Preface

Almost a century has gone by since the discovery of general relativity and quantum mechanics, yet the goal of finding a consistent theory of quantum gravity nonetheless remains elusive. After the two major triumphs of modern quantum field theory, quantum electrodynamics and the quantization of non-abelian gauge theories (including quantum chromodynamics and the electro-weak theory) the early seventies provided high hopes that a quantum treatment of general relativity might be around the corner. However, to the dismay of many, the results of t' Hooft and Veltman conclusively established that quantum gravity is not perturbatively renormalizable, thus confirming earlier suspicions based on purely dimensional arguments. Disturbingly, the divergences which appear in gravity at one loop order in the semiclassical expansion, involving curvature squared terms, cannot be re-absorbed into a redefinition of the coupling constants, thereby making it difficult to derive unambiguous statements about the properties of the underlying quantum theory. More importantly, the now exhaustively explored examples of quantum electrodynamics and non-abelian gauge theories have established that until these ultraviolet renormalization effects are consistently and systematically brought under control, it will be very difficult to make any sort of physically relevant predictions. To this day, the ultraviolet problems of quantum gravity border on the speculative for many: after all, if quantum gravity effects are relevant at distances of the order of the Planck length (10^{-33}cm), then these might very well have little relevance for laboratory particle physics in the foreseeable future. But how could one so conclude without actually doing the relevant calculations? What if new, non-perturbative scales arise in the renormalization procedure, as occurs in non-abelian gauge theories?

Since the seventies, strategies that deal with the problem of ultraviolet divergences in quantum gravity have themselves diverged. Some have advocated the search for a new theory of quantum gravity, a theory which does not suffer from ultraviolet infinity problems. In supersymmetric theories, such as supergravity and ten-dimensional superstrings, new and yet unobserved particles are introduced thus reducing the divergence properties of Feynman amplitudes. In other, very restricted classes of supergravity theories in four dimensions, proponents have claimed that enough conspiracies might arise thereby making these models finite. For superstrings, which live in a ten-dimensional spacetime, one major obstacle prevails to date: what dynamical mechanism would drive the compactification of spacetime from the ten dimensional string universe to our physical four-dimensional world, or for that matter, to any other dimension less than ten?

A second approach to quantum gravity has endeavored to pursue new avenues to quantization, by introducing new quantum variables and new cutoffs, which involve quantum Hamiltonian methods based on parallel transport loops, spacetime spin foam and new types of quantum variables describing quantum dust. It is characteristic of these methods that the underlying theory is preserved, it essentially remains a quantum version of Einstein's relativistic theory, yet the ideas employed are intended to go past the perturbative treatment. While some of these innovative tools have had limited success in exploring the much simpler non-perturbative features of ordinary gauge theories, proponents of such methods argue that gravity is fundamentally different, thereby necessitating the use of new methods.

The third approach to quantum gravity, which forms the underlying topic of this book, focuses on the application of modern methods of quantum field theory, whose cornerstones include the manifestly covariant Feynman path integral approach, Wilson's modern renormalization group ideas and the development of lattice methods to define a regularized form of the path integral, which then allows non-perturbative calculations. In non-ablelian gauge theories and in the standard model of elementary particle interactions, said methods are invariably the the tools of choice; the covariant Feynman path integral approach is crucial in proving the renormalizability of non-abelian gauge theories; modern renormalization group methods establish the core of the derivation of the asymptotic freedom result and the related discussion of momentum dependence of amplitudes in terms of running coupling constants; and finally, the lattice formulation of gauge theories, which thus far provides the only convincing theoretical evidence of confinement and chiral symmetry breaking in non-abelian gauge theories.

Therefore, this book shall cover key aspects and open issues related to a consistent regularized formulation of quantum gravity from the perspective of the covariant Feynman path integral quantization. In the author's opinion, such a formulation is an important and essential step towards a quantitative and controlled investigation of the physical content of the theory.

An Outline of the Book

This book is composed of three major sections. Part I introduces basic elements of the covariant formulation of continuum quantum gravity, with some emphasis on those issues which bear an immediate relevance for the remainder of the book. Discussion will include the nature of the spin-two field, its wave equation and possible gauge choices, the Feynman propagator, the coupling of a spin two field to matter and the implementation of a consistent local gauge invariance to all orders, ultimately leading to the Einstein gravitational action. Additional terms in the gravitational action, such as the cosmological constant and higher derivative contributions, are naturally introduced at this stage.

