Preface to the Second Edition

The second edition of this book contains both basic and more advanced material on non-life insurance mathematics. Parts I and II of the book cover the basic course of the first edition; this text has changed very little. It aims at the undergraduate (bachelor) actuarial student as a first introduction to the topics of non-life insurance mathematics. Parts III and IV are new. They can serve as an independent course on stochastic models of non-life insurance mathematics at the graduate (master) level.

The basic themes in all parts of this book are point process theory, the Poisson and compound Poisson processes. Point processes constitute an important part of modern stochastic process theory. They are well understood models and have applications in a wide range of applied probability areas such as stochastic geometry, extreme value theory, queuing and large computer networks, insurance and finance.

The main idea behind a point process is counting. Counting is bread and butter in non-life insurance: the modeling of claim numbers is one of the major tasks of the actuary. Part I of this book extensively deals with counting processes on the real line, such as the Poisson, renewal and mixed Poisson processes. These processes can be studied in the point process framework as well, but such an approach requires more advanced theoretical tools. Parts I and II of this text are kept at a level which requires basic knowledge of probability theory and stochastic processes. Such knowledge is typically available in the 2nd or 3rd year of a bachelor program with a mathematical, statistical, actuarial or econometric orientation.

The new parts of this text tell the story of point processes in non-life insurance mathematics. The concept of a point process requires some knowledge of measure and advanced probability theory. The student who is familiar with the topics of Parts I and II will not have difficulties in continuing with Parts III and IV. Those who read from cover to cover will discover that many results in the first parts will become much more transparent and elegant in the framework of point processes. The advanced student may immediately start with the parts on point and Poisson processes. I hope that not only the specialized actuarial student will benefit from this book, but anybody who wants to know about the wonderful world of counting points.

The main objective of writing this book was to produce lecture notes for my students. The material of this book grew out of courses I have been teaching in the bachelor and master actuarial programs at the University of Copenhagen. The interaction with the students kept me sufficiently realistic about the theoretical level which one may wish to approach. Through the years the success rate of my students has confirmed that the material of this book is accessible, both at the undergraduate and graduate levels.

Different ideas have inspired the process of writing the new chapters of this book. Norberg's [114] propagation of point process theory in non-life insurance mathematics has been a guide to Chapter 8. Since the beginning of the 1990s Norberg has pointed towards the enormous gains for actuarial science by using advanced point process theory.

Insurance mathematics is not a scientific island. Through its history actuarial science has interacted with mathematical, statistical and economic disciplines. The Cramér-Lundberg theory of collective risk is a fine example of how this applied stochastic theory gained from the ideas of renewal, queuing and large deviation theory. Various excursions will lead the reader of this book into different, but related fields of applied probability theory, including extreme value theory, teletraffic models and Lévy processes. Extreme value theory is a natural tool for analyzing the large claims in an insurance portfolio. In this text we will learn about the close relationship of extremes and point processes. We will also read about point process models which are used both in insurance mathematics and for modeling large computer networks. The compound Poisson process originates in Lundberg's thesis from 1903. This was the first use of a Lévy jump process. An excursion to Lévy processes will illustrate how a fundamental non-life insurance model was extended to a major class of stochastic processes.

In the process of writing the second edition of this book I have benefitted from discussions with students, colleagues and friends. Jeffrey Collamore, Richard A. Davis, Anders Hedegaard Jessen, Jesper Lund Pedersen and Gennady Samorodnitsky have read parts of this book and proposed various improvements. My sincere thanks go to Sid Resnick for a long series of colorful and insightful comments. I have been blessed by an effort whose helpfulness went beyond what one would expect from a colleague and friend. I am indebted to Catriona Byrne, Marina Reizakis and Jef Boys from Springer-Verlag for efficient and competent professional editorial support.

Thomas Mikosch Copenhagen, October 2008

Preface to the First Edition

To the outside world, insurance mathematics does not appear as a challenging topic. In fact, everyone has to deal with matters of insurance at various times of one's life. Hence this is quite an interesting perception of a field which constitutes one of the bases of modern society. There is no doubt that modern economies and states would not function without institutions which guarantee reimbursement to the individual, the company or the organization for its losses, which may occur due to natural or man-made catastrophes, fires, floods, accidents, riots, etc. The idea of insurance is part of our civilized world. It is based on the mutual trust of the insurer and the insured.

