3 Flow properties of bulk solids

The flow properties of bulk solids depend on many parameters, e.g.:

- particle size distribution,
- particle shape,
- chemical composition of the particles,
- moisture,
- temperature.

It is not possible to determine theoretically the flow behavior of bulk solids as a function of all of these parameters. Even if this were possible, the expense for the determination of all parameters of influence would be very high. Thus it is necessary, and also simpler, to determine the flow properties in appropriate testing devices.

To approach the subject of this section, at first a simple test procedure, the uniaxial compression test, will be considered. Following this the measurement of flow properties with shear testers based on Jenike’s contributions [3.1] will be explained.

3.1 Uniaxial compression test

3.1.1 Consolidation of bulk solids

The phrase “good flow behavior” usually means that a bulk solid flows easily, i.e., it does not consolidate much and flows out of a silo or a hopper due to the force of gravity alone and no flow promoting devices are required. Products are “poorly flowing” if they experience flow obstructions or consolidate during storage or transport. In contrast to these qualitative statements, a quantitative statement on flowability is possible only if one uses an objective characteristic value that takes into account those physical characteristics of the bulk solid that are responsible for its flow behavior.

“Flowing” means that a bulk solid is deformed plastically due to the loads acting on it (e.g. failure of a previously consolidated bulk solid sam-
The magnitude of the load necessary for flow is a measure of flow-ability. This will be demonstrated first with the uniaxial compression test.

In Fig. 3.1 a hollow cylinder filled with a fine-grained bulk solid is shown (cross-sectional area $A$; internal wall of the hollow cylinder assumed as frictionless). The bulk solid is loaded by the stress $\sigma_1$ – the consolidation stress or major principal stress – in the vertical direction and, thus, compressed. The more the volume of the bulk solid is reduced, the more compressible the bulk solid.

With an easy-flowing, dry bulk solid with large, hard particles (e.g., wheat grains or glass beads), bulk density will increase very little. With a fine and/or moist bulk solid (e.g. flour, moist sand), one will observe a clear increase in bulk density.

In addition to the increase in bulk density from consolidation stress, one will observe also an increase in strength of the bulk solid specimen. Hence, the bulk solid is both consolidated and compressed through the effect of the consolidation stress.

After consolidation, the bulk solid specimen is relieved of the consolidation stress and the hollow cylinder is removed. If subsequently the consolidated cylindrical bulk solid specimen is loaded with an increasing vertical compressive stress, the specimen will break (fail) at a certain stress. The stress causing failure is called compressive strength, cohesive strength, or unconfined yield strength, $\sigma_c$ (another common designation is $f_c$ as introduced by Jenike [3.1])

In bulk solids technology one calls the failure “incipient flow”, because at failure the consolidated bulk solid specimen starts to flow. Thereby the bulk solid dilates somewhat in the region of the surface of the fracture, since the distances between individual particles increase. Therefore incipient flow is plastic deformation with decrease of bulk density (see Sect. 2.5). Since the bulk solid fails only at a sufficiently large vertical stress, which is equal to its compressive strength, there must exist a material-
specific yield limit for the bulk solid. Only when this yield limit is reached does the bulk solid start to flow.

The yield limits of many solid materials (e.g. metals) are listed in tables. However, the yield limit of a bulk solid is dependent also on its stress history, i.e., previous consolidation: The greater the consolidation stress, $\sigma_1$, the greater are bulk density, $\rho_b$, and unconfined yield strength, $\sigma_c$.

Uniaxial compression tests (Fig. 3.1) conducted at different consolidation stresses, $\sigma_1$, yield different pairs of values ($\sigma_c$, $\sigma_1$) and ($\rho_b$, $\sigma_1$). Plotting these pairs of values as points in a $\sigma_c, \sigma_1$ diagram and a $\rho_b, \sigma_1$ diagram, respectively, and drawing in each diagram a curve through these points, usually results in curves like those for product A in Fig. 3.2, where bulk density, $\rho_b$, and unconfined yield strength, $\sigma_c$, typically increase with consolidation stress, $\sigma_1$. Very rarely a curve $\sigma_c(\sigma_1)$ like curve B is obtained where within a limited range of the consolidation stress a progressive slope is observed. The curve $\sigma_c(\sigma_1)$ is called the flow function, or instantaneous flow function to emphasize that the strength is measured directly after consolidation.

Fig. 3.2. Bulk density, $\rho_b$, and unconfined yield strength, $\sigma_c$, in dependence on consolidation stress, $\sigma_1$
3.1.2 Time consolidation

Some bulk solids increase in strength if they are stored for a period of time at rest under a compressive stress (e.g. in a silo or an IBC). This effect is called time consolidation or caking. Time consolidation is the result of the increase of interparticle adhesive forces with time based on different mechanisms (see Sect. 2.6). If particles are moved relative to each other, these adhesive forces diminish and can build up again during further storage at rest.