A section on the perturbative weak field expansion will later introduce the main aspects of background field method as applied to gravity, including issues such as the choice of field parametrization and gauge fixing. Then, results related to the structure of one- and two-loop divergences in pure gravity shall be discussed, leading up to the statement of perturbative non-renormalizability for the Einstein theory in four dimensions. One important aspect that will be stressed is that perturbative methods generally rely on a weak field expansion for the metric fluctuations, and are therefore not necessarily well suited for the investigation of potentially physically relevant regime of large metric fluctuations.

Next, the Feynman path integral for gravitation will be introduced by closely analogizing the theory with the related Yang-Mills case. The discussion brings up the thorny issue of the gravitational functional measure, a threshold requirement used to define Feynman's sum over histories, as well some other important aspects related to the convergence of the path integral and derived quantum averages, along with the origin of the conformal instability affecting the Euclidean case. Emphasis will be drawn to the strongly constrained nature of the theory, which depends on the absence of matter, and its close analogy to pure Yang-Mills theories, on a single dimensionless parameter $G\lambda$, besides the required usual short distance cutoff.

Since quantum gravity is not perturbatively renormalizable, the next question arises naturally: what other theories are not perturbatively renormalizable, and what can be derived from those theories? The following sections will thus summarize the methods of Wilson's $2+\varepsilon$ expansion as applied to gravity, expanding the deviation of the space-time dimensions from two; in such a dimension the gravitational coupling becomes dimensionless and the theory is therefore power-counting renormalizable. As an initial motivation, but also for illustrative and pedagogical purposes, the non-linear sigma model is introduced first. The latter represents a reasonably well understood perturbatively non-renormalizable theory above two dimensions which is characterized by a rich two-phase structure, and whose scaling properties in the vicinity of the fixed point can nevertheless be accurately computed in three dimensions (via the $2+\varepsilon$ expansion, as well as by other methods which include the strong coupling expansions and a variety of other lattice approximation techniques), and whose universal predictions are known to compare favorably with experiments. Within the context of gravity, which is discussed next the main results of the perturbative expansion are the existence of a nontrivial ultraviolet fixed point close to the origin above two dimensions (a phase transition in statistical field theory language), and the predictions of non-trivial universal scaling exponents in the vicinity of the new fixed point.

Generally, discussion of the quantization of gravity without referring in some detail to the Hamiltonian formulation is not possible. As in ordinary non-relativistic quantum mechanics, there are a number of important physical results which are obtained much more readily using this approach. Particularly notable, in the case of gravity, involves the nature of the Hamiltonian constraint, which implies that the total energy of a quantum gravitational system is zero, and the Wheeler-DeWitt equation, a Schrödinger-like equation for the vacuum functional, whose solution in some simple cases can be obtained using reduced phase space (minisuperspace) methods. In addition, the Hamiltonian method can be used as a starting point for a lattice description of quantum gravity, whose results may be regarded as complementary to those obtained via the Feynman path integral approach. The ambiguities that appear here as operator ordering problems have their correspondence in the path integral approach, under the rubric of issue associated with the choice of functional measure. The Hamiltonian approach also presents additional problems, including the lack of covariance due to the choice of time coordinate, and the difficulty of doing practical approximate non-perturbative calculations. Closely related to Hamiltonian approach is an array of semiclassical methods which have been used to obtain approximate cosmological solutions to the Wheeler-DeWitt equations, which are discussed later in some detail. The section ends with the exposition of some physically relevant results such as black hole radiance, and some more general issues which arise in a semiclassical treatment of quantum gravity.

Part II discusses the lattice theory of gravity based on Regge's simplicial formulation, with a primary focus on the physically relevant four-dimensional case. The starting point is a description of discrete manifolds in terms of edge lengths and incidence matrices, then moving on to a description of curvature in terms of deficit angles, thereby offering a re-formulation of continuum gravity in terms of a discrete action and a set of lattice field equations. The direct and clear correspondence between lattice quantities (edges, dihedral angles, volumes, deficit angles, etc.) and continuum operators (metric, affine connection, volume element, curvature tensor etc.) allows one to define, as an example, discrete versions of curvature squared terms which arise in higher derivative gravity theories, or more generally as radiatively induced corrections. An important element in the lattice-to-continuum correspondence is the development of the lattice weak field expansion, allowing a clear and precise identification between lattice and continuum degrees of freedom, as well as their gauge invariance properties, as illustrated for example in the weak field limit by the arbitrariness in the assignments of edge lengths used to cover a given physical geometry. On the lattice one can then see how the lattice analogs of gravitons arise naturally, and how their transverse-traceless nature is made evident.

