

# Contents

Suggestions for the Reader . . . . .	xvii
Basic Notation . . . . .	xxi
Brief Survey of Stochastic Numerical Methods . . . . .	xxiii

## Part I. Preliminaries

<b>Chapter 1. Probability and Statistics . . . . .</b>	<b>1</b>
1.1 Probabilities and Events . . . . .	1
1.2 Random Variables and Distributions . . . . .	5
1.3 Random Number Generators . . . . .	11
1.4 Moments . . . . .	14
1.5 Convergence of Random Sequences . . . . .	22
1.6 Basic Ideas About Stochastic Processes . . . . .	26
1.7 Diffusion Processes . . . . .	34
1.8 Wiener Processes and White Noise . . . . .	40
1.9 Statistical Tests and Estimation . . . . .	44
<b>Chapter 2. Probability and Stochastic Processes . . . . .</b>	<b>51</b>
2.1 Aspects of Measure and Probability Theory . . . . .	51
2.2 Integration and Expectations . . . . .	55
2.3 Stochastic Processes . . . . .	63
2.4 Diffusion and Wiener Processes . . . . .	68

## Part II. Stochastic Differential Equations

<b>Chapter 3. Ito Stochastic Calculus . . . . .</b>	<b>75</b>
3.1 Introduction . . . . .	75
3.2 The Ito Stochastic Integral . . . . .	81
3.3 The Ito Formula . . . . .	90
3.4 Vector Valued Ito Integrals . . . . .	96
3.5 Other Stochastic Integrals . . . . .	99
<b>Chapter 4. Stochastic Differential Equations . . . . .</b>	<b>103</b>
4.1 Introduction . . . . .	103
4.2 Linear Stochastic Differential Equations . . . . .	110

4.3	Reducible Stochastic Differential Equations . . . . .	113
4.4	Some Explicitly Solvable Equations . . . . .	117
4.5	The Existence and Uniqueness of Strong Solutions . . . . .	127
4.6	Strong Solutions as Diffusion Processes . . . . .	141
4.7	Diffusion Processes as Weak Solutions . . . . .	144
4.8	Vector Stochastic Differential Equations . . . . .	148
4.9	Stratonovich Stochastic Differential Equations . . . . .	154
<b>Chapter 5. Stochastic Taylor Expansions . . . . .</b>		<b>161</b>
5.1	Introduction . . . . .	161
5.2	Multiple Stochastic Integrals . . . . .	167
5.3	Coefficient Functions . . . . .	177
5.4	Hierarchical and Remainder Sets . . . . .	180
5.5	Ito-Taylor Expansions . . . . .	181
5.6	Stratonovich-Taylor Expansions . . . . .	187
5.7	Moments of Multiple Ito Integrals . . . . .	190
5.8	Strong Approximation of Multiple Stochastic Integrals . . . . .	198
5.9	Strong Convergence of Truncated Ito-Taylor Expansions . . . . .	206
5.10	Strong Convergence of Truncated Stratonovich-Taylor Expansions . . . . .	210
5.11	Weak Convergence of Truncated Ito-Taylor Expansions . . . . .	211
5.12	Weak Approximations of Multiple Ito Integrals . . . . .	221
 <b>Part III. Applications of Stochastic Differential Equations</b>		
<b>Chapter 6. Modelling with Stochastic Differential Equations . . . . .</b>		<b>227</b>
6.1	Ito Versus Stratonovich . . . . .	227
6.2	Diffusion Limits of Markov Chains . . . . .	229
6.3	Stochastic Stability . . . . .	232
6.4	Parametric Estimation . . . . .	241
6.5	Optimal Stochastic Control . . . . .	244
6.6	Filtering . . . . .	248
 <b>Chapter 7. Applications of Stochastic Differential Equations</b>		<b>253</b>
7.1	Population Dynamics, Protein Kinetics and Genetics . . . . .	253
7.2	Experimental Psychology and Neuronal Activity . . . . .	256
7.3	Investment Finance and Option Pricing . . . . .	257
7.4	Turbulent Diffusion and Radio-Astronomy . . . . .	259
7.5	Helicopter Rotor and Satellite Orbit Stability . . . . .	261
7.6	Biological Waste Treatment, Hydrology and Air Quality . . . . .	263
7.7	Seismology and Structural Mechanics . . . . .	266
7.8	Fatigue Cracking, Optical Bistability and Nematic Liquid Crystals . . . . .	269
7.9	Blood Clotting Dynamics and Cellular Energetics . . . . .	271

7.10 Josephson Tunneling Junctions Communications and Stochastic Annealing . . . . .	273
---	-----

## Part IV. Time Discrete Approximations

<b>Chapter 8. Time Discrete Approximation of Deterministic Differential Equations . . . . .</b>	<b>277</b>
---	------------

