi, G.; Moura, A. (Eds.)
2010, XV, 293 p. 16 illus. in color., Hardcover
ISBN: 978-3-642-04628-5