
Contents

Preface	V
1 Mathematical Statistics and Information Theory	1
1.1 Probability Functions for Discrete Variables	2
1.1.1 Bernoulli Distribution	3
1.1.2 Binomial Distribution	3
1.1.3 Poisson Distribution	4
1.2 Probability Density Functions for Continuous Variables	6
1.2.1 Uniform Distribution	8
1.2.2 Exponential Distribution	8
1.2.3 Gaussian, or Normal Distribution	9
1.3 Joint Probability Density Functions	9
1.3.1 Bivariate Gaussian Distributions	10
1.4 Information Theory	11
1.4.1 Gamma Distribution	13
2 Introduction to Riemannian Geometry	19
2.0.2 Manifolds	20
2.0.3 Tangent Spaces	20
2.0.4 Tensors and Forms	22
2.0.5 Riemannian Metric	25
2.0.6 Connections	26
2.1 Autoparallel and Geodesic Curves	29
2.2 Universal Connections and Curvature	29
3 Information Geometry	31
3.1 Fisher Information Metric	32
3.2 Exponential Family of Probability Density Functions	33
3.3 Statistical α -Connections	34
3.4 Affine Immersions	35
3.4.1 Weibull Distributions: Not of Exponential Type	36

3.5	Gamma 2-Manifold \mathcal{G}	37
3.5.1	Gamma a -Connection	38
3.5.2	Gamma a -Curvatures	39
3.5.3	Gamma Manifold Geodesics	40
3.5.4	Mutually Dual Foliations	42
3.5.5	Gamma Affine Immersion	42
3.6	Log-Gamma 2-Manifold \mathcal{L}	42
3.6.1	Log-Gamma Random Walks	45
3.7	Gaussian 2-Manifold	45
3.7.1	Gaussian Natural Coordinates	47
3.7.2	Gaussian Information Metric	47
3.7.3	Gaussian Mutually Dual Foliations	48
3.7.4	Gaussian Affine Immersions	48
3.8	Gaussian a -Geometry	49
3.8.1	Gaussian a -Connection	49
3.8.2	Gaussian a -Curvatures	50
3.9	Gaussian Mutually Dual Foliations	50
3.10	Gaussian Submanifolds	51
3.10.1	Central Mean Submanifold	51
3.10.2	Unit Variance Submanifold	52
3.10.3	Unit Coefficient of Variation Submanifold	52
3.11	Gaussian Affine Immersions	52
3.12	Log-Gaussian Manifold	53
4	Information Geometry of Bivariate Families	55
4.1	McKay Bivariate Gamma 3-Manifold M	55
4.2	McKay Manifold Geometry in Natural Coordinates	58
4.3	McKay Densities Have Exponential Type	59
4.3.1	McKay Information Metric	59
4.4	McKay a -Geometry	60
4.4.1	McKay a -Connection	60
4.4.2	McKay a -Curvatures	61
4.5	McKay Mutually Dual Foliations	64
4.6	McKay Submanifolds	65
4.6.1	Submanifold M_1	65
4.6.2	Submanifold M_2	68
4.6.3	Submanifold M_3	69
4.7	McKay Bivariate Log-Gamma Manifold \tilde{M}	71
4.8	Generalized McKay 5-Manifold	72
4.8.1	Bivariate 3-Parameter Gamma Densities	72
4.8.2	Generalized McKay Information Metric	73
4.9	Freund Bivariate Exponential 4-Manifold \mathcal{F}	74
4.9.1	Freund Fisher Metric	75
4.10	Freund Natural Coordinates	76

