Altruistic Utility Functions for Joint Decisions

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1 Introduction

All of us make decisions that are not entirely self-centered; we voluntarily anticipate what we think to be the preferences of others and incorporate them into our decision making. We do this, not because of legal requirements or social norms, but because we are altruistic; we care intrinsically about the welfare of others. In this paper, we illustrate for these types of decisions how confusion may arise because the distinction between our personal (egotistical) preferences and our altruistic concerns is not carefully distinguished. We first define the distinction between personal and altruistic preferences, and then show how to use both of these kinds of preferences in prescriptive decision making methodologies.

We confine ourselves to the class of problems where two or more people must select a common course of action. The following story illustrates a simple example. Joan and Dan have decided to have dinner and must choose a restaurant. They quickly specify three possibilities: a Brazilian restaurant, a French restaurant, and a Thai restaurant. Joan is thoughtful and wishes to choose a restaurant that Dan will really like. Similarly, Dan wants to choose a restaurant that pleases Joan. So what happens? Joan, thinking about what might be Dan's preferences, decides that Dan would like the French restaurant, followed by the Brazilian restaurant, followed by the Thai restaurant. Dan, thinking about what Joan would like, also decides that the French restaurant would be best, followed by the Brazilian restaurant, and then the Thai restaurant. Joan speaks first and suggests the French restaurant. Dan, thinking that this is what Joan wants, agrees and off they go. During dinner discussion, Joan mentions that she would have preferred the Thai restaurant to the French restaurant. Somewhat surprised, Dan then says that he also would have preferred the Thai restaurant. They wonder how this state of affairs came about.

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Two compounding errors led to an inferior choice. First, each person guessed at the others preferences. Second, the stated preferences are mistakenly interpreted as those of the speaker. How could this have been avoided? Clearly both Dan and Joan could have written down their personal preference order for restaurants, assuming that they did not care about the other's preferences, and then compared notes. In our illustration, this would have led immediately to a mutually satisfactory decision. Our experience is that even decision analysts are rarely that explicit. What often happens instead is that through discussion, or generalized experience with the other person, each person informally updates their own preferences to take account of the other's likes and dislikes. There are many ways this informal approach can produce inadequate solutions.

There are many situations where a group of individuals must collectively choose among alternatives and where each individual wishes to please the others. Examples include parents making choices with their children, decisions by boards of directors, decisions by departments or groups within organizations, decisions by legislative or regulatory bodies, choices made by families, and decisions among friends. In many of these cases, parties to the decision will take account of the preferences of the others, not only for the expediency of arriving at a consensus, but often out of an altruistic interest in their happiness. An altruistic decision maker will be willing to forgo some direct personal gain to help others achieve their objectives.

The general problem of combining preferences of individuals into group preferences is not new. There is a large body of published work on this topic ((Arrow, 1951), (Harsanyi, 1955), (Diamond, 1967), (Sen, 1979), (Broome, 1984), and many others). Much of the work prior to 50 years ago is summarized in (Luce & Raiffa, 1957). Since that time, there has been work on risk sharing (e.g. Raiffa, 1968), group utility functions (e.g. Keeney & Raiffa, 1976), and utility functions where a seller incorporates the preferences of a buyer (Edgeworth, 1881), (Raiffa, 1982), (Keeney & Lilien, 1978), and (Keeney & Oliver, 2005). There has also been work on preference dependencies in multiattribute utility functions (Fishburn, 1965), (Bell, 1977), (Meyer, 1977), (Keeney, 1981). Several authors have discussed the adaptation of preferences in a group context (Zizzo, 2005), (Sobel, 2005), and (Cubitt & Sugden, 1998). Trautmann (2006) proposes a similar approach to ours, but his proposal is based on the descriptive criterion suggested by (Fehr & Schmidt, 1999), whereas ours is consistent with standard multiattribute approaches, and amenable to assessment as we discuss later.