Time consolidation can be determined with the test shown in Fig. 3.1, in order, for example, to simulate long-term storage in a silo. To conduct this test one loads the specimen with consolidation stress, $\sigma_1$, not just for a short moment, but for a defined period of time, $t$. Then the unconfined yield strength is determined following the principle explained above (right part of Fig. 3.1).

In Fig. 3.3 the instantaneous flow function $\sigma_c(\sigma_1)$ of product A (see Fig. 3.2) is shown. The instantaneous flow function represents the unconfined yield strength without influence of time consolidation, i.e., for a storage period $t = 0$. Additionally, examples of curves $\sigma_c(\sigma_1)$ for storage periods $t > 0$ (curves A1, A2) are drawn. The curves $\sigma_c(\sigma_1)$ for the storage periods $t > 0$ are called time flow functions. Here each curve results from the connection of several pairs of values $(\sigma_c, \sigma_1)$, which were measured at identical storage periods, $t$, but at different consolidation stresses, $\sigma_1$.

![Fig. 3.3. Instantaneous flow function and time flow functions](image)

For the example of bulk solid A, its unconfined yield strength, $\sigma_c$, increases with increasing storage time. This result is true for many bulk solids, but not for all. There are bulk solids which undergo no or only very slight consolidation over time; i.e., $\sigma_c$ do not increase or increase only very slightly with increasing storage period (e.g. dry quartz sand). Other bulk solids un-
dergo a large increase in unconfined yield strength after storage periods of only a few hours, whereas after longer storage periods their unconfined yield strength does not increase further. These differences are due to the different physical, chemical, or biological effects that are the causes of consolidation over time, e.g. chemical processes, crystallizations between the particles, enlargement of the contact areas through plastic deformation, capillary condensation, or biological processes such as fungal growth (see Sect. 2.6).

With measurement of time consolidation, a “time-lapse effect” is not realizable, i.e., one must store a bulk solid specimen at the consolidation stress, $\sigma_1$, for exactly that period of time for which one would like to get data on time consolidation. Without such a test no quantitative statement can be made regarding time consolidation.

3.1.3 Representation of stresses using Mohr stress circles

The uniaxial compression test presented in Fig. 3.1 is shown below in a $\sigma, \tau$ diagram (Fig. 3.4). If one neglects the force of gravity acting on the bulk solid specimen and assumes that no friction is acting between the wall of the hollow cylinder and the bulk solid, both vertical stress as well as horizontal stress are constant within the entire bulk solid specimen. Therefore at each position within the bulk solid sample the state of stress, which can be represented by a Mohr stress circle, is identical.

During consolidation the normal stress, $\sigma_1$, acts on the top of the bulk solid specimen. Perpendicular to the vertical stress the smaller horizontal stress, $\sigma_2$, prevails according to the stress ratio, $K$ (see Sect. 2.3). At neither the top nor bottom of the specimen, nor at the internal wall of the hollow cylinder, which is assumed to be frictionless, will shear stresses be found; i.e., $\tau = 0$. The pairs of values ($\sigma, \tau$) for vertical and horizontal cutting planes within the bulk solid specimen are plotted in the $\sigma, \tau$ diagram (Fig. 3.4). Both points are located on the $\sigma$-axis because $\tau = 0$. The Mohr stress circle, which describes the stresses in the bulk solid sample at consolidation, is thus well defined (because each stress circle has exactly two intersections with the $\sigma$-axis defining the principal stresses). The stress circle representing the stresses at consolidation is shown in Fig. 3.4 (stress circle A).

In the second part of the test shown in Fig. 3.1, the specimen is loaded with increasing vertical stress after it has been relieved of the consolidation stress and the hollow cylinder has been removed. Here vertical stress and horizontal stress are principal stresses.
During the increasing vertical load in the second part of the test, the stress states at different load steps are represented by stress circles with increasing diameter (stress circles $B_1$, $B_2$, $B_3$ in Fig. 3.4). The minor principal stress, which is equal to the horizontal stress, is equal to zero at all stress circles, since the lateral surface of the specimen is unrestrained and not loaded.