When coupling matter fields to lattice gravity one needs to introduce new fields localized on vertices, as well as appropriate dual volumes which enter the definition of the kinetic terms for those fields. In the fermion case, it is necessary (as in the continuum) to introduce vierbein fields within each simplex, and then use an appropriate spin rotation matrix to relate spinors between neighboring simplices. In general the formulation of fractional spin fields on a simplicial lattice is useful in formulating a lattice discretization of supergravity. At this point it will be useful to compare and contrast the simplicial lattice formulation to other discrete approaches to quantum gravity such as, the hypercubic (vierbien-connection) lattice formulation and simplified fixed-edge-length approaches such as dynamical triangulations.

Subsequent sections deal with the interesting problem of what gravitational observables should look like, that is which expectation values of operators (or ratios thereof) have meaning and physical interpretation in the context of a manifestly covariant formulation, specifically in a situation where metric fluctuations are not necessarily bounded. Such averages naturally include expectation values of the integrated scalar curvature and other related quantities (involving for example curvature squared terms), as well as correlations of operators at fixed geodesic distance, sometimes referred to as bi-local operators. Another set of physical averages refer to the geometric nature of space-time itself, such as the fractal dimension. Finally, one more set of physical observables correspond to the gravitational analog of the Wilson loop, which provides information about the parallel transport of vectors, and therefore on the effective curvature, around large near-planar loops, and the correlation between particle world-lines, which gives the static gravitational potential. These quantities play an important role in the physical characterization of the two phases of gravity, as seen both in the $2+\varepsilon$ expansion and in the lattice formulation in four dimensions.

Part III of the book discusses applications of the lattice theory to non-perturbative gravity. Ultimately, investigations of the strongly coupled regime of quantum gravity where metric fluctuations cannot be assumed to be small, requires the use of numerical methods applied to the lattice theory. A discrete formulation combined with numerical tools can therefore be viewed as an essential step towards a quantitative and controlled investigation of the physical content of the theory: that is, in the same way that a discretization of a complicated ordinary differential equation can be viewed as a mean to determine the properties of its solution with arbitrary accuracy. These methods are outlined next, together with a summary of the main lattice results, showing the existence of two phases, depending on the value of the bare gravitational coupling, and in good agreement with the qualitative predictions of the $2+\varepsilon$ expansion. Specifically, lattice gravity in four dimensions is characterized by two phases: a weak coupling degenerate polymer-like phase, and a strong coupling smooth phase with small average curvature. The somewhat technical aspect of the determination of universal critical exponents and non-trivial scaling dimensions, based on finite size methods, is outlined, together with a brief discussion of how the lattice continuum limit has to be approached in the vicinity of a non-trivial ultraviolet fixed point.

The determination of non-trivial scaling dimensions in the vicinity of the fixed point leads to a discussion of the renormalization group properties of fundamental couplings, that is their scale dependence, as well as the emergence of physical renormalization group invariant quantities, such as the fundamental gravitational correlation length and the closely related gravitational condensate. These are discussed next, with an eye towards physical applications. This includes a discussion of the physical nature of the expected quantum corrections to the gravitational coupling, based, in part on an analogy to QED and QCD, on the effects of a virtual graviton cloud, and of how the two phases of lattice gravity relate to the two opposite scenarios of gravitational screening (for weak coupling, and therefore unphysical due to the branched polymer nature of this phase) versus anti-screening (for strong coupling, and therefore physical).

A final section touches on the general problem of formulating running gravitational couplings in a context that does not assume weak gravitational fields. The discussion includes a brief presentation on the topic of covariant running of G based on the formalism of non-local field equations, with the scale dependence of G expressed through the use of a suitable covariant d'Alembertian. Simple applications to standard metrics (static isotropic and homogeneous isotropic) are briefly summarized and their potential physical consequences and interpretation elaborated. The book ends with a general outlook on future prospects for lattice studies of quantum gravity, some open questions and work that can be done to help elucidate the relationship between discrete and continuum models, such as extending the range of problems addressed by the lattice, and providing new impetus for further developments in covariant continuum quantum gravity.

One final comment on the notation used in this book. Unless stated otherwise, the same notation is used as in (Weinberg, 1972), with the sign of the Riemann tensor reversed; the signature in the Lorentzian case is therefore $-,+,+,+$. In the Euclidean case $t = -i\tau$ on the other hand the flat metric is of course the Kronecker $\delta_{\mu\nu}$, with the same conventions as before for the Riemann tensor.

Berlin, May 2008 *Herbert W. Hamber*

Springer

http://www.springer.com/978-3-540-85292-6

Quantum Gravitation
The Feynman Path Integral Approach
Hamber, H.W.
2009, XVIII, 342 p. 60 illus., Hardcover
ISBN: 978-3-540-85292-6