We focus in this paper on one particular type of joint decision. One could think of this type as altruistic joint decisions, because each of the individuals has a fundamental preference for the other individuals being pleased. Section 2 defines an altruistic joint decision and discusses its relevance. As conceptual distinctions are extremely important in discussing problems with interpersonal dependence of preferences, Sect. 3 outlines the relevant concepts and terminology used to analyze altruistic joint decisions. In Sect. 4, we focus on altruistic joint decisions involving two individuals and illustrate the main results that collectively characterize a reasonable set of altruistic utility functions to use in analyzing joint decisions. Section 5 elaborates on the foundations for altruistic utility functions. Section 6 suggests how one might assess these utility functions, and Sect. 7 is a discussion of the insights from and uses of the concepts and results of the paper.

2 Altruistic Joint Decisions

The altruistic joint decisions that we investigate in this paper are characterized by six properties:

1. A group of individuals have a decision that they must make jointly,
2. The alternatives are exogenously given,
3. All individuals in the group bear the same consequences,
4. Each individual has utilities for the alternatives,
5. Each individual is altruistic about the others; they prefer them to be happy even at some cost to themselves and,
6. Each person is honest in revealing their preferences.

Property 3 rules out decisions that involve risk sharing or somehow dividing the consequences among the individuals. With regard to Property 4, the individuals may have utility functions over the consequences which can be used to derive utilities for the alternatives. Property 5 is the one that states the altruism assumption. Without it, we would have the more general situation sometimes referred to as the group decision problem. Property 6 eliminates the need to worry about strategic gaming; Property 2 is included to give the additional "safeguard" that individuals do not introduce irrelevant alternatives to skew the decision making procedures.

It is useful to analyze altruistic joint decisions for many reasons. First, as suggested above, they occur often in the real world. Second, the consequences are frequently important. Poor choices increase the likelihood of a disastrous vacation or a poor business outcome. Such consequences can contribute to dissolve what was previously a wonderful group of friends, a terrific marriage, or an exciting and productive business relationship. Third, *ad hoc* choices on altruistic decisions may contribute to poor choices and hence less desirable consequences. The reason this may occur is because there are sophisticated concepts necessary to take into account in altruistic joint decisions. Self-centered preferences for consequences can get confused or be confused with altruistic concerns for those same consequences. A little analysis can help define and distinguish these aspects.

3 Concepts and Terminology

We characterize an altruistic decision as follows: There are J alternatives a_j, $j = 1,\ldots,J$, one of which must be chosen by an altruistic group. The group has N individuals, referred to as $I_1,\ldots,I_N$. Each individual I_i has a personal utility function u_i

over the alternatives. This *egotistical utility function* only incorporates the value of the alternative directly to the individual and does not include any value to I_i due to his or her altruistic feelings for the happiness of others. Thus, for each alternative a_j, individual I_i assigns an egotistical utility $u_i(a_j)$.

Each individual also has what we refer to as an *altruistic utility function* $U_i,\ i = 1,\ldots,N$ which is the function that describes the preferences the person announces or acts upon, which takes into account both his or her personal concerns and concerns for the welfare of the others. For example, I_1's evaluation of alternative a_j might be expressed as $U_1(u_1(a_j), U_2(a_j), \ldots, U_N(a_j))$. An example of an individual altruistic utility function for individual I_1 is the additive form

$$U_1\left(u_1\left(a_j\right), U_2\left(a_j\right), \ldots, U_N\left(a_j\right)\right) = k_1 u_1\left(a_j\right) + \sum_{i=2}^{N} k_i U_i\left(a_j\right), \tag{1}$$

where u_1 and $U_i, i = 1,\ldots,N$ are scaled 0 to 1, $k_1 > 0$ (the person is not totally altruistic) and the scaling factors $k_2,\ldots,k_N$ are also non-negative to incorporate the altruism that individual I_1 feels for individuals $I_i, i = 2,\ldots,N$.

The *group altruistic utility function* U_G is a utility function that incorporates the preferences of each of the individuals in the group. In general the arguments in this utility function can be each individual's egotistical and/or altruistic utility function. A possible example is the additive utility function

$$U_G\left(a_j\right) = \sum_{i=1}^{N} K_i U_i\left(a_j\right), \tag{2}$$

where the scaling factors $K_i, i = 1,\ldots,N$ must be positive to incorporate altruism of each individual for the other individuals.

