

---

# Contents

<b>I</b>	<b>Differential Calculus in the Complex Plane <math>\mathbb{C}</math></b> . . . . .	9
I.1	Complex Numbers . . . . .	9
I.2	Convergent Sequences and Series . . . . .	24
I.3	Continuity . . . . .	36
I.4	Complex Derivatives . . . . .	42
I.5	The CAUCHY-RIEMANN Differential Equations . . . . .	47
<b>II</b>	<b>Integral Calculus in the Complex Plane <math>\mathbb{C}</math></b> . . . . .	69
II.1	Complex Line Integrals . . . . .	70
II.2	The CAUCHY Integral Theorem . . . . .	77
II.3	The CAUCHY Integral Formulas . . . . .	92
<b>III</b>	<b>Sequences and Series of Analytic Functions, the Residue Theorem</b> . . . . .	103
III.1	Uniform Approximation . . . . .	104
III.2	Power Series . . . . .	109
III.3	Mapping Properties of Analytic Functions . . . . .	124
III.4	Singularities of Analytic Functions . . . . .	133
III.5	LAURENT Decomposition . . . . .	142
A	Appendix to III.4 and III.5 . . . . .	155
III.6	The Residue Theorem . . . . .	162
III.7	Applications of the Residue Theorem . . . . .	170
<b>IV</b>	<b>Construction of Analytic Functions</b> . . . . .	191
IV.1	The Gamma Function . . . . .	192
IV.2	The WEIERSTRASS Product Formula . . . . .	210
IV.3	The MITTAG-LEFFLER Partial Fraction Decomposition . . . . .	218
IV.4	The RIEMANN Mapping Theorem . . . . .	223
A	Appendix : The Homotopical Version of the CAUCHY Integral Theorem . . . . .	233
B	Appendix : A Homological Version of the CAUCHY Integral Theorem . . . . .	239

C	Appendix : Characterizations of Elementary Domains . . . .	244
<b>V</b>	<b>Elliptic Functions</b> . . . . .	251
V.1	LILOVILLE's Theorems . . . . .	252
A	Appendix to the Definition of the Period Lattice . . . . .	259
V.2	The WEIERSTRASS $\wp$ -function . . . . .	261
V.3	The Field of Elliptic Functions . . . . .	267
A	Appendix to Sect. V.3 : The Torus as an Algebraic Curve . . . . .	271
V.4	The Addition Theorem . . . . .	278
V.5	Elliptic Integrals . . . . .	284
V.6	ABEL's Theorem . . . . .	291
V.7	The Elliptic Modular Group . . . . .	301
V.8	The Modular Function $j$ . . . . .	309
<b>VI</b>	<b>Elliptic Modular Forms</b> . . . . .	317
VI.1	The Modular Group and Its Fundamental Region . . . . .	318
VI.2	The $k/12$ -formula and the Injectivity of the $j$ -function . . . . .	326
VI.3	The Algebra of Modular Forms . . . . .	334
VI.4	Modular Forms and Theta Series . . . . .	338
VI.5	Modular Forms for Congruence Groups . . . . .	352
A	Appendix to VI.5 : The Theta Group . . . . .	363
VI.6	A Ring of Theta Functions . . . . .	370
<b>VII</b>	<b>Analytic Number Theory</b> . . . . .	381
VII.1	Sums of Four and Eight Squares . . . . .	382
VII.2	DIRICHLET Series . . . . .	399
VII.3	DIRICHLET Series with Functional Equations . . . . .	408
VII.4	The RIEMANN $\zeta$ -function and Prime Numbers . . . . .	421
VII.5	The Analytic Continuation of the $\zeta$ -function . . . . .	429
VII.6	A TAUBERIAN Theorem . . . . .	436
<b>VIII</b>	<b>Solutions to the Exercises</b> . . . . .	449
VIII.1	Solutions to the Exercises of Chapter I . . . . .	449
VIII.2	Solutions to the Exercises of Chapter II . . . . .	459
VIII.3	Solutions to the Exercises of Chapter III . . . . .	464
VIII.4	Solutions to the Exercises of Chapter IV . . . . .	475
VIII.5	Solutions to the Exercises of Chapter V . . . . .	482
VIII.6	Solutions to the Exercises of Chapter VI . . . . .	490
VIII.7	Solutions to the Exercises of Chapter VII . . . . .	498
	<b>References</b> . . . . .	509
	<b>Symbolic Notations</b> . . . . .	519



<http://www.springer.com/978-3-540-93982-5>

Complex Analysis

Freitag, E.; Busam, R.

2009, X, 532 p. 112 illus., 2 illus. in color., Softcover

ISBN: 978-3-540-93982-5