# Contents

Introduction .......................................................... 1
References ............................................................. 7

**Part One. The Nonlinear Schrödinger Equation (NS Model) ........ 9**

Chapter I. Zero Curvature Representation .......................... 11
 § 1. Formulation of the NS Model .................................. 11
 § 2. Zero Curvature Condition ...................................... 20
 § 3. Properties of the Monodromy Matrix in the Quasi-Periodic Case ........................................... 26
 § 4. Local Integrals of the Motion .................................. 33
 § 5. The Monodromy Matrix in the Rapidly Decreasing Case 39
 § 6. Analytic Properties of Transition Coefficients .......... 46
 § 7. The Dynamics of Transition Coefficients ................. 51
 § 8. The Case of Finite Density. Jost Solutions ............... 55
 § 9. The Case of Finite Density. Transition Coefficients .... 62
 § 10. The Case of Finite Density. Time Dynamics and Integrals of the Motion ........................................ 72
 § 11. Notes and References ........................................ 78
References .............................................................. 80

Chapter II. The Riemann Problem ................................... 81
 § 1. The Rapidly Decreasing Case. Formulation of the Riemann Problem ............................................... 81
 § 2. The Rapidly Decreasing Case. Analysis of the Riemann Problem ....................................................... 89
 § 3. Application of the Inverse Scattering Problem to the NS Model ....................................................... 108
 § 4. Relationship Between the Riemann Problem Method and the Gelfand-Levitan-Marchenko Integral Equations Formulation .......... 114
 § 5. The Rapidly Decreasing Case. Soliton Solutions .......... 126
 § 7. Solution of the Inverse Problem in the Case of Finite Density. The Gelfand-Levitan-Marchenko Formulation .... 146
 § 8. Soliton Solutions in the Case of Finite Density .......... 165
§ 9. Notes and References ............................................. 177
References ...................................................................... 182

Chapter III. The Hamiltonian Formulation ......................... 186
§ 1. Fundamental Poisson Brackets and the $r$-Matrix .......... 186
§ 2. Poisson Commutativity of the Motion Integrals in the Quasi-
Periodic Case ............................................................... 194
§ 3. Derivation of the Zero Curvature Representation from the Fun-
damental Poisson Brackets ............................................. 199
§ 4. Integrals of the Motion in the Rapidly Decreasing Case and in
the Case of Finite Density ............................................... 205
§ 5. The $A$-Operator and a Hierarchy of Poisson Structures ... 210
§ 6. Poisson Brackets of Transition Coefficients in the Rapidly
Decreasing Case ............................................................. 222
§ 7. Action-Angle Variables in the Rapidly Decreasing Case ... 229
§ 8. Soliton Dynamics from the Hamiltonian Point of View ..... 241
§ 9. Complete Integrability in the Case of Finite Density ........ 249
§ 10. Notes and References .............................................. 267
References ...................................................................... 274

Part Two. General Theory of Integrable Evolution Equations ..... 279
Chapter I. Basic Examples and Their General Properties ........ 281
§ 1. Formulation of the Basic Continuous Models ............... 281
§ 2. Examples of Lattice Models ........................................ 292
§ 3. Zero Curvature Representation as a Method for Constructing
Integrable Equations ....................................................... 305
§ 4. Gauge Equivalence of the NS Model ($\varepsilon = -1$) and the HM
Model ............................................................................. 315
§ 5. Hamiltonian Formulation of the Chiral Field Equations and
Related Models ............................................................... 321
§ 6. The Riemann Problem as a Method for Constructing Solutions
of Integrable Equations .................................................... 333
§ 7. A Scheme for Constructing the General Solution of the Zero
Curvature Equation. Concluding Remarks on Integrable Equa-
tions .............................................................................. 339
§ 8. Notes and References .............................................. 345
References ...................................................................... 350

Chapter II. Fundamental Continuous Models ....................... 356
§ 1. The Auxiliary Linear Problem for the HM Model .......... 356
§ 2. The Inverse Problem for the HM Model ....................... 370
§ 3. Hamiltonian Formulation of the HM Model ................. 384
§ 4. The Auxiliary Linear Problem for the SG Model .......... 393
§ 5. The Inverse Problem for the SG Model ....................... 407
§ 6. Hamiltonian Formulation of the SG Model ................. 431
Hamiltonian Methods in the Theory of Solitons
Faddeev, L.D.; Takhtajan, L.
2007, IX, 592 p., Softcover
ISBN: 978-3-540-69843-2