4 Main Results for Altruistic Decisions

In this section, we present our main analytical results. To focus on the conceptual ideas, all of the work in this section concerns a joint altruistic decision made by two individuals. We begin by stating our most important analytical results, though the assumptions we use for Result 1 are stronger than necessary. In Sect. 5, these assumptions are weakened. The ideas also extend to altruistic groups of more individuals as discussed in Sect. 7.

Result 1. An individual's altruistic utility function should have two attributes which are the egotistical utility functions of the two individuals. The resulting two-attribute function, should be multiplicative (or additive) in those attributes. Thus,

$$U_1\left(a_j\right) = k_1 u_1\left(a_j\right) + k_2 u_2\left(a_j\right) + k_3 u_1\left(a_j\right) u_2\left(a_j\right) \tag{3}$$

and

$$U_2\left(a_j\right) = k_4 u_1\left(a_j\right) + k_5 u_2\left(a_j\right) + k_6 u_1\left(a_j\right) u_2\left(a_j\right), \tag{4}$$

where all utility functions are scaled 0 to 1, all k_i scaling factors are positive, and $k_1+k_2+k_3=1$ and $k_4+k_5+k_6=1$. The scaling factors indicate the relative importance of the ranges of consequences possible on the corresponding utility function as discussed in the assessment Sect. 6.

Argument. As we will discuss in Sect. 5, it might be tempting to think that one person's altruistic utility function should be a function of the other person's altruistic function, but, as we shall see, this leads to problems. We believe that a fundamental property of altruism is that if individual I_1, say, is personally indifferent among the available alternatives then he or she would wish to select the alternative that maximizes the other individual's egotistical utility function. For example, if Dan personally regards all of the restaurant alternatives as equally preferable, surely he would wish to select the one that Joan most prefers. One might imagine that if Dan dislikes the available restaurants, then he might be jealous if Joan is delighted, but that does not meet our sense of altruism. Similarly if Joan is personally indifferent among the available restaurants, then surely Dan should feel comfortable selecting his own favorite, especially since he knows Joan is altruistic towards him (we assume all parties are altruistic). In the language of multiattribute utility, we have therefore concluded that individual I_1's altruistic utility function should have the two attributes and each should be utility independent of the other. Thus his altruistic utility function should have the form (3), and by symmetry, individual I_2's should have the form (4).

The factors k_1 and k_5 are positive as each individual certainly cares about their own direct consequences. Factors k_2 and k_4 are positive as both individuals are altruistic. We argue below that k_3 and k_6 should at least be non-negative and more likely are positive.

Suppose individual I_1 has a choice between two alternatives, one with $(u_1,u_2)=(x,y)$ and the other with $(u_1,u_2)=(x-b,y+c)$. Hence, I_1 must decide if for her the sacrifice of an amount of utility b is worth the improvement of an amount of utility c to individual I_2. Using her altruistic utility function (3), we see the answer is yes if $U_1(x-b,y+c)>U_1(x,y)$ which implies $k_1(x-b)+k_2(y+c)+k_3(x-b)(y+c)>k_1x+k_2y+k_3xy$, so

$$-k_1b+k_2c+k_3\,(cx-by-bc)>0. \tag{5}$$

Dividing (5) by bc yields

$$-k_1/c+k_2/b-k_3+k_3\,(x/b-y/c)>0. \tag{6}$$

If $k_3=0$, then this preference is independent of x and y. If $k_3>0$, then I_1 is relatively more altruistic when x is high or y is low. We believe this is more in line with how altruistic people will like to behave than when $k_3<0$. Thus, in general, it seems reasonable to have $k_3>0$, so $k_6>0$ also by the same argument. It is worth mentioning that all of our results hold for the cases when $k_3=0$ and/or $k_6=0$

though this is not required. It is quite possible that a person's level of altruism could vary depending on the actual disparity in egotistical utility each person derives from a consequence.

Result 2. The group altruistic utility function should be additive over the two arguments of the individual's altruistic utility functions, so

$$U_G\left(a_j\right) = C_1U_1\left(a_j\right) + C_2U_2\left(a_j\right), \tag{7}$$

where all utility functions are scaled 0 to 1 and $C_1 + C_2 = 1$.

