The goal of this book is to present the results of Classical Invariant Theory (abbreviated CIT) and those of Standard Monomial Theory (abbreviated SMT) in such a way as to bring out the connection between the two theories. Even though there are many recent books on CIT, e.g., [25, 35, 53, 97, 99], none of them discusses SMT: there is but only a passing mention of the main papers of SMT towards the end of [53]. Details about the connection are also not to be found in the comprehensive treatment of SMT [59] that is in preparation. Hence the need was felt for a book that describes in some detail this natural and beautiful connection.

After presenting SMT for Schubert varieties—especially, for those in the ordinary, orthogonal, and symplectic Grassmannians—it is shown (using SMT) that the categorical quotients appearing in CIT may be identified as “suitable” open subsets of certain Schubert varieties. Similar results are presented for certain canonical actions of the special linear and special orthogonal groups. We have also included some important applications of SMT: to the determination of singular loci of Schubert varieties, to the study of some affine varieties related to Schubert varieties—ladder determinantal varieties, quiver varieties, variety of complexes, etc.—and to toric degenerations of Schubert varieties.

Prerequisite for this book is some familiarity with commutative algebra, algebraic geometry and algebraic groups. A basic reference for commutative algebra is [27], for algebraic geometry [37], and for algebraic groups [7]. We have also included a brief review of GIT (geometric invariant theory), a reference for which is [87] (and also [96]).

We have mostly used standard notation and terminology, and have tried to keep notation to a minimum. Throughout the book, we have numbered Theorems, Lemmas, Propositions etc., in order according to their section and subsection; for example, 3.2.4 refers to fourth item in the second subsection of third section of the present chapter. The chapter number is also mentioned if the item appears in another chapter.

This book may be used for a year long course on invariant theory and Schubert varieties. The material covered in this book should provide adequate preparation for
graduate students and researchers in the areas of algebraic geometry and algebraic groups to work on open problems in these areas.

A Homage & an acknowledgement: In the original plan for this book, Musili was one of the co-authors. Unfortunately, Musili passed away suddenly on Oct 9, 2005. We dedicate this book to the memory of Musili. We also would like to thank Ms. Bhagyavati (Musili’s wife) and Ms. Lata (Musili’s daughter) for providing us with the files that Musili had prepared.

We wish to thank the referees for their comments.

Boston, Trieste

October 2007

V. Lakshmibai
K. N. Raghavan
Standard Monomial Theory
Invariant Theoretic Approach
Lakshmibai, V.; Raghavan, K.N.
2008, XIV, 266 p., Hardcover
ISBN: 978-3-540-76756-5