# Contents

**Introduction** ................................................ 1

**Chapter I. Hilbert Schemes and Chow Varieties** .......... 5
I.1 Construction of the Hilbert Scheme ....................... 8
I.2 Infinitesimal Study of the Hilbert Scheme ............... 20
I.3 Chow Varieties .......................................... 40
I.4 Chow Varieties in Positive Characteristic .......... 62
I.5 Related Moduli Problems ............................... 73
I.6 Comparing Chow Varieties and Hilbert Schemes ......... 78
I.7 Appendix: Some Results from Commutative Algebra .......... 83

**Chapter II. Curves on Varieties** ............................ 90
II.1 Deformations of Morphisms of Curves ................ 92
II.2 Families of Rational Curves ............................ 103
II.3 Free Morphisms of Curves ............................. 113
II.4 Cone of Curves ....................................... 121
II.5 Bend and Break ....................................... 134
II.6 Application: A Vanishing Theorem for $H^1$ ........ 148
II.7 Smoothing of Morphisms of Curves .................... 154

**Chapter III. The Cone Theorem and Minimal Models** ...... 159
III.1 The Cone Theorem ..................................... 160
III.2 Minimal Models of Surfaces ........................... 167
III.3 Del Pezzo Surfaces ................................... 171

**Chapter IV. Rationally Connected Varieties** ................ 179
IV.1 Ruled and Uniruled Varieties .......................... 181
IV.2 Minimizing Families of Rational Curves .......... 192
IV.3 Rationally Connected Varieties ....................... 198
IV.4 Growing Chains of Rational Curves ................... 209
IV.5 Maximal Rationally Connected Fibrations ............ 222
IV.6 Rationally Connected Varieties over Nonclosed Fields .... 228
Chapter V. Fano Varieties ........................................ 238
  V.1 Low Degree Rational Curves on Fano Varieties ............ 239
  V.2 Boundedness of Fano Varieties ............................ 250
  V.3 Characterizations of \( \mathbb{P}^n \) .............................. 257
  V.4 Lines on Fano Hypersurfaces .............................. 265
  V.5 Nonrational Fano Varieties ................................. 272

Chapter VI. Appendix ............................................. 285
  VI.1 Rational Curves on Exceptional Loci ...................... 286
  VI.2 Intersection Numbers of Divisors and
      Asymptotic Riemann-Roch .................................... 292

References .......................................................... 308
Index ................................................................. 318
Rational Curves on Algebraic Varieties
Kollár, J.
1996, VIII, 321 p., Hardcover
ISBN: 978-3-540-60168-5