Argument. The utility function U_G represents how the pair of individuals should evaluate joint decisions. It seems reasonable to suppose that if individual I_1 is indifferent among alternatives using U_1, then both individuals would be happy to let the joint decision be consistent with U_2. By symmetry the reverse would be true. Hence U_G should be multiplicative or additive in U_1 and U_2: $U_G(a_j) = C_1U_1(a_j) + C_2U_2(a_j) + C_3U_1(a_j)U_2(a_j)$.

Now we argue that C_3 should be zero. Consider two gambles involving lotteries over the alternatives. Suppose that individual I_1 has the same expected altruistic utility under either gamble. Suppose this is also true for individual I_2. Then both individuals are indifferent between the two gambles so it seems reasonable that U_G should reflect that indifference. As proven in (Harsanyi, 1955) and (Fishburn, 1984), this only occurs when $C_3 = 0$. If C_3 were greater than zero, for example, it would mean that the group might prefer an alternative with lower values of U_1 and U_2 in order to achieve more concordance between U_1 and U_2. But since U_1 and U_2 already, respectively, incorporate all of I_1's and I_2's altruistic concerns, any further sacrifice is counter-productive.

The conclusion that $C_3 = 0$, while not obvious, is consistent with the observation of (Keeney, 1981), namely that when the objectives are fundamental, complete, and do not overlap, an additive utility function is appropriate. The two individual altruistic utility functions are fundamental and a complete set in that they consider all objectives relevant to the decision (e.g. individual I_1's concerns are completely expressed by U_1) and do not overlap. Each individual altruistic utility function addresses both direct and altruistic preference concerns. It is also consistent with (Edgeworth, 1881) and (Harsanyi, 1955) who both argued that an altruistic solution could be determined by maximizing the sum of the affected individual's utilities.

Result 3. The group altruistic utility function is the multiplicative utility function with the egotistical utility functions of the individuals as the arguments, so

$$U_G\left(a_j\right) = K_1u_1\left(a_j\right) + K_2u_2\left(a_j\right) + K_3u_1\left(a_j\right)u_2\left(a_j\right), \tag{8}$$

where K_i, $i = 1, 2, 3$ are positive and $K_1 + K_2 + K_3 = 1$.

Argument. The argument in this case is a proof using Results 1 and 2. Substituting (3) and (4) into (7) and dropping the a_j's for clarity yields

$$\begin{aligned} U_G &= C_1(k_1u_1 + k_2u_2 + k_3u_1u_2) + C_2(k_4u_1 + k_5u_2 + k_6u_1u_2) \\ &= (C_1k_1 + C_2k_4)u_1 + (C_1k_2 + C_2k_5)u_2 + (C_1k_3 + C_2k_6)u_1u_2. \end{aligned} \tag{9}$$

Equation (9) is (8) with $K_1 = C_1k_1 + C_2k_4$, and so on.

The group utility function (8) is not necessarily additive in the individual's personal utilities. This is because if the altruism of any member of the group (their willingness to give up utility to help someone else) depends on their own level of satisfaction, then the multiplicative term will be present in their individual altruistic utility function, and therefore in the group function also. The functional form (8) is mathematically identical to an analysis of the group decision problem (Keeney and Kirkwood, 1975) that posited a possible concern by the group for equity. In that development the multiplicative term reflects the desire by the group not to have disparate outcomes. It is possible for both phenomena to occur at the same time; someone could be altruistic but also concerned about equity.

5 Personal Utilities are Fundamental to Altruistic Decisions

In Result 1, we made a strong assumption that the arguments in an individual's altruistic utility function should be the egotistical utilities of the individuals. We did this so the important results in Sect. 4, and the logic supporting them would be clear. Here, from more basic reasoning, we provide support for having egotistical utilities as arguments in altruistic utility functions.

Result 4. The egotistical utility functions should be the arguments in the altruistic utility functions.

Argument. We asserted the truth of Result 4 in stating our Result 1. But why is that the case? It might seem reasonable to think that the altruistic utility function of individual I_1 might depend on hers and on I_2's altruistic utility functions. But that is circular. For example, if

$$U_1(U_1, U_2) = h_1U_1 + h_2U_2 + h_3U_1U_2, \tag{10}$$

where we have deleted the a_j's for clarity, it is evident that $h_1 = 1$ and $h_2 = h_3 = 0$ is the only viable solution.