It was realized early on that this mutual trust must be based on science, not on belief and speculation. In the 20th century the necessary tools for dealing with matters of insurance were developed. These consist of probability theory, statistics and stochastic processes. The Swedish mathematicians Filip Lundberg and Harald Cramér were pioneers in these areas. They realized in the first half of the 20th century that the theory of stochastic processes provides the most appropriate framework for modeling the claims arriving in an insurance business. Nowadays, the Cramér-Lundberg model is one of the backbones of non-life insurance mathematics. It has been modified and extended in very different directions and, moreover, has motivated research in various other fields of applied probability theory, such as queuing theory, branching processes, renewal theory, reliability, dam and storage models, extreme value theory, and stochastic networks.

The aim of this book is to bring some of the standard stochastic models of non-life insurance mathematics to the attention of a wide audience which, hopefully, will include actuaries and also other applied scientists. The primary objective of this book is to provide the undergraduate actuarial student with an introduction to non-life insurance mathematics. I used parts of this text in the course on basic non-life insurance for 3rd year mathematics students at the Laboratory of Actuarial Mathematics of the University of Copenhagen. But I am convinced that the content of this book will also be of interest to others who have a background on probability theory and stochastic processes and would like to learn about applied stochastic processes. Insurance mathematics is a part of applied probability theory. Moreover, its mathematical tools are also used in other applied areas (usually under different names).

The idea of writing this book came in the spring of 2002, when I taught basic non-life insurance mathematics at the University of Copenhagen. My handwritten notes were not very much appreciated by the students, and so I decided to come up with some lecture notes for the next course given in spring, 2003. This book is an extended version of those notes and the associated weekly exercises. I have also added quite a few computer graphics to the text. Graphs help one to understand and digest the theory much easier than formulae and proofs. In particular, computer simulations illustrate where the limits of the theory actually are.

When one writes a book, one uses the experience and knowledge of generations of mathematicians without being directly aware of it. Ole Hesselager's 1998 notes and exercises for the basic course on non-life insurance at the Laboratory of Actuarial Mathematics in Copenhagen were a guideline to the content of this book. I also benefitted from the collective experience of writing EKM [46]. The knowledgeable reader will see a few parallels between the two books. However, this book is an *introduction* to non-life insurance, whereas EKM assume that the reader is familiar with the basics of this theory and also explores various other topics of applied probability theory. After having read this book, the reader will be ready for EKM. Another influence has been Sid Resnick's enjoyable book about Happy Harry [123]. I admit that some of the mathematical taste of that book has infected mine; the interested reader will find a wealth of applied stochastic process theory in [123] which goes far beyond the scope of this book.

The choice of topics presented in this book has been dictated, on the one hand, by personal taste and, on the other hand, by some practical considerations. This course is the basis for other courses in the curriculum of the Danish actuarial education and therefore it has to cover a certain variety of topics. This education is in agreement with the Group Consultatif requirements, which are valid in most European countries.

As regards personal taste, I very much focused on methods and ideas which, in one way or other, are related to renewal theory and point processes. I am in favor of methods where one can see the underlying probabilistic structure without big machinery or analytical tools. This helps one to strengthen intuition. Analytical tools are like modern cars, whose functioning one cannot understand; one only finds out when they break down. Martingale and Markov process theory do not play an important role in this text. They are acting somewhere in the background and are not especially emphasized, since it is the author's opinion that they are not really needed for an introduction to non-life insurance mathematics. Clearly, one has to pay a price for this approach: lack of elegance in some proofs, but with elegance it is very much like with modern cars.

According to the maxim that non-Bayesians have more fun, Bayesian ideas do not play a major role in this text. Part II on experience rating is therefore rather short, but self-contained. Its inclusion is caused by the practical reasons mentioned above but it also pays respect to the influential contributions of Hans Bühlmann to modern insurance mathematics.

Some readers might miss a chapter on the interplay of insurance and finance, which has been an open subject of discussion for many years. There is no doubt that the modern actuary should be educated in modern financial mathematics, but that requires stochastic calculus and continuous-time martingale theory, which is far beyond the scope of this book. There exists a vast specialized literature on financial mathematics. This theory has dictated most of the research on financial products in insurance. To the best of the author's knowledge, there is no part of insurance mathematics which deals with the pricing and hedging of insurance products by techniques and approaches genuinely different from those of financial mathematics.

It is a pleasure to thank my colleagues and students at the Laboratory of Actuarial Mathematics in Copenhagen for their support. Special thanks go to Jeffrey Collamore, who read much of this text and suggested numerous improvements upon my German way of writing English. I am indebted to Catriona Byrne from Springer-Verlag for professional editorial help.

If this book helps to change the perception that non-life insurance mathematics has nothing to offer but boring calculations, its author has achieved his objective.

Thomas Mikosch Copenhagen, September 2003

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