4.11	Freund a -Geometry	77
4.11.1	Freund a -Connection	77
4.11.2	Freund a -Curvatures	78
4.12	Freund Foliations	80
4.13	Freund Submanifolds	81
4.13.1	Independence Submanifold F_1	81
4.13.2	Submanifold F_2	82
4.13.3	Submanifold F_3	83
4.13.4	Submanifold F_4	84
4.14	Freund Affine Immersion	87
4.15	Freund Bivariate Log-Exponential Manifold	87
4.16	Bivariate Gaussian 5-Manifold \mathcal{N}	88
4.17	Bivariate Gaussian Fisher Information Metric	89
4.18	Bivariate Gaussian Natural Coordinates	90
4.19	Bivariate Gaussian a -Geometry	91
4.19.1	a -Connection	91
4.19.2	a -Curvatures	94
4.20	Bivariate Gaussian Foliations	98
4.21	Bivariate Gaussian Submanifolds	99
4.21.1	Independence Submanifold \mathcal{N}_1	99
4.21.2	Identical Marginals Submanifold \mathcal{N}_2	101
4.21.3	Central Mean Submanifold \mathcal{N}_3	103
4.21.4	Affine Immersion	105
4.22	Bivariate Log-Gaussian Manifold	106
5	Neighbourhoods of Poisson Randomness, Independence, and Uniformity	109
5.1	Gamma Manifold \mathcal{G} and Neighbourhoods of Randomness	110
5.2	Log-Gamma Manifold \mathcal{L} and Neighbourhoods of Uniformity	111
5.3	Freund Manifold \mathcal{F} and Neighbourhoods of Independence	112
5.3.1	Freund Submanifold F_2	113
5.4	Neighbourhoods of Independence for Gaussians	114
6	Cosmological Voids and Galactic Clustering	119
6.1	Spatial Stochastic Processes	120
6.2	Galactic Cluster Spatial Processes	121
6.3	Cosmological Voids	125
6.4	Modelling Statistics of Cosmological Void Sizes	126
6.5	Coupling Galaxy Clustering and Void Sizes	130
6.6	Representation of Cosmic Evolution	132
7	Amino Acid Clustering With A.J. Doig	139
7.1	Spacings of Amino Acids	139
7.2	Poisson Spaced Sequences	141

7.3	Non-Poisson Sequences as Gamma Processes	142
7.3.1	Local Geodesic Distance Approximations	145
7.4	Results	148
7.5	Why Would Amino Acids Cluster?	151
8	Cryptographic Attacks and Signal Clustering	153
8.1	Cryptographic Attacks	153
8.2	Information Geometry of the Log-gamma Manifold	154
8.3	Distinguishing Nearby Unimodular Distributions	155
8.4	Difference From a Uniform Distribution	157
8.5	Gamma Distribution Neighbourhoods of Randomness	157
9	Stochastic Fibre Networks	
	With W.W. Sampson	161
9.1	Random Fibre Networks	161
9.2	Random Networks of Rectangular Fibres	164
9.3	Log-Gamma Information Geometry for Fibre Clustering	168
9.4	Bivariate Gamma Distributions for Anisotropy	169
9.5	Independent Polygon Sides	171
9.5.1	Multiplanar Networks	175
9.6	Correlated Polygon Sides	179
9.6.1	McKay Bivariate Gamma Distribution	182
9.6.2	McKay Information Geometry	184
9.6.3	McKay Information Entropy	188
9.6.4	Simulation Results	191
10	Stochastic Porous Media and Hydrology	
	With J. Scharcanski and S. Felipussi	195
10.1	Hydrological Modelling	195
10.2	Univariate Gamma Distributions and Randomness	196
10.3	McKay Bivariate Gamma 3-Manifold	196
10.4	Distance Approximations in the McKay Manifold	198
10.5	Modelling Stochastic Porous Media	200
10.5.1	Adaptive Tomographic Image Segmentation	201
10.5.2	Mathematical Morphology Concepts	203
10.5.3	Adaptive Image Segmentation and Representation	209
10.5.4	Soil Tomographic Data	214
11	Quantum Chaology	223
11.1	Introduction	223
11.2	Eigenvalues of Random Matrices	226
11.3	Deviations	229
	References	235
	Index	247



<http://www.springer.com/978-3-540-69391-8>

Information Geometry

Near Randomness and Near Independence

Arwini, K.; Dodson, C.T.J.

2008, X, 260 p. 103 illus., Softcover

ISBN: 978-3-540-69391-8