Another way to think about the appropriate attributes for U_1 is that it can be a function of u_1 and U_2 so individual I_1's altruistic utility function could be represented by

$$U_1(u_1, U_2) = h_1u_1 + h_2U_2 + h_3u_1U_2, \tag{11}$$

and similarly for individual I_2,

$$U_2(U_1, u_2) = h_4U_1 + h_5u_2 + h_6U_1u_2. \tag{12}$$

But (11) and (12) together lead to problems of double counting. One way to see this is by substituting (12) into (11) which yields

$$U_1(u_1, U_2) = h_1u_1 + h_2(h_4U_1 + h_5u_2 + h_6U_1u_2) + h_3u_1(h_4U_1 + h_5u_2 + h_6U_1u_2). \tag{13}$$

Substituting (11) into (13) results in squared terms of u_1 if either h_3 or h_6 is not zero, and squared terms are unreasonable. The problem stems from the fact that individual I_1, in trying to please individual I_2, who is trying to please individual I_1, ends up double counting his own interests. The intent of any utility function is to maximize its expected value, which is a simple calculation, not to maximize its square. Hence, h_3 and h_6 would necessarily have to be zero if (11) and (12) were reasonable.

Even if $h_3 = h_6 = 0$ in (11) and (12), there are still difficulties. Substituting (12) into (11) yields

$$U_1(u_1, U_2) = h_1u_1 + h_2(h_4U_1 + h_5u_2). \tag{14}$$

Solving (14) for U_1, we find

$$U_1 = (h_1u_1 - h_2h_5u_2)/(1 - h_2h_4). \tag{15}$$

As a numeric example suppose that $h_1 = h_4 = 0.2$, $h_2 = h_5 = 0.8$, and $h_3 = h_6 = 0$. That is, individual I_1 is very altruistic and assigns 80% of the weight to the preferences of individual I_2, whereas individual I_2 is less altruistic but does assign a 20% weight to individual I_1's preferences.

Substituting the values for the h's into (15) we find

$$U_1 = \frac{5}{21}u_1 + \frac{16}{21}u_2. \tag{16}$$

Similar calculations for I_2 yield

$$U_2 = \frac{1}{21}u_1 + \frac{20}{21}u_2. \tag{17}$$

Thus, although both individuals agree, in a sense, that 80% of the weight should be on the preferences of individual I_2, the calculations show that the double counting leads to a different outcome. It is possible that in selecting weights for (10), individual I_1 correctly anticipates the effect of the double counting, but we believe that for most individuals this would be challenging.

If we consider the group altruistic utility function (7) in this case, any choice of C_1 and C_2 necessarily leads to a weighting of individual I_1's personal utility of less than 20%. We conclude that altruistic utility functions should be based on individuals' egotistical utility functions rather than on other altruistic utility functions.

6 Assessment Issues

Based on the results in Sect. 4, the group altruistic utility function could be assessed based on either Result 2 or 3. The best way to make the necessary assessments is to use Result 2. This requires first assessing the two individuals' egotistical utility functions, then both individuals' altruistic utility functions (3) and (4), and then the scaling factors C_1 and C_2 in (7).

The individuals' egotistical utility functions should be assessed using standard procedures as outlined in Keeney and Raiffa (1976) and many other sources. There is nothing special about these utility functions as they are simply utility functions for an individual concerned with consequences directly to that individual.

Assessing the individuals' altruistic utility functions are also just individual assessments. Relative weights on the individual egotistical utility functions in (3) and (4) incorporate two separate issues. One of these is the well-known interpersonal comparison of utility problem (Luce & Raiffa, 1957) and the other is the altruistic value to each individual for pleasing each other. To make these assessments requires understanding the relative desirability of the impacts for each individual of going from the worst of their alternatives to the best of their alternatives. For instance, if the two individuals are selecting a restaurant together, the range for individual I_1 may be in qualitative terms from a poor restaurant that would be "acceptable food and pleasant atmosphere" to a best restaurant that would be "good food and pleasant atmosphere." For the second individual I_2, the range could go from "very unappealing food and objectionable atmosphere" to "excellent food and perfect atmosphere." In such a situation, each individual may decide to place greater weight on I_2's utilities as I_2 seems to have a much more significant difference in desirability of the worst and the best of restaurants.

