Modern science has abstracted, as compensation for establishing rigorousness, the complexity of the real world, and has inclined toward oversimplified fictitious narratives; as a result, a disjunction has emerged between the wisdom of science and reality. Reflecting on this, we see the need for science to recover reality; can it reveal new avenues for thought and investigation of the complexity?

The study of science is the pursuit of clarity and distinctness. Physics, after Galilei placed it in the realm of mathematics, has been trying to establish clearness by mathematical logic. While physics and mathematics, respectively, have different intellectual incentives, they have intersected in history on countless occasions and have woven a flawless system of wisdom. The core of rigorous science is always made of mathematical logic; the laws of science cannot be represented without the language of mathematics. Conversely, it is undoubtedly difficult to stimulate mathematical intellect without a reference to the interests of science that are directed to the real world.

However, various criticisms have been raised against the discourses of sciences that explain the events of the real world as if they are “governed” by mathematical laws. Sciences, being combined with technologies, have permeated, in the form of technical rationalism, the domain of life, politics, and even the psychological world. The criticisms accuse seemingly logical scientific narratives of being responsible for widespread destruction and emergence of crises, unprecedented suffering of humanity. Such arguments are based on the objection to the oversimplified perspective that deems the real world to be “machines” and sees a common “mechanism” behind all phenomena. Here we notice the fact that many of the theories of physics and mathematics have become very improper “metaphors,” and have diffused; what have become the targets of criticisms as mechanistic barbarism are these ghosts.

In the history going back to Galilei and Newton, physics gained triumphs in describing the “cosmos”—the periodic movements of planets and similar regular motions in various systems. However, we have yet to write the theory of the other form of motion—a more general actuality of events in nature and society, that is the so-called “chaos.” We must speak of what the theories of sciences have understood; we must speak of the limits of their legitimacy; we shall have to speak of what these theories are leaving in abeyance. Then, and only then, we can determine the realm in which the cosmos and chaos are not disjunct: the complexity is not eliminated from
the scope of studies. We will also have to dispute the validity of the contemporary rhetoric “complex system” that has already begun to pervade scientific dialogue.

Beneath the complexity of actual phenomena, there is a mathematical structure that is called nonlinearity—this is the main theme of this book. Any mathematical law cannot be expected to hold unrestrictedly, even if it is exact within a certain range. The law changes with respect to the scale of variables (parameters)—this is the simple meaning of nonlinearity. For instance, how much is the price of 5 apples when one apple is 70 cents? Children learn to solve this problem by the proportionality relation and calculate the price to be $3.50. If the price for 50,000 apples is asked, however, $35,000 is not necessarily correct in economics even if exact as the arithmetic answer. We have to change the rule of pricing according to the scale of the variable (number of apples); the complexity of reality stems from this metamorphose.

The category of mathematical laws which is represented by the proportionality relation is called linear, because the graph of a proportionality relation is given by a “line”—nonlinearity is the distortion of the linear graph, the proportionality relation. To study the complexity of the real world, we have to renounce the convenience of assuming the simplest linear relationship between parameters. In the previous example of price calculation, the metamorphose of the linear relation is not simply formulated by varying the unit price; the problem reverts to the question of how the unit price is determined, and this question retrieves the conjunction of this problem to the surrounding “complex system” that consists of the producer, market, consumers, and so on.

The term nonlinear is worded by a negation form—it is not a descriptive (deictic) word characterizing a particular property, but it is a distinctive word indicating opposition to linear. The meaning of nonlinear is infinitely wide as a vague area and is not bound to a concrete frame. Therefore, when we say “mathematical structure that is nonlinear,” we do not mean that there is a prescribed structure giving a framework of the theory, but we are paying attention to the unboundedly developing “differences” from linearity. We will critically analyze the structure of linear theory and reveal its limitations. By this process, the meaning of nonlinear (and, simultaneously, linear) will become more clear and precise. It is hoped that partly through these arguments, the complexity that linear theory has abandoned might be revived on the horizon of science.

This book is written for readers who have a wide interest in science. It aims to provide an explicit explanation about what nonlinear science is. Because nonlinearity is primarily a mathematical concept, a simple list of so-called “nonlinear phenomena” will not suffice for a proper understanding. We will not evade mathematical considerations; we are going to analyze the “mathematical structure” of the theory. This will be a deliberate work of deciphering the manipulations that develop in the world of mathematical symbols.

It is not possible, however, to touch upon technically complicated subjects in this short book. In order to give hints about the methods to approach the problems of contemporary sciences, appendices are given as Notes at the end of each chapter.
Some related materials are also given as Problems. For extended studies, the reader is referred to textbooks listed at the end of each chapter.

The basic plan of this book evolved from discussions with Uichi Yoshida, the editor of Iwanami-Shoten. We wanted to publish a pedagogical book on nonlinear science, which would be sufficiently elementary but with a solid mathematical backbone. It took about five years before the first version of this book was published. I had to totally rewrite the manuscript three times, because I myself struggled with the question, “What is nonlinear?”, the very title of the planned book. I think I have been able to give an answer to this question, with the help of my friends and collaborators. I wish to record my special thanks to Swadesh Mahajan, Yoshikazu Giga, Vazha Berezhiani, Nana Shatashvili, Robert L. Dewar, Hamid Saleem, Vinod Krishan, and Akira Hasegawa. I had an opportunity to organize a research project entitled “Creation and Sustenance of Diversity” at the International Institute for Advanced Studies in Kyoto, where an extremely interdisciplinary group (consisting of researchers of mathematics, physics, meteorology, medicine, sociology, economics, and philosophy) was built up around the common theme “diversity”. This book draws heavily on the products of the project. Especially, I have benefited from discussions with Kamon Nitagai and Mitsuhiro Toriumi. I am also very grateful to all members as well as the directors and staff of the Institute.

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