At failure of the specimen the Mohr stress circle $B_3$ represents the stresses in the bulk solid sample. Since the load corresponding to this Mohr stress circle causes incipient flow of the specimen, the yield limit of the bulk solid must have been attained in one cutting plane of the specimen. Thus, Mohr stress circle $B_3$ must reach the yield limit in the $\tau,\sigma$ diagram. In Fig. 3.4 a possible yield limit is shown. A simplified interpretation of the yield limit is that it gives for every normal stress, $\sigma$, a shear stress, $\tau$, which is necessary to initiate flow, i.e., to move particles relative to each other.

The Mohr stress circles $B_1$ and $B_2$, which are completely below the yield limit, cause only an elastic deformation of the bulk solid specimen, but no failure and/or flow. Stress circles larger than stress circle $B_3$, and thus partly above the yield limit, are not possible: The specimen would already be flowing when the Mohr stress circle reaches the yield limit (failure), so that no larger load could be exerted on the specimen.
In Fig. 3.5 the connection between the Mohr stress circle, the yield limit and the orientation of the failure plane is shown. The point where the Mohr stress circle touches the yield limit defines the pair of values of shear stress, \( \tau \), and normal stress, \( \sigma \), that initiates flow. These stresses are attained only in cutting planes of two particular orientations. One orientation can be identified with the Mohr stress circle in Fig. 3.5. The angle from the point on the Mohr stress circle representing the horizontal plane, where the vertical stress, \( \sigma_v \), is acting, to the point where it touches the yield limit is \( 2\alpha \). Thus, a failure plane in the bulk solid specimen is inclined by the angle \( \alpha \) to the horizontal, whereby \( \alpha \) is measured in the opposite direction as in the Mohr circle (see Sect. 2.2). The second orientation of the failure planes is defined by the point where the Mohr stress circle touches the yield limit mirrored along the \( \sigma \)-axis (not visible in Fig. 3.5).

![Fig. 3.5. Relationship between Mohr stress circle, yield limit and failure plane in the bulk solid specimen](image)

If, during the second part of the experiment shown in Fig. 3.1 (measurement of compressive strength), one were to apply also a horizontal stress greater than zero (\( \sigma_2 > 0 \)) on the specimen (in addition to the vertical stress, \( \sigma_v \)), one would likewise find stress circles that indicate failure of the specimen and reach the yield limit (e.g. stress circle C in Fig. 3.4). Thus the yield limit is the envelope of all stress circles that indicate failure of a bulk solid sample.

### 3.1.4 Numerical characterization of flowability

Flowability of a bulk solid is characterized mainly by its unconfined yield strength, \( \sigma_c \), as a function of the consolidation stress, \( \sigma_1 \), and the storage period, \( t \). Usually the ratio \( \frac{\sigma_c}{\sigma_1} \) of consolidation stress, \( \sigma_1 \), to unconfined yield strength, \( \sigma_c \), is used to characterize flowability numerically:
The larger $ff_c$ is, i.e., the smaller the ratio of the unconfined yield strength to the consolidation stress, the better a bulk solid flows. Similar to the classification used by Jenike [3.1], one can define flow behavior as follows:

- $ff_c < 1$ not flowing
- $1 < ff_c < 2$ very cohesive
- $2 < ff_c < 4$ cohesive
- $4 < ff_c < 10$ easy-flowing
- $10 < ff_c$ free-flowing

In Fig. 3.6 the instantaneous flow function $A$ taken from the $\sigma_c, \sigma_1$ diagram in Fig. 3.2 is shown. Additionally, the boundaries of the ranges of the classifications listed above are shown as straight lines, each representing a constant value of flowability, $ff_c$. This diagram clearly shows that the ratio $ff_c$ of a specific bulk solid is dependent on the consolidation stress, $\sigma_1$ (in most cases $ff_c$ increases with $\sigma_1$ as with bulk solid A). Therefore, with each consolidation stress at which $\sigma_c$ and thus $ff_c$ were determined one obtains a different value of flowability: The flowability of a bulk solid depends on the stress level (= consolidation stress); thus for most bulk solids one will obtain a larger value of flowability (= better flowability) at a greater consolidation stress. For most bulk solids one will find a (possibly extremely low) consolidation stress at which the bulk solid flows poorly. Because of the dependence of flowability on consolidation stress, it is not possible (unfortunately) to describe the flowability of a bulk solid with only one numerical value.
The increase in flowability with increased consolidation stress at first glance may seem inconsistent, since unconfined yield strength, $\sigma_c$, increases with increasing consolidation stress, $\sigma_1$; i.e., the bulk solid increasingly gains strength. How then can it flow better? The answer is to be found in the definition of flowability in Eq. (3.1): Flowability is the ratio of consolidation stress to unconfined yield strength, and this ratio becomes greater with increasing consolidation stress for most bulk solids (e.g. see curve A in Fig. 3.2).

