Preface

An outstanding problem of theoretical physics is the incorporation of gravity into quantum physics. After the increasing experimental evidence for the validity of Einstein's theory of general relativity, a theory based on the differential geometry of Lorentzian manifolds, and the discovery of the standard model of elementary particle physics, relying on the formalism of quantum field theory, the question of mutual compatibility of these theoretical concepts gains more and more importance. This becomes in particular urgent in modern cosmology where both theories have to be applied simultaneously.

Early attempts of incorporating gravity into quantum field theory by treating the gravitational field as one of the quantum fields run into conceptual and practical problems. This fact led to rather radical new attempts going beyond the established theories, the most prominent ones being string theory and loop quantum gravity. But after some decades of work a satisfactory theory of quantum gravity is still not available; moreover, there are indications that the original field theoretical approach may be better suited than originally expected.

In particular, due to the weakness of gravitational forces, the back reaction of the spacetime metric to the energy momentum tensor of the quantum fields may be neglected, in a first approximation, and one is left with the problem of quantum field theory on Lorentzian manifolds. Surprisingly, this seemingly modest approach leads to far-reaching conceptual and mathematical problems and to spectacular predictions, the most famous one being the Hawking radiation of black holes.

Quantum field theory on Minkowski space is traditionally based on concepts like vacuum, particles, Fock space, S-matrix, and path integrals. It turns out that these concepts are, in general, not well defined on Lorentzian spacetimes. But commutation relations and field equations remain meaningful. Therefore the algebraic approach to quantum field theory proves to be especially well suited for the formulation of quantum field theory on curved spacetimes.

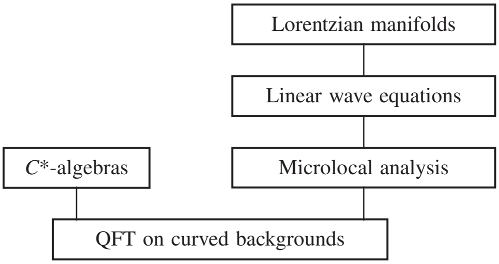
Ingredients of this approach are the formulation of quantum physics in terms of C^*-algebras, the geometry of Lorentzian manifolds, in particular their causal structure, and linear hyperbolic differential equations where the well posedness of the Cauchy problem plays a distinguished role. These ingredients, however, are sufficient only for the treatment of so-called free fields which satisfy linear field equations. The breakthrough for the treatment of nonlinear theories (on the level of formal power series which is also the state of the art in quantum field theories on Minkowski space) relies on the insight (due to M. Radzikowski) that concepts of microlocal analysis are suited for an incorporation of those features of quantum field theory which are on Minkowski space related to the requirement of positivity of energy.

Another major open problem for long time was to find a replacement for the property of symmetry under the isometry group of Minkowski space which plays a crucial role in traditional quantum field theory. The solution to this problem turned out to require means from category theory. Roughly speaking, symmetry has to be replaced by functoriality, and field theoretical constructions can be considered as natural transformations between appropriate functors. From the point of view of physics, the leading idea is that globally hyperbolic subregions of a spacetime have to be considered as spacetimes in their own right, and the allowed constructions apply to all spacetimes (of the class considered) such that they restrict correctly to sub-spacetimes. This was termed the principle of local covariance. It contains the traditional requirement of covariance under spacetime symmetries and the principle of general covariance of general relativity.

Based on it, the perturbative renormalization of quantum field theory on curved spacetime could be carried through. Perturbative renormalization solves the problem of divergences of naive perturbation theory in interacting quantum field theory. In its standard formulation for Minkowski space it heavily relies on translation symmetry. Its combinatorial, algebraic, and analytic structures have been a source of inspiration for mathematics; in recent times in particular the Connes–Kreimer approach found much interest. For curved spacetime the causal perturbation theory of Epstein and Glaser is better suited. As a result, perturbatively renormalized quantum field theory on curved spacetimes has now the status of a proper generalization of quantum field theory on Minkowski space; and it should be able to describe physics on almost all presently accessible scales. Moreover, compared to the Minkowski space theory which often appears to consist of more or less well-defined cooking recipes, the theory becomes more transparent and its fundamental features become visible.

In October 2007 we organized a compact course on quantum field theory on curved spacetimes at the University of Potsdam. More than 40 participants with varying backgrounds came together to learn about the subject including its mathematical prerequisites. Assuming some basic knowledge of differential geometry and functional analysis on the part of the audience we offered several lecture series introducing C^*-algebras, Lorentzian geometry, the classical theory of linear wave equations, and microlocal analysis. Thus prepared the participants then attended the lecture series on the main topic itself, quantum field theory on curved backgrounds.

This book contains the extended lecture notes of this compact course. The logical dependence is as follows:



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