# Chapter 1
# Overview of Music Theories

The title of this chapter states *Music Theories* in the plural and not the singular *Music Theory* or *Theory of Music*. Probably no single theory will ever cover the enormous richness of music in the world, although there have been promising endeavors to extract certain principles common to most. This is a boon and a bane at the same time: a bane, because all attempts at reducing music to a single set of rules have failed so far to satisfy adherents of universally valid systems; a boon, since it relieves music lovers from the potential danger of the dreaded reduction of music to a mere system of rules, devoid of all mysticism cherished by many. Even putting aside all aspects of the subjective, this state of affairs promises new vistas for future musical activity, since the reservoir of new and modified theories for generating new music seems inexhaustible.

The role of theory in the analysis as well as the production of music has however not been the same at all times in the history of music. To better understand how theories and what theories fit into contemporary music, it is necessary to shed some light on the last few hundred years during which music theories have been perceived as such by musicians.

It is necessary to restrict the discussion to theories about Western music. Of course theories of music from other cultural environments such as India exist and its literature is very broad indeed, but including it would by far break the tight limits set by this text.

Certainly the most famous theory linking music and natural science known from Antiquity is the description of music intervals using of chord ratios by Pythagoras. However from the post-Antiquity treatise by Boethius through the Middle Ages until 19th century, music theories were rather enshrouded in the philosophical (or theological) frameworks of the day. Those parts relevant to the analysis and composition of music did not much more than describe the state of the art as it was a decade before the publication of the treatises.

Thus the *Gradus ad Parnassum* by Johann Joseph Fux published in 1725 [25] laid out the rules of counterpoint, even if those rules did not accurately describe the practice of most advanced contemporary composers, such as Johann Sebastian Bach. This work, well used even into the present, also illustrates features common to similar theoretical treatises: they delivered a set of recipes justified by empirical or psychological phenomena, thus providing weak theories, even in comparison to the studies pursued by Greek mathematicians.

The schema behind the history of these theories can be summarized in the phrase: *Theory follows Composition*. There is hardly any theoretical proposal that would provide the composer with new ways and guidelines to drive his work. Therefore, it is mostly the case that "avant-garde" composers, such as Beethoven in his late string quartets, produce the material that musicologists try to put into theory *after the event*.

The beginning of the 20th century saw new developments of music theorists trying to give the various aspects of music, such as harmonic and formal structure, a more exact and scientific character. This is certainly connected to the rise of abstract mathematics in the 19th century, which relies on new rigorous types of notation, as well as the use of diagrammatic methods. Such theories include the *Funktionstheorie* by Hugo Riemann and the structural theories by Heinrich Schenker.

During the second half of the 20th century, finally, modern mathematics found its way into theoretical considerations of music [5]. Combinatorics is seen as playing a major role throughout the history of musical composition. Dodecaphonic composition and its generalization to serial techniques provide the telling example of this trend. Driven by the popularity of structuralist views, generative theories as they were first proposed by Noam Chomsky for the formalization of language have been applied to musical form as well [38].

This has been a breakthrough in that, for the first time, theories have been devised not least with a view to provide new means for composition. Composers start complementing their musical works by theoretical studies explicating the methods that led to their construction, for example *Musiques formelles* (1962) by the Greek composer Iannis Xenakis [74]. Several other eminent composers, such as Arnold Schoenberg, Ferruccio Busoni or Paul Hindemith, have published theoretical works that give insight into their works. It has eventually become possible to state that: *Composition follows Theory*.

In the last decades, the role of mathematics in music has been on a steady increase. The invention of digital computers in the late 1940s was soon followed by the creation of music with the help of computers (for example the famous Illiac Suite from 1957 by Hiller and Isaacson [31]) and along with it the development of mathematical theories quite specific to musical applications. It was Milton Babbitt, Hillers teacher of composition at

Princeton University, who promoted the use of mathematics, its terminology and, thus, scientific precision in studying theories of music. A music theory should be objective and be stated as a body of definitions, axioms, and theorems, just as in the case of other mathematical investigations. An analogy can be made to the relation of physics to engineering: mathematical music theories play the role of physics, and musical composition the role of engineering. It is obvious that without the theoretical work provided by physics, engineering would be impossible, or, even worse, engineering with a disregard of physics would result in bad work. Cautiously applying the analogy, we would say that contemporary composition without regard to mathematics results in bad music. . . .

Two types of composition by electronic means have been predominant. One type is exemplified by the computer programs *Project 1* (1964) and *Project 2* (1966) developed by the composer Gottfried Michael Koenig. They implement composition algorithms and operate on the level of musical structure. On the mathematical side, aleatoric and combinatorial methods play an important role [36].

Another line of development involving the use of computers for musical composition concerns the production of *sound* by means of digital processing. In this case music is controlled on a very low level, effectively down to the sound wave. Many parameters, which before were constrained by acoustic instruments, become available. These parameters allow qualities of sound never heard before and thus fruitful for new principles of composition. The pioneering work has flown into the MUSIC-N family of synthesis languages, the prototype program MUSIC having been written by Max Mathews in 1957 at Bell Labs [42]. The most recent descendant in the family, widely used today on many computer platforms, is Csound [10].

The import from mathematics to musical theories comes from all domains of abstract algebra, such as group theory. More recently, category theory has provided comprehensive tools for both analysis and composition. Mathematical studies have been conducted for fields such as counterpoint or the theory of the string quartet. A main event in this direction has been the publication of *The Topos of Music* by Guerino Mazzola [47], which uses extensive knowledge from category theory to provide theoretical treatments of a broad selection of branches in musical theory.

The mathematical approach to music analysis and composition also allows new means for joining the musical past to the future. In accordance with the idea of "analyse créatrice (creative analysis)" put forward by Pierre Boulez [49], Guerino Mazzola undertook the ambitious project of a new composition based on the analysis of the first movement of Beethoven's sonata op. 106 "Hammerklavier" [43]. The process involved the discovery of mathematical principles buried in the piano sonata and the extraction of significant parameters ("analysis"). The result is an analytical model. After changing the analytical coordinates of this model using mathematical

transformations, the model was remapped to produce a new musical work that fundamentally based on, but different from, Beethoven's composition ("creative"). The process itself is not entirely new, as similar principles have been employed in a vague manner throughout the history of music. But the mathematical approach, the exactness of the procedure, and the awareness of its happening are aspects of a modern phenomenon and open new vistas for future experimentation. The result of the project is the piano sonata "L'Essence du Bleu" which has also been recorded [48].

The sonata has been constructed completely by hand, using the traditional tools available to the composer and mathematician, such as pencil and graph paper. To manage this kind of composition, computer software implementations of the methods used would provide an ideal tool. The development of such software is of paramount importance for future experimentation in this direction. The RUBATO COMPOSER presented in this thesis is the result of this insight. It is the attempt to bring the mathematical and computational tools to a level that a composer is comfortable with. The mathematics it is based on is category theory, specifically the category of modules. Module theory is very important in mathematical music theory, since it allows the representation of the common objects in music, such as notes and chords, and the description of the usual transformations in the theory of counterpoint and composition, such as transposition, inversion and retrograde, among others. The theory of modules contains the complete theory of linear algebra, and thus Euclidean spaces, which ensures the availability of general geometric manipulations at the hands of the composer. Categorical considerations led to the development of a very general data model, called denotators, which is used throughout RUBATO COMPOSER for the transport of concepts and information.