The following will clarify that the definition of flowability is meaningful. In Fig. 3.7 the lower part of a hopper filled with bulk solid is shown. In a hopper the major principal stress (= consolidation stress, $\sigma_1$) decreases towards the hopper apex (see Sect. 9.1.2). The diagrams in Fig. 3.7 show the consolidation stress, $\sigma_1$, the unconfined yield strength, $\sigma_c$, resulting from the consolidation stress, and the flowability, $ff_c$, determined according to Eq.(3.1).

Poor flowability of the bulk solid in the hopper means that flow obstructions due to arching occur. From experience it is known that arching can be avoided if the outlet opening of the hopper is sufficiently large. This finding is supported by the hopper in Fig. 3.7: In the lower part of the hopper the flowability, $ff_c$, is close to 1, thus the bulk solid can be characterized as nearly not flowing. With increasing distance from the hopper apex the consolidation stress increases and the flowability becomes better, so that in the upper part of the hopper stable arches cannot form. Thus, if the outlet opening is sufficiently large, everywhere in the hopper the flowability of the bulk solid is high enough to avoid flow obstructions.

If a bulk solid is set to flow, its yield limit must be attained. Often only stresses of the same order of magnitude as at the previous consolidation can be applied (e.g. the force of gravity). In the above example (bulk solid in a hopper, Fig. 3.7) the bulk solid should flow after the outlet is opened. After opening of the outlet approximately the same stresses act on the bulk solid as previously (with the outlet closed). The bulk solid can flow out only if the stress acting on the bulk solid is greater than the strength of the bulk solid. A measure of this strength is the unconfined yield strength, $\sigma_c$. One can also say that the greater the ratio of the stress acting on the bulk solid to the unconfined yield strength, the more easily will the bulk solid flow. Precisely this relationship is indicated through the value of flowability, $ff_c$, according to Eq.(3.1). This value of flowability, being the ratio of the consolidation stress, $\sigma_1$, to the unconfined yield strength, $\sigma_c$, is therefore in many cases the criterion that determines whether a bulk solid flows in a certain situation or not.
With the results of time consolidation tests, flowability can be determined with Eq. (3.1), using the unconfined yield strength, $\sigma_c$, which was measured after the corresponding storage period. If the bulk solid shows a time consolidation effect, one will measure a larger unconfined yield strength with increasing storage period, so that from Eq. (3.1) lesser flowability will follow. This is logical: If a bulk solid gains strength with an increasing period of storage at rest at a certain consolidation stress, it will be more difficult to get this bulk solid to flow; i.e., its flowability decreases with increasing storage period.

In Fig. 3.3, Sect. 3.1.2, an instantaneous flow function and two time flow functions are shown. The instantaneous flow function represents unconfined yield strength, $\sigma_c$, as a function of consolidation stress, $\sigma_1$, without influence of a storage period, i.e., for the storage period $t = 0$. A time flow function represents the unconfined yield strength which emerges after storage at the consolidation stress over a period of time, $t$ (see Sect. 3.1.2). The instantaneous flow function and time flow functions from Fig. 3.3 are shown in Fig. 3.8.a along with the boundaries of the ranges, which follow from the classification of flowability as outlined above. It can be seen that flowabilities, $ff_c$, measured at identical consolidation stress, but after different consolidation periods, decrease with increasing consolidation time. For the consolidation stress $\sigma_{Example}$ (Fig. 3.8.a) chosen as an example, one obtains measurement points in areas of decreasing flowability when increasing consolidation period $t$ (see arrow in Fig. 3.8.a). The decrease of flowability with consolidation period is shown in Fig. 3.8.b.
From the dependence of flowability, $f_{fc}$, on consolidation stress, $\sigma_{1}$, it follows that one can compare the flow behavior of several bulk solids quantitatively using $f_{fc}$ only if all measurements have been performed at identical consolidation stresses. Otherwise, totally different (incorrect) statements might result. See the example in Fig. 3.9, where the instantaneous flow functions of two bulk solids, A and B, are shown. The flowability, $f_{fc}$, of the better flowing bulk solid A was measured at a very low consolidation stress, so that a relatively low flowability value emerged. The flowability of the more poorly flowing bulk solid B was measured at a clearly greater consolidation stress, where this bulk solid has a better flowability than the better flowing bulk solid A at the lower consolidation stress. Therefore, a comparison of $f_{fc}$ values would result in an incorrect ranking of flowabilities. If one had measured flowability at identical consolidation stresses, one would have determined flowabilities that represent the true relationships.