To determine appropriate relative scaling factors (the k's) for the individuals' altruistic utility functions given by (3) and (4), each individual should consider the range of the desirability of the various consequences to each individual as well as how much weight she wants to place on pleasing the other individual. Consider the scaling factors k_1, k_2, and k_3 in (3). The best way to assess these factors is to compare specific alternatives in terms of their egotistical utilities to both individuals I_1 and I_2 and look for pairs of alternatives that the individual feels are equally desirable. Once two pairs of such joint consequences described by (u_1, u_2) are found indifferent, the individual's altruistic utility function (3) should equate the utilities of the pairs. This provides two equations with three unknowns, namely k_1, k_2, and k_3. The fact that $k_1 + k_2 + k_3 = 1$ is a third equation. These three equations can be solved to yield specific values for the three scaling factors. Note that the altruistic function just assessed is the function the individual would use if he or she were to make the group decision unilaterally. Put another way, it represents the preferences that this individual would use if the decision were left up to her.

Assessing C_1 and C_2 in the group altruistic utility function (7) is the only value judgment in the assessment process requiring agreement of the two individuals. The value judgments about C_1 and C_2 are basically assessments about the relative significance of each person to the group. With individuals who have altruistic feelings for each other, it seems reasonable to select $C_1 = C_2 = 0.5$. That is because all of the more conceptually difficult value judgments concerning altruism and strength of preferences are incorporated into each individual altruistic utility function. As a specific example, suppose individual I_1 selected $k_1 = 0.6$ and $k_2 = 0.4$ in her utility function (3), so $k_3 = 0$. This would mean that she thought her personal utility function counted 1.5 times as much as I_2's personal utility function. It would not then seem reasonable to underweight her altruistic preferences relative to those of individual I_2, with $C_1 < 0.5$, or to overweight them, with $C_2 > 0.5$. Obviously, similar reasoning holds for individual I_2.

7 Insights and Uses

The insights in this paper can be used informally or formally in making joint decisions. Indeed, we would expect that the more common use of the concepts would be in making decisions informally, but thoughtfully.

One basic finding is that the informal notion of agreement through discussion and iterated compromise, while intuitively attractive, is fraught with difficulty: even if the process converges, the compromise solution might not be the appropriate solution.

Though it may appear to be selfish, it is important for altruistic decision makers to focus initially on what they know best, their own personal (egotistical) utilities. These are the utilities the individual has for the direct consequences of an alternative. Each individual naturally knows much more about his or her own preferences than about the other individual's preferences. There is no reason for the guessing effort to occur in altruistic decisions. Each individual should honestly first express their own preferences for themselves. Once these are clearly laid out for both individuals to understand, then any appropriate weighting by each individual to account for the personal utilities and the altruistic concerns can more effectively occur.

An important insight from this work is that an altruistic utility function should be over the egotistical utility functions. In particular, a multiplicative utility model is a general model that can address these concerns for individuals and for joint decisions of two individuals. The altruistic values that each of the individuals have can be addressed in assessing the scaling factors in the multiplicative utility function.

So how would one use this theory on a simple decision like Joan and Dan's choice of a restaurant? First Joan and Dan should express their personal preferences for the restaurants to each other. If they agree on their first choice, choose it. If they disagree, eliminate any dominated alternatives. Then they should discuss their personal strengths of preference among the remaining contenders, and then jointly decide based on that information. Either the choice should be obvious or it should not matter as they are about equally desirable in the joint sense.

Results 2 and 3 together state that the group altruistic utility function is additive over the two individual's altruistic utility functions and also multiplicative over those two individual's egotistical utility functions. This demonstrates the significance of how framing a decision, in this case specifying the objectives explicitly included in the analysis of a decision, can and should influence the functional form of the appropriate utility function.

The insights discussed above generalize to joint decisions involving more than two individuals. Specifically, the multiplicative utility function is an appropriate formulation for a joint altruistic utility function and the arguments of that function should be the egotistical utility functions. The altruistic values of each of the individuals are addressed in assessing the scaling factors in that altruistic utility function.

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