Preface

In this monograph we review numerical computation methods for multivariate normal and t probabilities while focusing on recent computer intensive integration methods. This monograph has two main objectives. First, we aim to convince the reader that these type of integration problems are computationally feasible for moderate accuracies and dimensions. Numerical results suggest that typical computation problems require only a few seconds of workstation time. Generalizations are available to singular problems, to domains other than the classical rectangular regions and to related integrals having multivariate normal and t weights. Special cases such as lower dimensional integrals or specific correlations structures allow quick and accurate computations for most practical purposes. Second, this monograph gives the reader a glimpse into current multidimensional integration techniques. This work might therefore also serve as a guide for other integration problems.

The monograph is organized as follows. In Chapter 1 we motivate the problem and offer a historical perspective to show how methodological approaches have evolved over time. We then introduce the integration problems and some related notation. We conclude this chapter with several numerical examples, which are used later to illustrate the methods. Chapter 2 is devoted to special cases, for which the integration problem can be simplified. We first consider the work done on bivariate and trivariate normal and t probabilities. We then consider calculating probabilities over special integrations regions, such as orthants, ellipsoids, and hyperboloids. Finally, we review multivariate normal and t probability problems for special correlation structures. In Chapter 3 we describe inexact methods for approximating general multivariate normal and t probabilities by one or more integration problems which have easier solutions than the original problem. This includes, for example, methods that are based on Boole's formula, which uses combinations of lower dimensional problems to approximate the original problem. In Chapter 4 we describe approximation methods for the general integration problems that can lead to exact results given sufficient computational resources. These methods are based on reparameterizing the integrals of the original problems, thus enabling the use of efficient numerical integration methods, including stochastic and deterministic methods. In Chapter 5, some related and application specific topics are considered, such as singular distributions, integration problems having an application specific expectation function with multivariate normal or t weight, and a review of numerical test results. A description of current software implementations in MATLAB and R is also given. Finally, in Chapter 6 we illustrate the theoretical results from the previous chapters with numerical examples from different applications, including multiple comparison procedures, Bayesian statistics and computational finance.

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