

# Table of Contents

## Number Theory \_\_\_\_\_ 1

1. Six proofs of the infinity of primes ..... 3
2. Bertrand's postulate ..... 7
3. Binomial coefficients are (almost) never powers ..... 13
4. Representing numbers as sums of two squares ..... 17
5. The law of quadratic reciprocity ..... 23
6. Every finite division ring is a field ..... 31
7. Some irrational numbers ..... 35
8. Three times  $\pi^2/6$  ..... 43

## Geometry \_\_\_\_\_ 51

9. Hilbert's third problem: decomposing polyhedra ..... 53
10. Lines in the plane and decompositions of graphs ..... 63
11. The slope problem ..... 69
12. Three applications of Euler's formula ..... 75
13. Cauchy's rigidity theorem ..... 81
14. Touching simplices ..... 85
15. Every large point set has an obtuse angle ..... 89
16. Borsuk's conjecture ..... 95

## Analysis \_\_\_\_\_ 101

17. Sets, functions, and the continuum hypothesis ..... 103
18. In praise of inequalities ..... 119
19. The fundamental theorem of algebra ..... 127
20. One square and an odd number of triangles ..... 131

21. A theorem of Pólya on polynomials .....	139
22. On a lemma of Littlewood and Offord .....	145
23. Cotangent and the Herglotz trick .....	149
24. Buffon's needle problem .....	155

## **Combinatorics** \_\_\_\_\_ **159**

25. Pigeon-hole and double counting .....	161
26. Tiling rectangles .....	173
27. Three famous theorems on finite sets .....	179
28. Shuffling cards .....	185
29. Lattice paths and determinants .....	195
30. Cayley's formula for the number of trees .....	201
31. Identities versus bijections .....	207
32. Completing Latin squares .....	213

## **Graph Theory** \_\_\_\_\_ **219**

33. The Dinitz problem .....	221
34. Five-coloring plane graphs .....	227
35. How to guard a museum .....	231
36. Turán's graph theorem .....	235
37. Communicating without errors .....	241
38. The chromatic number of Kneser graphs .....	251
39. Of friends and politicians .....	257
40. Probability makes counting (sometimes) easy .....	261

## **About the Illustrations** \_\_\_\_\_ **270**

## **Index** \_\_\_\_\_ **271**