8.1 Introduction . . . . .	277
8.2 Taylor Approximations and Higher Order Methods . . . . .	286
8.3 Consistency, Convergence and Stability . . . . .	292
8.4 Roundoff Error . . . . .	301

<b>Chapter 9. Introduction to Stochastic Time Discrete Approximation . . . . .</b>	<b>305</b>
--	------------

9.1 The Euler Approximation . . . . .	305
9.2 Example of a Time Discrete Simulation . . . . .	307
9.3 Pathwise Approximations . . . . .	311
9.4 Approximation of Moments . . . . .	316
9.5 General Time Discretizations and Approximations . . . . .	321
9.6 Strong Convergence and Consistency . . . . .	323
9.7 Weak Convergence and Consistency . . . . .	326
9.8 Numerical Stability . . . . .	331

## Part V. Strong Approximations

<b>Chapter 10. Strong Taylor Approximations . . . . .</b>	<b>339</b>
---	------------

10.1 Introduction . . . . .	339
10.2 The Euler Scheme . . . . .	340
10.3 The Milstein Scheme . . . . .	345
10.4 The Order 1.5 Strong Taylor Scheme . . . . .	351
10.5 The Order 2.0 Strong Taylor Scheme . . . . .	356
10.6 General Strong Ito-Taylor Approximations . . . . .	360
10.7 General Strong Stratonovich-Taylor Approximations . . . . .	365
10.8 A Lemma on Multiple Ito Integrals . . . . .	369

<b>Chapter 11. Explicit Strong Approximations . . . . .</b>	<b>373</b>
---	------------

11.1 Explicit Order 1.0 Strong Schemes . . . . .	373
11.2 Explicit Order 1.5 Strong Schemes . . . . .	378
11.3 Explicit Order 2.0 Strong Schemes . . . . .	383
11.4 Multistep Schemes . . . . .	385
11.5 General Strong Schemes . . . . .	390

<b>Chapter 12. Implicit Strong Approximations</b> . . . . .	395
12.1 Introduction . . . . .	395
12.2 Implicit Strong Taylor Approximations . . . . .	396
12.3 Implicit Strong Runge-Kutta Approximations . . . . .	406
12.4 Implicit Two-Step Strong Approximations . . . . .	411
12.5 A-Stability of Strong One-Step Schemes . . . . .	417
12.6 Convergence Proofs . . . . .	420
<b>Chapter 13. Selected Applications of Strong Approximations</b> . . . . .	427
13.1 Direct Simulation of Trajectories . . . . .	427
13.2 Testing Parametric Estimators . . . . .	435
13.3 Discrete Approximations for Markov Chain Filters . . . . .	442
13.4 Asymptotically Efficient Schemes . . . . .	453
 <b>Part VI. Weak Approximations</b>	
<b>Chapter 14. Weak Taylor Approximations</b> . . . . .	457
14.1 The Euler Scheme . . . . .	457
14.2 The Order 2.0 Weak Taylor Scheme . . . . .	464
14.3 The Order 3.0 Weak Taylor Scheme . . . . .	468
14.4 The Order 4.0 Weak Taylor Scheme . . . . .	470
14.5 General Weak Taylor Approximations . . . . .	472
14.6 Leading Error Coefficients . . . . .	480
<b>Chapter 15. Explicit and Implicit Weak Approximations</b> . . . . .	485
15.1 Explicit Order 2.0 Weak Schemes . . . . .	485
15.2 Explicit Order 3.0 Weak Schemes . . . . .	488
15.3 Extrapolation Methods . . . . .	491
15.4 Implicit Weak Approximations . . . . .	495
15.5 Predictor-Corrector Methods . . . . .	501
15.6 Convergence of Weak Schemes . . . . .	506
<b>Chapter 16. Variance Reduction Methods</b> . . . . .	511
16.1 Introduction . . . . .	511
16.2 The Measure Transformation Method . . . . .	513
16.3 Variance Reduced Estimators . . . . .	516
16.4 Unbiased Estimators . . . . .	522
<b>Chapter 17. Selected Applications of Weak Approximations</b> . . . . .	529
17.1 Evaluation of Functional Integrals . . . . .	529
17.2 Approximation of Invariant Measures . . . . .	540
17.3 Approximation of Lyapunov Exponents . . . . .	545

*CONTENTS*

XV

Solutions of Exercises . . . . .	549
Bibliographical Notes . . . . .	587
Bibliography . . . . .	599
Index . . . . .	629



<http://www.springer.com/978-3-540-54062-5>

Numerical Solution of Stochastic Differential Equations

Kloeden, P.E.; Platen, E.

1992, XXXVI, 636 p., Hardcover

ISBN: 978-3-540-54062-5