[Figure 3.8. Influence of storage period on flowability; a. instantaneous flow function A and time flow functions A1, A2; b. flowability vs. time]

[Figure 3.9. Influence of consolidation stress on flowability]
When providing the flowability value, \( f_{fc} \), of a bulk solid, the consolidation stress at which flowability was measured must also be given. Otherwise the flowability value is meaningless because - as outlined above - depending on the consolidation stress selected one can measure very different values of flowability for any product.

In addition, the time consolidation of different bulk solids can be compared only at identical consolidation stress and identical consolidation time. Therefore, besides the \( f_{fc} \) value, one must provide the consolidation stress and the consolidation period values used.

The consolidation stress and the consolidation period selected for testing should reflect, as much as possible, the actual process conditions in which the problem occurs (e.g. stress and storage period for storage in bags on pallets, stress near the outlet opening of a silo, see Sect. 4.2). This is the most reliable way to achieve realistic data. Since the flowability ranking of several bulk solids is in many cases largely independent of stress level (example: Fig. 3.9: Product A flows better than product B throughout the consolidation stress range), it is often sufficient that the consolidation stresses measured roughly approximate the actual stresses. Very much more important than the absolute magnitude of the consolidation stress is that during comparative tests all products be tested at the same consolidation stress. This has already been discussed in detail above.

In exceptional cases a different ranking of the flowability of two products may result based on stress level (\( \sigma_1 \)). This is shown by the flow functions of products A and B in Fig. 3.2. Due to the intersection of the flow functions there is a range of \( \sigma_1 \) where the flow function of A is located above that of product B, so that product B flows better in this range of stresses. But there is also a range of \( \sigma_1 \), where product A flows better. In such a case it is very important to use a consolidation stress close to the stress acting in the practical application under consideration.

With many applications the bulk solid flows by gravity, e.g. when flowing out from a bin or silo. Two bulk solids with the same flowability value, \( f_{fc} \), but a different bulk density, \( \rho_b \), will flow differently because a larger gravitational force acts on the bulk solid with the larger bulk density. This means, for all applications utilizing gravity flow of bulk solids (e.g. flow out of a silo), that flow of a bulk solid with greater bulk density is supported by a greater force. Because of the greater force of gravity at equal flowability the strength of the bulk solid with the greater bulk density can be overcome more easily.

With comparative tests on similar samples of a product, often individual samples differ only slightly in their bulk density, so the influence of bulk density can be neglected. Then the flowability value, \( f_{fc} \), is sufficiently accurate. Sometimes one finds very strong differences between the bulk den-
3.2 Principles of shear testing

The use of the uniaxial compression test with fine-grained, cohesive bulk solids is problematic, since one obtains unconfined yield strength values that are too low (Chap. 5) [3.2-3.6], and since preparation of the hollow cylinder to obtain frictionless walls is very time-consuming if not impossible to achieve (Sect. 6.3.10). In addition, further important parameters (e.g. internal friction and wall friction) cannot be determined with this test. Therefore shear testers are used in advanced bulk solids technology. The first shear tester designed for bulk solids is the translational shear tester developed by Jenike around 1960 (Jenike shear tester) [3.1]. Some years later the first ring shear testers designed for bulk solids followed [3.7]. In the following the test procedure (called “shear test”) is described in general, i.e., independent of a specific type of shear tester. The implementation of the procedure with different types of shear testers follows in Chap. 4.

3.2.1 Test procedure

The test procedure described in the following corresponds to the test procedure recommended by Jenike for the translational shear tester [3.1,3.8–3.12]. Similar to a uniaxial compression test, the test procedure is done in two steps: First the bulk solid specimen is consolidated, what is called “preshear”. Subsequently a point on the yield limit is measured. This step is called “shear” or “shear to failure”.

\[ ff' \rho = ff_c \cdot \frac{\rho_b}{\rho_w} \]  

(3.2)

with \( \rho_w = 1000 \text{ kg/m}^3 \) (liquid water at 0°C, 1 bar).

In order to obtain a non-dimensional term, the bulk density, \( \rho_b \), is divided by the density of liquid water \( (\rho_w = 1000 \text{ kg/m}^3 \text{ at } 0°C \text{ and } 1 \text{ bar}, \text{ rounded}) \). Of course, this is not physically meaningful, since the density of water plays no role in the gravity flow of a bulk solid, but in this way one obtains a non-dimensional term which is called the “density-weighed flowability”.

sities of samples to be compared, particularly with very fine-grained products. Here bulk density must be considered in order to reach conclusions about flowability under gravity flow. It might be useful then to consider the product of flowability, \( ff_c \), and bulk density, \( \rho_b \):

with \( \rho_w = 1000 \text{ kg/m}^3 \) (liquid water at 0°C, 1 bar).
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